PRINCIPLES OF LOGIC

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PRINCIPLES OF LOGIC

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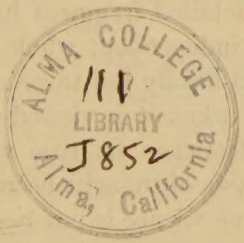

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PREFACE TO SECOND EDITION

In issuing this new edition of my *Principles of Logic* I welcome the opportunity it affords me of expressing my thanks to my various critics for the very kind manner in which the work was received. The friendly notices accorded to it, not only by such as were in sympathy with its general standpoint, but by those whose philosophical position was altogether different, tend to show that at the present time a new interest is felt in that Scholastic system, whose principles it was my purpose to defend.

Since the publication of the first edition of this work the Traditional Logic has been the object of hostile criticism. More than one writer of mark has maintained that, as an analysis of our mental operations, it is entirely worthless : that it is destitute of any claim to be regarded as a science. Such an estimate, we are convinced, is only possible because Logic has for so long been studied out of all relation to that philosophy of which it forms an integral part. A detailed fragment has been treated as though it were an independent whole. Under such circumstances it could hardly fail to be misinterpreted and undervalued. But when it is viewed in the light of the principles of Scholasticism its true value is seen. Its validity as an analysis of thought becomes apparent, and its claim to be a true science is put beyond all dispute.

For the purpose of the present edition the work has been carefully revised, certain corrections have been made, and a few sections have been re-written.

INTRODUCTION

THIS work is an attempt at a presentment of what is frequently termed the Traditional Logic, and is intended for those who are making acquaintance with philosophical questions for the first time. Yet it is impossible, even in a text-book such as this, to deal with logical questions save in connexion with definite metaphysical and epistemological principles. Logic, as the theory of the mind's rational processes in regard of their validity, must necessarily be part of a larger philosophical system. Indeed when this is not the case, it becomes a mere collection of technical rules, possessed of little importance and of less interest. The point of view adopted in this book is that of the Scholastic philosophy; and as far as is compatible with the size and purpose of the work, some attempt has been made to vindicate the fundamental principles on which that philosophy is based.

From one point of view, this position should prove a source of strength. The thinkers who elaborated our system of Logic, were Scholastics. With the principles of that philosophy, its doctrines and its rules are in full accord. In the light of Scholasticism, the system is a connected whole; and the subjects, traditionally treated in it, have each of them its legitimate place.

When a writer adopts some other standpoint, it is inevitable that his Logic must be remodelled, if it is to be in harmony with his philosophical principles. For some parts of the traditional science, there will be no place in his scheme. And though these subjects may find treatment in his work, yet it will be manifest that

they are present as unwelcome guests, only tolerated out of deference to custom and the exigencies of a popular demand. In such a case, a young student may well be excused, if he fails to grasp the bearing of the question at issue.

From another point of view, it might seem that Scholastic principles must be a source of weakness. Have not, it will be asked, the universities, one and all, long since discarded Scholasticism ?

That this is true of all those universities which have submitted to secular influences, must be frankly admitted. At our ancient seats of learning, there has been a complete neglect of the great mediæval philosophers, the representatives of that once famous school. The names of Albert the Great, of St. Thomas Aquinas, of Duns Scotus are never mentioned. It is not that they are weighed and found wanting. They are ignored. It is assumed that there is nothing in them worth knowing. The practice of what certain German writers have termed ' the leap over the middle ages (*der Sprung über das Mittelalter*),' has been universal. From Plotinus to Bacon has been regarded as a blank in the history of philosophy.[1]

Yet by common consent the period thus ignored was one of intense philosophic activity. Metaphysical problems were discussed with an interest, a zeal, an acumen since unknown ; and some of the greatest intellects the world has ever seen were nurtured in the schools of the day. Nor was the philosophy of the Scholastics one of those immature systems, which arise when the mind of man is called to grapple for the first time with the great problems of the universe. These men had inherited the two streams of Greek and Arabian thought. They had

[1] It is with pleasure we notice that at Oxford the *Summa Theologica* of St. Thomas is now recommended to candidates for theological honours, and his *Summa contra Gentiles* has been made an optional subject in the school of *Litteræ humaniores*. But this welcome change only testifies to the complete neglect which has so long prevailed.

set themselves to master and to develop the conclusions of Plato, of Aristotle, of Avicenna, of Averroes. They were influenced not by the Peripatetic school alone, but further by Stoicism, Neo-platonism, Augustinianism.[1] It is significant that nearly every thinker, even of those occupying a hostile position, who has devoted enough time and attention to understand the matter, has expressed his admiration for the great synthesis effected by the Scholastic philosophers.[2]

When, therefore, the Neo-Scholastics of to-day avail themselves of the results attained in that epoch, no wise man will consider that this is likely to impair the value of their conclusions. They are but claiming their share in the great inheritance of the past.

The deliberate ignoring of so famous a period, and one so fruitful for the civilization of Europe, may well provide matter for reflection. For continuity is the law of human progress. Advance must ever be won by building on the foundations laid by our predecessors. The nature of man, as essentially social, involves his subjection to this law.

[1] Cf. de Wulf, *Scholasticism Old and New*, trans. by P. Coffey, p. 45. Picavet, *Esquisse d'une histoire des philosophies médiévales.*

[2] The opinions of two authors neither of whom can be accused of sympathy with Scholasticism may be of interest. Professor Huxley writes as follows: "The Scholastic philosophy is a wonderful monument of the patience and ingenuity with which the human mind toiled to build up a logically consistent theory of the Universe. . . . And that philosophy is by no means dead and buried as many vainly suppose. On the contrary, numbers of men of no mean learning and accomplishment and sometimes of rare power and subtlety of thought, hold by it as the best theory of things which has yet been stated. And what is still more remarkable, men who speak the language of modern philosophy, nevertheless think the thoughts of the Schoolmen." *Science and Culture*, Lect. 2. *Universities*, p. 41. Somewhat similarly von Hartmann speaks of Scholasticism as "a wonderful and close-knit system of thought, of which none can think lightly save those who have not yet overcome the bias of party-feeling nor learnt to view things from an objective standpoint." *Die Selbstzersetzung des Christenthums*, p. 75.

It would not be difficult to multiply such testimonies from the great minds of every century. Thus Hugo Grotius writes, 'Ubi in re morali consentiunt [Scholastici] vix est ut errent.' *De Jure Belli et Pacis*, Proleg: § 52. Cf. also Leibniz, *Epist. ad Thomasium*, 49, and *Trois Lettres à M. de Montmort*, Lettre III. On the other hand the atheist Diderot says of Scholasticism, 'Cette philosophie a été une des plus grandes plaies de l'esprit humain." *Œuvres, tom.* xix. p. 372.

Pascal has well said, " C'est grace à la tradition que toute ' la suite des hommes pendant le cours de tant de siècles ' doit être considérée comme un même homme, qui sub- ' siste toujours et qui apprend continuellement." [1] The attempt to break with the past, to dispense with what former generations have accomplished, to pull down what they have laboriously built and to make a fresh beginning, has ever ended in failure. No forward step has ever been taken in that way ; for so to act is to violate a fundamental law of our nature. Movements thus initiated have been retrograde, not progressive.

Yet this is what the men of the Renaissance strove to do in regard to the Christian civilization of the middle ages. They put aside as valueless the hardly-won results of five centuries of strenuous effort. Of that great revolt against the past, the repudiation of the traditional philosophy was an integral part.

It was Descartes who first framed a new synthesis, which in some measure filled the place once held by the Scholastic system. He, more than any other man, has a right to be regarded as the father of modern philosophy. And it is not without its lesson to note that he assigns as his reason for holding the philosophy of the School to be worthless, that it was the work not of a single mind but of many minds.[2]

Since the days of Descartes, many another philosophical system, idealist, sensationalist and materialist, has been offered as a solution of the world's riddles. These systems differ widely among themselves. But one common feature differentiates them all alike from Scholasticism. They simplify the problem by the omission of some essential element. This characteristic seems inseparable from any system which severs itself from that which has been aptly termed the main stream of European thought.

[1] Preface to the treatise *Du Vide*.
[2] Descartes, *Meditation* I.

The factors, with which philosophy must deal, are three—God, the world, and the human soul. The Scholastic philosophy faced the problem in its completeness: it shirked no element of it. It is creationist, thus distinguishing between God as Absolute and Unconditioned Being on the one hand, and the soul and the world as Contingent Being on the other. As regards the soul, it is spiritualist, not materialist; and in relation to the problem of knowledge, it is objectivist, teaching that the intellect is capable of valid cognition in regard of that external order with which it is brought into contact by the senses.[1]

In the novel philosophies proposed as substitutes for Scholasticism, sometimes one of the three factors, sometimes another, is omitted; and thus the solution remains unsatisfactory and inadequate. Some, as e.g. Materialism, dispense both with God and the soul. In others, as in that Neo-Hegelianism which finds the only conscious life of the Divine in the human consciousness, God is set aside, and the soul alone is kept. In others again, as in the philosophy of Berkeley, the world is eliminated; God and the soul alone remain.

The rapid rise and fall of systems is but the natural result of this. Men will not long rest satisfied with any scheme which does not account for all the facts. The pendulum of thought swings, with more, or it may be with less velocity, but as surely as it is biassed by a single prejudice withholding it from any truth, it will continually change the curve it traces, and move in succession to all points of the compass.

Of the multiplicity of modern philosophies, which offer themselves to our acceptance, not one can claim to be

[1] Cf. de Wulf, *Histoire de la Philosophie Médiévale*, p. 222. "Toute théorie négatrice de la spiritualité de l'âme ou de la personalité humaine, ou de la distinction essentielle entre Dieu et la créature, est à nos yeux, subversive des principes fondamentaux de la scolastique. Voilà pourquoi nous n'hésitons pas à ranger parmi les adversaires de la scolastique quiconque enseigne le matérialisme, la migration des âmes, l'athéisme ou le panthéisme."

more than the shibboleth of a school. We may recognize and honour to the full, the great ability of the thinkers who propounded them. But the task, which they set themselves, was too great for their powers. As it is not given to any man to reconstruct the whole of physical science from its first foundations, so it is not given to any to reconstruct philosophy.

Evil of Mod. Philosophy

It is not one of the least evils that have arisen from this state of things, that many now look on philosophy as a body of doctrines purely relative to a particular age. Philosophical systems, they hold, must come and go like the fashions of our dress. We should not regard them as more than a convenient mode of representing facts. As men at one period interpreted the universe on a basis of Aristotelianism, so at the present age they do well to adopt the thought-forms of other systems, and interpret it in accordance with the doctrines of Kant or of Hegel.

Against the corrosive scepticism of such a view, Neo-Scholasticism utters its protest. Philosophy is a science—the highest of the sciences. Just as in the natural sciences, the long line of investigators gradually pushes forward the frontiers of human knowledge, and age by age increases the number of those truths which are the permanent conquests of the human mind, so it is in philosophy. Wherever a real advance has been made, that advance is true for all time. The point has been well put by Professor de Wulf. "The endeavour," he writes, "of 'Neo-Scholasticism to re-establish and plant down deeply 'among the controversies of the twentieth century, the 'principles which animated the Scholasticism of the thir-'teenth, is in itself an admission that philosophy cannot '*completely* change from epoch to epoch: that the truth 'of seven hundred years ago, is true to-day: that out and 'out relativism is an error: that down through all the 'oscillations of historical systems, there is ever to be met 'with a *philosophia perennis*—a sort of atmosphere of truth

'pure and undiluted whose bright clear rays have lighted 'up the centuries even through the shadows of the darkest 'and gloomiest clouds . . . For 'if reason be aught but a 'deceptive aspiration after the absolutely inaccessible, 'surely whatever has been brought to light, whatever 'our ancestors have unearthed, and acquired in their 'pioneer labours, cannot have proved entirely worthless 'to posterity.' " [1]

It is not of course to be supposed that the Neo-Scholasticism of to-day is in all points identical with the Scholasticism of the middle ages. The astronomical physics of the mediæval doctors were theoretically erroneous. Moreover new questions have arisen, new difficulties been suggested, new discoveries have been made. The adversaries of to-day are not the adversaries against whom the mediæval doctors were called to contend. In adapting our methods to the needs of the day, we do not discard the principles of the Scholastics. But Neo-Scholasticism belongs to the twentieth century, not to the thirteenth; and it employs the weapons of a new age.

It has seemed advisable to make this brief apology for the standpoint adopted in this book, since in England comparatively little is known of the reaction towards Scholasticism which has taken place in recent years. Abroad its strength is better understood. The importance of the philosophical works of men such as Mercier, de Régnon, de Wulf, Nys, Farges, Domet de Vorges, Carra de Vaux, Mandonnet, Seeberg, Asin y Palacios, is acknowledged by all competent judges.[2] In England, for obvious

[1] De Wulf, *Scholasticism Old and New*, trans. by Coffey, p. 161.

[2] Mercier, *Cours de Philosophie*, Louvain 1897–1903; de Régnon, *Métaphysique des Causes*, Paris 1886; de Wulf, *Histoire de la Philosophie Médiévale*, Louvain 1900; Nys, *Cosmologie*, Louvain 1900; Farges, *Études Philosophiques*, Paris 1892–1907; Domet de Vorges, *Abrégé de Metaphysique*, Paris 1906; Carra de Vaux, *Avicenne, Gazali*, Paris 1900, 1904; Mandonnet, *Siger de Brabant et l'Averroisme latin au xiiie Siècle*, Fribourg-en-Suisse 1899; Seeberg, *Die Theologie des Johannes Duns Scotus*, Leipzig 1900: Asin y Palacios, *El Averroismo teologico de S. Thomas de Aquino*, Saragoza 1904. To these we must add the most valuable work *Beiträge zur Geschichte der Philosophie des Mittelalters* edited by C. Bäumker and G. von Hertling, Münster. Ozanam's *Dante et la philosophie*

reasons, the movement has been less felt. But some at least of those who have noticed it have not underrated its significance. In regard to it Professor Case writes as follows in his article on *Metaphysics* in the Encyclopedia Britannica : " One cannot but feel regret at seeing the 'Reformed Churches blown about by every wind of doc-'trine, and catching at straws, now from Kant, now from 'Hegel, and now from Lotze : or at home from Green, 'Caird, Martineau, Balfour and Ward in succession, with-'out ever having considered the basis of their faith : while 'the Roman Catholics are making every effort to ground 'a Universal Church on a sane system of metaphysics. 'However this may be, the power of the movement is 'visible enough from the spread of Thomism over the 'civilized world." [1]

My sincerest thanks are due to the Rev. G. S. Hitchcock alike for many valuable criticisms and suggestions, and for his kindness in reading the proof-sheets. I must further gratefully acknowledge my obligations to the Rev. M. Maher and the Rev. T. Rigby. If my work prove of service to those for whose use it has been written, the result will be in no small measure due to the generous assistance accorded me by these and other friends.

*catholique au xiii*e *siècle*, Paris 1855, may also be mentioned, as a book appealing to a wider circle of readers than the others we have named.

[1] M. Picavet, professor at Sorbonne, no friend of the movement, writes as follows in the *Revue Philosophique* for 1896 : " Les catholiques, unis par le Thomisme, qu'ils complètent avec une ample information scientifique, sont devenus les maîtres de la Belgique : on compte avec eux en Amerique et en Allemagne : leur influence grandit en France, même en Hollande et en Suisse." cited by Mercier, *Origines de la Psych. contemp.* c. 8.

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PRINCIPLES OF LOGIC

PART I

THE LOGIC OF THOUGHT

N.B. The student is recommended to reserve the passages marked with an asterisk (*) for a second reading of the work.

CHAPTER I.

THE NATURE AND AIM OF LOGIC.

§ 1. **Definition of Logic.** Logic may be defined as *the science which directs the operations of the mind in the attainment of truth.*

What do we mean by truth? An assertion is said to be true when it corresponds to the reality of which the assertion is made. But the verbal statement is merely the outward expression of the thought within. It is our thoughts which are properly said to be true or erroneous. For present purposes, therefore, we may define truth as the conformity of the intellect with its object. Thus if I see a white horse, and judge 'That horse is white,' my judgment is said to be true, because my thought corresponds with the thing about which I am judging.

The aim of all our mental operations is to attain true judgments. If I endeavour to establish a geometrical proposition, my object is to arrive in the end at a judgment, which is in conformity with reality. Now there are certain definite ways in which, and in which alone, our thinking faculty must proceed if it is to achieve

its task of faithfully representing the real order. Reflection enables us to observe the operations of the mind; and hence we are able to know and to catalogue these common types of mental action. In this way we learn the rules, which we must needs observe in reasoning, if we are to arrive at a true result. For, as experience shows us, it is very easy to argue in a way that will bring us, not to truth, but to error. It was a boast of the Sophists in ancient Greece that they could make the worse appear to be the better cause. They effected this end by skilfully violating the rules which men must observe, if their conclusions are to be true.

Another definition may be given of Logic, in which the science is considered in a different aspect. Logic is *the science which treats of the conceptual representation of the real order*; in other words, which has for its subject-matter things as they are represented in our thought. The difference between this definition and that which we gave in the first instance, is that this definition expresses the subject-matter of Logic, the former its aim. We shall find as we proceed that the science can scarcely be understood, unless both these aspects are kept in view.

The work of Logic therefore is not to teach us some way of discovering new facts. This belongs to the special sciences, each in its own sphere. It assists us in the attainment of truth, because it treats of the way in which the mind represents things, and thus shows us what are those general conditions of right thinking, which must be observed whatever be the subject which occupies us.

Where we have a systematic body of securely established principles and of conclusions legitimately drawn from these principles, there we have a science. Thus in the science of Astronomy we start from certain general laws, and have a body of conclusions derived from these. Mere facts not brought under general laws do not constitute a science. We are rightly said to have a science of Logic, for, as we shall see, it consists of a body of principles and legitimate conclusions, such as we have described.

§ 2. **Divisions of Logic.** The simplest act of the mind in which it can attain truth is the **judgment**—the act by which the mind affirms or denies something of something else. That which is affirmed (or denied) of the other is called an **attribute:** that to which it is said to belong (or not to belong) is called a **subject.** Hence we may define a judgment as **the act by which the mind affirms or denies an attribute of a subject.**

A judgment however gives the mind a complex object: for it involves these two parts—subject and attribute. We must therefore take account of a more elementary act of the mind than judgment, viz.: **Simple Apprehension.** Simple apprehension is the act by which the mind without judging, forms a concept of something. Thus if I should conceive the notion of a triangle, without however making any judgment about it, I should be said to have formed a simple apprehension of a triangle. The words true or false cannot be applied to simple apprehensions, just as we cannot say that the words in a dictionary are true or false. Some philosophers indeed deny that the mind ever forms a simple apprehension; they hold that in every case some judgment is made. We need not enter into this question. We can at least analyse the judgment into simple apprehensions: for every judgment requires two concepts, one in which the mind expresses the subject, and the other in which it expresses the attribute. Thus in the example given above, I must have a concept of *horse*, and one of *whiteness*, in order to say 'The horse is white.' These are the elements which go to constitute the complex act of judgment, and they can be considered in isolation from it. Logic therefore must deal with the concept.

There is a third process of the mind, namely **Reasoning** or **Inference.** This is defined as, **the act by which from two given judgments, the mind passes to a third judgment distinct from these, but implicitly contained in them.** Thus if I say:—

All roses wither in the autumn;
This flower is a rose;

Therefore : *This flower will wither in the autumn ;*

or if I argue :—

Whatever displays the harmonious ordering of many parts is due to an intelligent cause ;
The world displays the harmonious ordering of many parts ;

Therefore : *The world is due to an intelligent cause ;*

I am said in each case to **infer** the third judgment. An inference of the form which we have employed in these examples, is called a **syllogism.** The two judgments given are known as the **premisses.** The judgment derived from them is the **conclusion.**

It is of these three acts of the mind that Logic treats : and the science falls correspondingly into three main divisions,—the Logic (1) of the Concept, (2) of the Judgment, (3) of Inference.

Since Logic deals with thought, it necessarily takes account to some extent of language—the verbal expression of thought. It does so however from quite a different point of view to that of **Grammar.** Grammar is concerned with words as such. It is *the art by which the words employed in significant speech are combined according to the conventional rules of a language.* Hence in it each of the nine parts of speech is treated independently, and rules are given for their respective use. On the other hand, the simplest object of which Logic takes account is the Concept. In its consideration of words, therefore, it does not deal with any of those parts of speech, which taken by themselves are incapable of giving us an independent concept. It is conversant not with nine, but with two forms only of significant utterance, viz.: the **Name,** the verbal expression of the Concept, and the **Proposition,** the verbal expression of the Judgment.[1] The proposition consists of three parts. These

[1] Cf. Boethius, *Introduct. ad Syll. Cat.* (Migne P. L. vol. 64. col. 766. A), and *De Syll. Cat.* lib. I. (*ibid.* col. 766. D). Another important difference between Logic and Grammar is to be found in the fact that Logic is concerned with but one mood—the Indicative, Grammar with all the moods equally. See below, Ch. 3, § 1.

are, (1) the **Subject** — that of which the assertion is made: (2) the **Predicate** — that which is affirmed or denied of the Subject: and (3) the **Copula** — the verb *is* or *are* which connects the Subject and the Predicate. The Subject and the Predicate are called the **Terms** (from the Latin *terminus* — a boundary) of the proposition: and the predicate is said to be *predicated* of the subject.

§ 3. **The Place of Logic in Philosophy.** The sciences fall into two broad divisions, viz.: the *speculative* and the *regulative* (or *normative*) sciences. In the speculative sciences, philosophic thought deals with those things which we find proposed to our intelligence in the universe: such sciences have no other immediate end than the contemplation of the truth. Thus we study Mathematics, not primarily with a view to commercial success, but that we may *know*. In the normative sciences, on the other hand, the philosopher pursues knowledge with a view to the realization of some practical end. 'The object of philosophy,' says St. Thomas of Aquin, 'is order. This order may be such as we find already existing; but it may be such as we seek to bring into being ourselves.' [1] Thus sciences exist, which have as their object the realization of order in the acts both of our will and of our intellect. The science which deals with the due ordering of the acts of the will, is Ethics, that which deals with order in the acts of the intellect is Logic.

The question has often been raised, whether **Logic is a science or an art.** The answer to this will depend entirely on the precise meaning which we give to the word 'art.' The mediæval philosophers regarded the notion of an art as signifying a body of rules by which man

[1] St. Thomas *in Ethic.* I. lect. 1. Sapientis est ordinare. . . . Ordo autem quadrupliciter ad rationem comparatur. Est enim *quidam ordo* quem ratio non facit sed solum considerat, sicut est ordo rerum naturalium. *Alius autem est ordo* quem ratio considerando facit in proprio actu, puta cum ordinat conceptus suos ad invicem et signa conceptuum quae sunt voces significativae. *Tertius autem est ordo* quem ratio considerando facit in operationibus voluntatis. *Quartus autem est ordo* quem ratio considerando facit in exterioribus rebus, quarum ipsa est causa, sicut in arca et domo.

directs his actions to the performance of some work.[1] Hence they held Logic to be the art of reasoning, as well as the science of the reasoning process. Perhaps a more satisfactory terminology is that at present in vogue, according to which the term 'art,' is reserved to mean a body of precepts for the production of some external result, and hence is not applicable to the normative sciences.

Aesthetics, the science which deals with beauty and proportion in the objects of the external senses, is now reckoned with Ethics and Logic, as a normative science. By the mediæval writers it was treated theoretically rather than practically, and was reckoned part of Metaphysics.

It may be well to indicate briefly the distinction between Logic and two other sciences, to which it bears some affinity.

Logic and Metaphysics. The term Metaphysics sometimes stands for philosophy in general : sometimes with a more restricted meaning it stands for that part of philosophy known as Ontology. In this latter sense Metaphysics deals not with thoughts, as does Logic, but with things, not with the conceptual order but with the real order. It investigates the meaning of certain notions which all the special sciences presuppose, such as *Substance*, *Accident*, *Cause*, *Effect*, *Action*. It deals with principles which the special sciences do not prove, but on which they rest, such as e.g., *Every event must have a cause*. Hence it is called the science of Being, since its object is not limited to some special sphere, but embraces all that is, whether material or spiritual. Logic on the other hand deals with the conceptual order, with thoughts. Its conclusions do not relate to things, but to the way in which the mind represents things.

Logic and Psychology. The object of Psychology is the human soul and all its activities. It investigates

[1] St. Thomas *in An. Post. I.*, lect. 1. 'Nihil enim aliud ars esse videtur, quam certa ordinatio rationis qua per determinata media ad debitum finem actus humani perveniunt.'

the nature and operations of intellect, will, imagination, sense. Thus its object is far wider than that of Logic, which is concerned with the intellect alone. And even in regard to the intellect, the two sciences consider it under different aspects. Psychology considers thought merely as an act of the soul. Thus if we take a judgment, such as e.g., 'The three angles of a triangle are together equal to two right angles,' Psychology considers it, merely in so far as it is a form of mental activity. Logic on the other hand, examines the way in which this mental act expresses the objective truth with which it deals; and if necessary, asks whether it follows legitimately from the grounds on which it is based. Moreover, Logic, as a regulative science, seeks to prescribe rules as to how we ought to think. With this Psychology has nothing to do: it only asks, What as a matter of fact is the nature of the mind's activity?

§ 4. **The Scope of Logic.** Logicians are frequently divided into three classes, according as they hold that the science is concerned (1) with names only, (2) with the form of thought alone, (3) with thought as representative of reality.

(1) The first of these views—that Logic is concerned with names only,—has found but few defenders. It is however taught by the French philosopher Condillac (1715–1780), who held that the process of reasoning consists solely in verbal transformations. The meaning of the conclusion is, he thought, ever identical with that of the original proposition.

(2) The theory that Logic deals only with the forms of thought, irrespective of their relation to reality, was taught among others by Hamilton (1788–1856) and Mansel (1820–1871). Both of these held that Logic is no way concerned with the truth of our thoughts, but only with their consistency.

In this sense Hamilton says: "Logic is conversant with the form of thought, to the exclusion of the matter" (*Lectures*, I. p. 15). By these logicians a distinction is

drawn between 'formal truth,' i.e., self-consistency and 'material truth,' i.e., conformity with the object; and it is said that Logic deals with formal truth alone.

On this view Mill well observes: "the notion of the 'true and false will force its way even into Formal Logic. ' . . . We may abstract from actual truth, but the 'validity of reasoning is always a question of condi-'tional truth,—whether one proposition must be true 'if the others are true, or whether one proposition can 'be true if others are true" (*Exam. of Hamilton*, p. 399).

(3) According to the third theory, Logic deals with thought as the means by which we attain truth. Mill, whom we have just quoted, may stand as a representative of this view. "Logic," he says, "is the theory of valid 'thought, not of thinking, but of correct thinking" (*Exam. of Hamilton*, p. 388).

To which class of logicians should Aristotle and his Scholastic followers be assigned? Many modern writers rank them in the second of these groups, and term them Formal Logicians. It will soon appear on what a misconception this opinion rests, and how completely the view taken of Logic by the Scholastics differs from that of the Formal Logicians. In their eyes, the aim of the science was most assuredly not to secure self-consistency, but theoretically to know how the mind represents its object, and practically to arrive at truth.

The terms **Nominalist, Conceptualist,** and **Realist Logicians** are now frequently employed to denote these three classes. This terminology is singularly unfortunate: for the names, Nominalist, Conceptualist and Realist, have for centuries been employed to distinguish three famous schools of philosophy, divided from each other on a question which has nothing to do with the scope of Logic. In this work we shall as far as possible avoid using the terms in their novel meaning.

§ 5. **History of Logic.** It was Aristotle (384–322 B.C.) who laid the foundations of the science by treating logical questions separately from other parts of

philosophy. Six of his treatises are concerned with the subject : they cover almost the whole ground.

1. *The Categories*,—a treatise on the ten primary classes into which our concepts of things are divided.
2. *De Interpretatione*,—a treatise on terms and propositions.
3. *Prior Analytics*,—a treatise on inference.
4. *Posterior Analytics*,—a treatise on the logical analysis of science.
5. *Topics*,—a treatise on the method of reasoning to be employed in philosophical questions, when demonstrative proof is not obtainable.
6. *Sophistical Refutations*,—an account of fallacious reasonings.

This group of treatises was afterwards known as the *Organon*. It should, however, be noticed that they are separate works. Aristotle himself had no single word to signify the whole of Logic, and it seems doubtful whether he viewed it as a single science. The name Logic was introduced by Zeno the Stoic (about 300 B.C.).

The successors of Aristotle added but little of permanent value to his great achievement. Enduring importance however attaches to a small treatise by the Neoplatonist Porphyry (233–304 A.D.) entitled the *Isagogé* or *Introduction to the Categories of Aristotle.*

In a certain sense the name of Boethius (B. Severinus Boethius 470–525 A.D.) constitutes a landmark in the history of Logic : for it was through the medium of his translation of the *Organon*, and his commentaries on the *Categories* and the *Isagogé*, that the works of Aristotle and Porphyry were available for educational purposes in Western Europe from the sixth to the thirteenth century.[1] Through this period some knowledge of Logic was widely possessed, as it was one of the seven liberal arts—Grammar, Dialectic i.e., Logic, Rhetoric, Geometry,

[1] Two works attributed to St. Augustine were also recognized authorities at this period. St. Augustine's interest in the science was not shared by all the fathers. We are told of St. Ambrose, that he used to exclaim *A Logica Augustini, libera me Domine.*

Arithmetic, Astronomy, and Music—of which higher education was held to consist.

At the beginning of the thirteenth century the numerous other treatises of Aristotle, and the works of his Arabian commentators Avicenna (Ibn Sina 980–1037 A.D.) and Averroes (Ibn Roshd 1126–1189 A.D.) were translated into Latin, and gave an immense impetus to philosophic study. The mediæval Scholastics availed themselves of these works, to build up a thoroughly systematic science of Logic. It may perhaps be said that their main advance on Aristotle's treatment lay in the greater accuracy with which they discriminated the respective spheres of Logic and Metaphysics, and in their more precise arrangement of the various parts of Logic itself.

The 15th and 16th centuries witnessed the decadence of Scholasticism, and in 1620 an attack was made on the very foundations of the Aristotelian Logic by Francis Bacon in his *Novum Organum*. Much of his criticism was ill-founded, since he believed that the purpose of Logic was to provide men with a means towards making discoveries regarding the laws and phenomena of nature. Yet it was of service in calling fresh attention to the theory of Induction, a part of Logic to which too little attention had been given by the later Scholastics.

Since the time of Bacon the whole question of Induction has been very fully discussed by writers on Logic. The most eminent of these among English thinkers was John Stuart Mill (1806–1873), whose treatment of the subject long held rank as the classical work on Induction. Many of the points, however, raised by these writers do not strictly speaking belong to the province of Logic. For, influenced by Bacon, they have dealt not merely with Induction as a process of thought, but also with a very different subject, namely the general theory of scientific investigation. It was indeed natural that a keen interest should be felt in this question. The rapid growth and multiplication of the physical sciences during the last three centuries could not but lead to the codi-

fication of their rules and to reflection on their methods, —in other words to the formation of a philosophy of evidence. Such a science was impossible in the middle ages, ere the great era of physical investigation had dawned. At the present day the treatment of this subject forms a part of every work on Logic. By many writers it is termed *Material* or *Inductive Logic*, the traditional part of the science receiving by way of distinction the name of *Formal* or *Deductive Logic*. These names are misleading. The traditional Logic was, as we have seen (§ 4), not purely formal. And though the treatment of Induction, properly so called, by many of the mediæval writers was inadequate, yet they all regarded it as falling within their scope. We have therefore preferred to designate the two portions of this volume *The Logic of Thought* and *Applied Logic or the Method of Science* respectively. Induction as a process of thought, finds its place in the first of these two divisions.

NOTE TO CHAPTER I.

Different views as to the scope of Logic.

* The difference of opinion as to the true scope of Logic is far wider than would appear from the triple division given in § 4, which is that usually recognized in logical text-books : and special names are now employed by logicians to indicate the point of view from which the science is treated. Moreover the threefold division is, as we have noticed, open to the further objection that it compels us to group the Scholastic logicians either with the school of Mansel or with that of Mill, though they have little enough in common with either of these. It seems, therefore, desirable to enter somewhat more into detail on the subject. In this note we give an explanation of the special designations referred to, viz. : *Scholastic Logic, Formal Logic, Symbolic Logic, Inductive Logic, Transcendental Logic, Logic of the Pure Idea, Modern Logic.* The notice in each case is necessarily very brief. The purpose of the present work would render a more detailed account of the systems out of place. As far as possible we have availed ourselves of citations from authors representative of the various views, in order to elucidate the meaning of the different terms.

(1) *Scholastic Logic.* We have explained above that the *Scholastic* or *Traditional Logic* holds the subject-matter of the

science to be the conceptual representation of the real order. This may be otherwise expressed by saying that it deals with things, not as they are in themselves, but as they are in thought. Cardinal Mercier says: "There are two sciences whose object is 'in the highest degree abstract, and hence universal in its applica-'bility. These are Metaphysics and Logic. The object of 'Metaphysics is Being considered in abstraction from all in-'dividual determinations and material properties, in other words the Real as such. . . . Logic also has Being for its object. . . . 'It must not however be thought that Logic and Metaphysics 'consider Being from the same point of view. . . . The object 'of Metaphysics is the *thing* considered as a real substance en-'dowed with real attributes. The object of Logic is likewise 'the *thing*, but considered as an object of thought endowed with 'attributes of the conceptual order" (*Logique*, § 23 [ed. 3]).

(2) *Formal Logic.* The characteristic of this school is to consider the mental processes in entire abstraction from the relation which the concept bears to the real order. Logic, says Dean Mansel, "accepts as valid, all such concepts, judgments and 'reasonings, as do not directly or indirectly imply contradictions: 'pronouncing them thus far to be legitimate as thoughts, 'that they do not in ultimate analysis destroy themselves" (*Proleg. Logica*, p. 265). Mr. Keynes abstains from deciding whether Formal Logic constitutes the whole of the science, but says in its regard: "The observance of the laws which Formal 'Logic investigates, will not do more than secure freedom from 'self-contradiction and inconsistency" (*Formal Logic*, § 1).

(3) *Symbolic Logic* is a further development of Formal Logic. It is thus defined in *Baldwin's Dict. of Philosophy*. 'Symbolic 'logic is that form of logic in which the combinations and relations 'of terms and of propositions, are represented by symbols, in 'such a way that the rules of a calculus may be substituted for 'actively conscious reasoning.' Mr. Venn, its ablest exponent in this country, claims for it the great advantages, that it "gene-'ralizes the processes of the ordinary Logic," showing them as particular cases of wider problems dealing with relation (*Symbolic Logic*, Introd. pp. xxi,–xxiii.). Though the subject has engaged the attention of several able men, it has no claim yet to be considered a science. Almost every investigator has adopted his own system of symbolic notation. It should further be mentioned, that many thinkers believe that although it affords scope for much ingenuity, it cannot possibly contribute in any way whatever to our knowledge of the reasoning process.

(4) *Inductive, Empirical, Material or Applied Logic* is a science developed on the basis of the views set forth in Bacon's *Novum Organum*. Mill terms it "a general theory of the sufficiency of 'Evidence," and "a philosophy of Evidence and of the Investi-

'gation of Nature" (*Exam. of Hamilton*, p. 402). "Every 'one,' he says, 'who has obtained any knowledge of the physical 'sciences from really scientific study, knows that the questions 'of evidence presented . . . are such as to tax the very highest 'capacities of the human intellect" (*ibid.*): and he severely calls to task those who hold "that the problem which Bacon 'set before himself, and led the way towards resolving, is an 'impossible one . . . that the study of Nature, the search for 'objective truth, does not admit of any rules." Granted that there be such a science it must belong, he urges, to Logic, "for 'if the consideration of it does not belong to Logic, to what science 'does it belong?" (*ibid.* p. 400). It is manifest that the science Mill here describes, differs essentially from Logic, as heretofore it had been understood. This philosophy of evidence deals, not with thought, but with things as they are in the real order; and its function is to prescribe the due methods of enquiry in each several science,—not merely in the physical sciences as this passage might suggest. H. Spencer with more consistency than Mill refuses altogether the name of Logic to the traditional science of that name, and prefers to term it the Theory of Reasoning (*Psychology*, pt. vi., c. viii.).

(5) *Transcendental Logic* is the name given by Kant to the most fundamental portion of his Critical Philosophy. Kant started with the assumption that all our knowledge, whether sensitive or intellectual, is internal to the mind,—that we have no immediate knowledge of the external world. He further assumed that the material of all our knowledge can be nought but successive pulses of sensation without unity of any kind. If these purely subjective feelings undergo such a transmutation within us as to present to our experience an orderly world of matter and motion, this must be in virtue of an *a priori* element—an internal mechanism providing certain 'forms' according to which we perceive, think and reason. In the *Transcendental Aesthetic*—the work in which he treats of sensible perception—he endeavours to shew that space and time are the 'forms' of our sensitive faculty, while the pulses of sensation constitute its matter. In the *Transcendental Logic* he deals with the 'forms' of intellect (*Transcendental Analytic*), and of reasoning (*Transcendental Dialectic*). The intellectual 'forms' will be noticed in Ch. 9 below. He gave them the name of Categories, a term employed in a very different sense by Aristotle and his followers. They are "the regular lines imposed by the intellect, on which 'sensations settle down with unities, orders, sequences, identi-'ties" (Wallace, *Kant*, p. 70). The problems raised in the *Transcendental Dialectic* fall outside the scope of the present work.

(6) *Logic of the Pure Idea.* This is the name given by Hegel

to his system. It will be sufficient for us to advert in the briefest manner to this philosophy, with which we are only concerned because of its bearing on Modern Logic. Its salient feature is the identification of Logic and Metaphysics. Hegel would not admit the existence of two orders—an order of thought and an order of reality. Thought, according to him, constitutes reality. Hence the science of the real—Metaphysics, is to be found in Logic—the science of thought: "Logic in our sense coincides 'with Metaphysics" (Wallace, *Logic of Hegel*, p. 38). The Universe has its origin in the inner necessity of the categories of thought. But thought in its fullest development—the thought of the Whole or the Absolute Idea passes over into reality. Thought becomes things, and realizes itself as the universe which we know.

Hegelianism is in fact a form of Pantheism. In it things are thoughts, and these thoughts are a Divine Mind evolving itself in the process of the Universe.

(7) *Modern Logic.* The treatment of logical problems known by this name owes its origin to the Hegelian philosophy. It is plain that thinkers who deny the distinction between the order of things external to us, and the order of thought within, were bound to institute a new enquiry into the nature of those mental acts, which had hitherto been regarded as representative of the real order. The principal exponents of Modern Logic in England are Mr. Bradley and Mr. Bosanquet: their work, however, is very largely based on that of the eminent German logicians Lotze and Sigwart. According to Mr. Bosanquet the only difference between Logic and Metaphysics lies in the aspect under which they view the same subject matter. "I make no doubt," he says, "that in content Logic is one with Metaphysics, and 'differs if at all simply in mode of treatment,—in tracing the 'evolution of knowledge in the light of its value and import, 'instead of attempting to summarize its value and import apart 'from the details of its evolution" (*Logic*, I. 248). The operations of the mind—judgment and reasoning—are according to this view regarded as *vital functions*, by which 'the totality we 'call the real world' is intellectually constituted. The task of Logic is to analyse the process of constitution (*ibid.* p. 3).

CHAPTER II.

THE CONCEPT: THE NAME: THE TERM.

§ 1. **The Concept.** We have already explained what are the grounds, on which Logic takes cognizance of the Concept. Considered in isolation, the concept is not an act by which the mind attains truth. It can neither be termed true nor false. But concepts are the material of which our mental acts, true and false, consist. Every judgment of necessity contains two concepts. Hence the treatment of the concept is fundamental in the science of Logic. And in every science it is of vital importance that the primary notions should be accurately grasped. There is truth in that saying of Aristotle's, which in the middle ages had passed into a proverb: "What is at the 'beginning but a small error, swells to huge proportions 'at the close." [1]

In the first place it is necessary to distinguish carefully between the Concept or intellectual idea properly so called, and the Phantasm or mental picture. Whenever I think of an object, I simultaneously form a sensible picture of it in my imagination. If for instance I judge that 'Fishes are vertebrate,' or that 'The sun is round,' I cannot do so without imagining to myself a sensible representation of a fish, or of the sun. Sometimes, indeed, as when I think of some abstract subject, such as 'virtue,' the image of the mere word 'virtue' will serve my purpose: but some image is requisite, nor does the intellect ever operate save in connexion with a phantasm.[2]

[1] *De Caelo*, I. c. 5. τὸ ἐν ἀρχῇ μικρὸν ἐν τῇ τελευτῇ γίνεται παμμέγεθες. *Parvus error in principio fit magnus in fine.*

[2] The term 'phantasm' (φάντασμα) is Aristotle's. Thus in *De Anima*, III. c. 8, he tells us that "when we contemplate any thing, we are forced to con-

This mental picture is however very different from the concept. This will be easily understood, if we notice that to judge 'The sun is round,' I must in *thought* have separated the attribute of 'roundness' from the thing I term 'the sun.' No sensible image can effect such a separation. It can only picture the single object 'a round sun.' If again I say, 'This glass is transparent,' I have in thought separated the attribute 'transparency' from the thing 'glass.' This power of separation requires a higher faculty than that of the imagination, namely the faculty of *thought* or *intelligence*. It is the intellect alone that has this wonderful power of distinguishing two things which in nature are inseparably conjoined, of severing its roundness from the sun, its transparency from the glass. Thus I can look at a single object, e.g., the paper I am using, and consider separately its whiteness, its smoothness, its oblong shape, its opacity, etc.

The mind's power of thus separating in thought things which in the real order are one, is known as its power of **abstraction.**

The characteristic feature by which the Concept differs from the Phantasm, is its **universality.** A Concept is equally representative of all objects of the same character. Thus if I see a circle drawn on a black-board, the concept which I form of that geometrical figure will express not merely the individual circle before me, but all circles. The figure I see is of a definite size, and is in a particular place. But my mind by an act of abstraction omits these individual characteristics, and forms the concept of a circle as it is enunciated in Euclid's definition. This concept is applicable to every circle that ever was drawn. When however I form the phantasm of a circle, my phantasm must necessarily represent a figure of particular dimensions. In other words the concept of the circle is universal: the phantasm is singular. Similarly, if I form a concept of 'man,' my concept is applicable to all

template it in conjunction with a phantasm" (ὅταν τε θεωρῇ, ἀνάγχη ἅμα φάντασμά τι θεωρεῖν): and he proceeds to distinguish carefully between the phantasm and the concept (*νόημα*).

men. But a phantasm of a man must represent him as possessed of a certain height, with certain features, with hair of a definite colour, etc.

We have only to consider any object to see that all the concepts which we can form of it, have this universal character. Thus, glancing out of the window, I see a garden-roller. My mind conceives it as a roller, as cylindrical, as iron, as dusty, cold and so on. Every one of these concepts is universal, and thus applicable to any other thing which resembles the roller in that one attribute, no matter how much it may differ from it in others. The concept of 'roller' is equally applicable to the ponderous machine with which the county-council repairs the highways: that of 'cylindrical' expresses perfectly the shape of the candle on my mantel-piece: while the concept of 'cold' is appropriate to the water in the neighbouring fountain. These various concepts are not, of course, so many isolated units in the mind. They unite to form a single composite concept. But that composite concept is a universal, and would express all similar rollers.[1]

It must not be thought that the intellect has no means of knowing individuals. It knows the individual by advertence to the phantasm from which the universal idea is abstracted.[2] But the work of the intellect, as distinct from that of sense, is to express the individual object of sense-perception in a series of universal concepts. Thus it is the intellectual faculty which enables us to conceive the individual Socrates as a 'man,' or as a 'vertebrate,' or as a 'father.'

[1] The doctrine of this section is insisted on by Aristotle. "The universal 'nature is evident to the intelligence, the individual to sense-perception. 'For the intelligence deals with what is universal, and sense-perception with 'what is particular (ὁ μὲν γὰρ λόγος τοῦ καθόλου, ἡ δὲ αἴσθησις τοῦ κατὰ 'μέρος)," *Physics*, I. c. 5, § 8. "The phantasm is as the perceptions of sense, 'save that it is without material embodiment (τὰ γὰρ φαντάσματα ὥσπερ 'αἰσθήματά ἐστι πλὴν ἄνευ ὕλης)," *De Anima*, III. c. 8. For a further consideration of the point, see Maher, *Psychology* (6th ed.), 235–238.

[2] *Vide* St. Thomas, *de Anima*, III. lect. 8. The interpretation here given by St. Thomas of the passage *de Anima*, III. c. 4, § 7, is not only in full accord with Aristotle's general doctrine, but is substantially the same as that proposed by Themistius and by Simplicius. Cf. Rodier, *Traité de l'âme*, *l.c.* Cf. also *De Veritate*, Q. II. Art. 6.

§ 2. **Repugnant Concepts.** Concepts are said to be repugnant, when, as mutually exclusive, they cannot be united in one composite concept.

It is not all concepts which can be brought together to form a composite concept such as those described in the last section. There are some which are incompatible, so that the one necessarily postulates the exclusion of the other. These are known as *repugnant concepts*. Thus it is impossible to form the concept of 'a thinking stone': for the concept of 'a stone' expresses lifeless matter, and the concept 'thinking' expresses living intelligence. Just as in the real order a thing cannot both live and not live, so in the order of thought such a thing is *inconceivable*.

The difference between what is *inconceivable* and what is *unimaginable* should be carefully noted. There are many things, that cannot be represented in our imagination, which nevertheless contain no repugnance—as for instance, colour to the blind from birth. On the other hand, some things are quite inconceivable, and hence impossible, of which I can form some kind of image. Perhaps, it is possible to imagine a thinking stone.

§ 3. **Adequate, Clear and Obscure Concepts.** A concept is said to be *adequate* when it represents distinctly all the notes which go to make up an object. It is plain that in regard to individual things such concepts are impossible. We cannot form a concept representing every characteristic of even the smallest flower. We can however have adequate concepts of mathematical figures in their universal aspect. Euclid's definitions express adequate concepts of the figures with which they deal. A concept is *clear* when, although it does not represent all the notes of an object, it yet contains a sufficient number to distinguish that object from any other. Thus our knowledge of nature may be very limited, but our concepts of the chief tribes of animals have a degree of definiteness amply sufficient to enable us accurately to distinguish one from another. A concept is *obscure* when our knowledge is insufficient to distinguish the object of thought from other things. Thus a man may have a general idea of what is meant by prudence and by fortitude and may nevertheless be incapable of distinguishing prudence from timidity, fortitude from rashness.

§ 4. **The Name and the Term.** We have intimated (Ch. 1, § 2) that Logic takes account not only of thought, but of language the verbal expression of thought. Hence after the consideration of the Concept, we must say something of the Name. **A Name is a word or group of words which by convention signifies the concept of the speaker, and the object of that concept.** It is of importance to observe that the name is immediately significative of the concept, and only mediately of the thing: that is to say, it is the name of the thing in question, because the concept, which it immediately expresses, is the concept of that thing. That this is so may be easily seen. I can give the same name to many different individuals, because the same universal concept expresses them all. Socrates, Plato, Peter, Paul are each of them termed *man*. For one and all, in virtue of similar characteristics, are truly represented by the same concept.

Names signify the particular characteristics contained in the concept, which they express, but they are the names of the thing, which the concept represents to us.

The Name is the expression of our thought, considered out of all relation to its position in a proposition. Since Logic considers names merely in so far as they are actual or possible terms, we shall, in dealing with the distinctions which we are about to enumerate, speak of them as distinctions of terms. The distinction immediately following constitutes an exception for reasons that will appear.

§ 5. **Categorematic and Syncategorematic Words.** It is plain, that it is not every word, which can be used as a term. Many can only be so used in conjunction with other words, e.g., *in, but, well, me*. Hence, we at once distinguish words into two classes :—

A Categorematic word is one, which can be used as a term without being accompanied by any other word.

A Syncategorematic word is one, that can only enter into a term in conjunction with other words.

These expressions are derived from the Greek words *κατηγορεῖν—to predicate*, and *συν—together with*.

Substantives, Pronouns, Adjectives and Participles

are categorematic words. It will be noticed, however, that adjectives and participles can only be predicates: they cannot stand as the subject of a sentence, except where there is ellipsis of the substantive. If we say, 'The unjust shall perish,' 'unjust' stands for 'unjust men.'

Adverbs, Prepositions, Conjunctions, and Interjections are syncategorematic. We can it is true employ them as the subjects of those propositions in which we speak of the mere words themselves; as for instance, if we say, 'When is an adverb of time,' 'When is a word of four letters.' But in this case we use them in a different sense, namely as signifying the mere vocal sound, or the written characters.

A term which is composed of categorematic and syncategorematic words, is spoken of as a **many-worded term.** Terms consisting of a single categorematic word are known as **single-worded terms.**

The student should be on his guard against speaking of syncategorematic *terms*. If a word is said to be syncategorematic, it is thereby affirmed to be incapable of being a *term*.

§ 6. **Divisions of Terms.** We enumerate here the various divisions of terms. It should be observed that every term may receive a place in each of the divisions, since the various divisions are based on different principles. What are called the *logical characteristics* of a term, are expressed by referring it to its due position under each head. All the divisions are of considerable logical importance.

(1) General and Singular Terms.
(2) Concrete and Abstract Terms.
(3) Connotative and Non-connotative Terms.
(4) Positive and Negative Terms.
(5) Absolute and Relative Terms.
(6) Terms of First Intention and Terms of Second Intention.
(7) Univocal, Equivocal and Analogous Terms.

§ 7. **Singular, General and Collective Terms.**

A Singular term is one which can be used in the same sense of only one individual thing. Such, for instance, are 'the present king of Spain,' 'the capital city of Italy,' 'Walter Scott.' To this class belong all proper names. **A Proper Name is a word whose sole purpose is to denote an individual thing.** Other Singular terms which are not proper names, are called **significant Singular terms,** since they tell us something about the object. They are formed by using General terms, and restricting them to designate but one of the many individuals to which they might be applied. Thus 'king' is a term applicable to a number of individuals in history: but if I say 'the present King of Spain,' I limit the sense of the term to one of the class. A frequent way in which terms are thus restricted, is by mentioning the time and place to which reference is made. In the instance just employed, the particular reference of the term 'king' is thus indicated. A proper name on the other hand tells us nothing about the object. Its one purpose is to serve as a distinguishing mark. Many proper names indeed once had a meaning. But in so far as they are used as proper names, the meaning they had is disregarded, and they are employed solely for the purpose of identification.

A General or Universal term is one which can be used in the same sense distributively of many things. A term is said to be used 'distributively', when it can be applied to *each* of the objects taken separately. Thus 'man,' 'soldier,' 'Englishman,' 'white,' 'black,' are affirmed of each of the objects to which they are applied.

It is not requisite that there should actually be a plurality of objects, to which the name is applied. It suffices, that it should be capable of such application. Thus 'six-masted steam-ship' is a term, which for very many years was applicable to one ship only, 'The Great-Eastern': but there was nothing in the nature of things to prevent the existence of an indefinite number of six masted steam-ships.

It will be well to give careful attention to the way in which these General terms are formed. The mind, when considering some object, attends to some feature in it, which is, or may be, precisely similar to features present in other objects : and it abstracts from all other features save this. Thus I may look at the ink in my bottle, and abstracting from its liquid state, its taste, etc., may consider it simply as black. The ink has many other qualities besides this. It is, moreover, an individual thing, and as such must have something in it, which is absolutely peculiar to itself, and is possessed by nothing else. But all this, my concept and the term which manifests the concept do not express. The term ' black ' may indeed be said to include *implicitly* all the other qualities, for it does not exclude the supposition that the subject which is black, possesses also many other attributes. If the word ' black,' like the word ' blackness,' involved the exclusion of everything save the attribute of blackness alone, we could not say, ' The ink is black,' just as we cannot say, ' The ink is blackness.' The term ' black ' neither excludes, nor *explicitly* expresses the other attributes.

Then follows another step. I observe that my concept of ' black ' is equally representative of all other black objects. Mentally there is nothing to distinguish the representations of an indefinite number of black things, in so far as they are black. The same concept expresses all. It is a *Universal concept*, and the word signifying it is a *General term.*

It is thus that all our General terms arise. For instance, experience shows us certain animals possessing peculiar characteristics common to all alike. We group the more marked of these characteristics into a concept, e.g., that of a horse : and wherever we see an animal possessed of these features, we term it a horse. The characteristics thus expressed in a common concept are styled the **Intension** or **Comprehension** of the General term : the individuals to which the term is applicable are said to constitute its **Extension.** Since the time of

Mill, English writers have more usually employed the words *Connotation* and *Denotation* to signify respectively Intension and Extension.

A Collective term is one that is applied to a group of similar objects, the term not being applicable to the objects taken singly. A Collective term may be either a proper name, or a significant Singular term, or a General term. Thus the term 'army' is collective, since it is predicable of the soldiers taken as a group, and not singly. It is also a General term, for it is applicable to many different armies. 'The National Portrait Gallery' is a Collective term signifying the pictures taken together as a group. It is also a significant Singular term. 'The Alps' is a Collective term, which is a proper name.

It is requisite to a Collective term that the objects should be capable of being considered from some point of view common to all. Thus the members of a family, though differing in many points, are alike as sharers in one home. For this reason, the definition requires that there should be 'similar objects.'

The names of substances, such as 'water,' 'gold,' 'lead,' are sometimes spoken of as **Substantial terms.** When they are used to signify all the water or gold that exists, they may be ranked as Singular terms. When they are employed to signify different portions of the substance, they become General terms.

§ 8. Abstract and Concrete Terms.

An Abstract term is the name of a nature or attribute considered in separation from the subject in which it inheres, e.g., whiteness, height, ebullition, humanity.[1]

A Concrete term is a name which expresses a nature or attribute as inherent in a subject, e.g., white, high, ebullient, man.

[1] Subject in these definitions is used in a different sense from that in which we have hitherto employed it. Hitherto we have been speaking of the subject of a proposition—*the subject of predication*, as it is termed by logicians. But real things are also termed the subjects of the qualities which inhere in them: they are *subjects of inhesion*. It is in this sense that the word is here used.

We have already when speaking of the Concept (§ 1) dealt with the abstractive power of the mind, in virtue of which it is able to sever an attribute from the subject in which it inheres, and to form such concepts as that of 'whiteness.' Light will be thrown on this mental operation, if we notice the distinction between a Substance and an Accident. A Substance is a thing which can possess independent existence, as e.g., Peter, Paul, man, lion. An Accident can only exist as inhering in a Substance, e.g., whiteness, prudence, transparency. In the Abstract concept we represent Accidents as though they were isolated from the subject in which they inhere.

The basis for this way of conceiving them, is found in the fact that the Accident which inheres in a Substance is not identical with the Substance. The whiteness of the animal is not the animal itself. The colour of the animal might alter, and yet the animal would be the same individual as before. Hence the mind is naturally led to conceive the quality as an independent entity, as something which the animal has, and by which it is determined and qualified.

But it is not Accidents alone which are expressed by Abstract terms. By an act of the mind the substantial nature itself can be represented in this abstract form, as though it were an accidental determination. Thus we form the concept of 'humanity,' though there is no real distinction between the individual Peter and his human nature.

Since these terms represent a single feature of the whole entity, as if it existed independently and in isolation, they possess a logical characteristic to which attention was called in the last section, viz.: that they cannot be predicated of the subject to which they belong. We can say, 'The horse is white': for the concrete term 'white,' though it expresses but one attribute, yet *implicitly includes the whole object.* We cannot say, 'The horse is whiteness,' for the abstract term positively excludes the subject in which it inheres. We can say

'Socrates is man.' But we cannot say 'Socrates is his humanity': for the abstract form of the word shows that the characteristics, which Socrates has in common with other men, in virtue of which he is called 'man,' are alone to be considered, to the exclusion of those which are proper to himself.

Abstract terms have no plural. The plurality that a quality may have in the real order, it acquires in virtue of the concrete individuals, in which it inheres. Hence when we conceive it in isolation, we have no means of conceiving it multiplied. Where plural forms are used, as when we speak of 'enthusiasms' or 'ineptitudes,' we merely mean the various instances in which the quality was realized. The quality as abstract, is incapable of multiplication. It follows from this that the only Abstract terms which are general, are those which embrace a whole group of qualities. Thus 'virtue' is a general Abstract term including 'justice,' 'prudence,' 'temperance,' 'fortitude,' etc. But these latter names are all singular.

Mill finds fault with "a practice, which if not introduced by 'Locke, has gained currency chiefly from his example, of apply-'ing the expression 'Abstract name' to all names, which are 'the result of abstraction or generalization, and consequently 'to all general names." The term 'Abstract name' is, as Mill notices, restricted by traditional usage to such as are considered in the present paragraph. Its application to General terms is a mistake. On the two kinds of abstraction see St. Thomas, *Summa Theol.* I. Q. 40, Art. 3.

§ 9. **Connotative and Non-Connotative Terms.** This distinction is one of the traditional divisions of terms. Its signification in recent English Logic is, however, altogether different from that which it used to bear. The change is due to Mill, who imposed a new meaning on the old terminology. The former sense of the words has fallen almost altogether into oblivion among English writers, and on the other hand much has been written on the distinction in its novel sense. We shall therefore deal in the first place with the terms

in their recent acceptation, and subsequently add an account of their traditional meaning.

The definitions given by Mill are as follows :—

A Connotative Term is one which denotes a subject and implies an attribute. A Non-connotative Term is one which denotes a subject only, or an attribute only.

(1) *Connotative Terms.* We have already mentioned (§ 6) that the word Connotation is understood by logicians to mean the characteristics signified by a name : the word Denotation, to mean the individual objects to which it is applicable. Every term which possesses both these features is termed Connotative. Thus ' horse,' ' man,' ' the Czar,' ' courageous,' are Connotative terms. They are applicable to individual objects, and they are given to these objects, because certain definite characteristics signified by the name are present in them.

Not all the characteristics possessed by the individuals in question enter into the connotation of the name. The characteristics which constitute the connotation are those only, because of which the name is given, and in the absence *of any of which* it would be refused. Thus, though before the discovery of Australia all swans of which the civilized world had experience were white, no one dreamed of refusing the name swan to the newly-discovered black variety. Nor do we deny the name of cat to Manx cats because they lack a tail. But the absence of a single *essential* attribute would deprive the object of its right to the class-name. Were an animal discovered, otherwise like the horse in appearance, but with cloven feet, it should not be termed a horse.

The explanation just given assumes that words possess a fixed and precise meaning. This is of course an ideal not altogether realized. The English language, perhaps even more than others, is lacking in this scientific precision. Yet it will usually be found that among the educated, there are definite and assignable characteristics implied by a name, in default of which it is not applied to an individual. It is true that this connotation may

vary. Our knowledge as to the real nature of things increases; and thus we insensibly alter the signification of a name. Mediæval writers would have understood the term 'whale' as connoting a special kind of fish. We have learnt that the whale is not a fish but a mammal. There are comparatively few things, as to which we are sure that we possess the ultimate connotation. In the case of mathematical figures, we know it, for the law determining the construction of a geometrical figure is within our mental grasp. But even when a connotation is susceptible of change, these changes are not arbitrary. Science only calls on us to alter the connotation of a term, when it has found some more fundamental characteristic, in relation to which, those hitherto reckoned as its connotation are merely derivative. Thus the advance of knowledge is ever tending to greater fixity of connotation,—to the discovery of what the name *ought* to mean.

Two other views as to the connotation of a term call for mention, though neither of them is defensible.

Some writers have held that all the attributes possessed by the object of the name, whether they are known or unknown, constitute its connotation. This is the view of Prof. Jevons: "A term taken in its intent (i.e. connotation) has for its meaning the whole infinite series of 'qualities and circumstances, which a thing possesses." It is plain that if the term primarily represents the *concept*, its connotation must concern the thing as it is known, not the thing in its objective reality. The meaning of the term is what the thought represents to us.

Others have held that it should be taken to signify all that we know about the thing at the present time. But many of these attributes are quite unimportant, and their absence would never cause us to refuse the name to an object. Hence it is manifestly inaccurate to speak of them as the meaning of the term. The only satisfactory account of connotation is that which takes it to signify the fundamental characteristics, which determine the application of the term.

It is frequently given as a rule that, 'As connotation 'increases, denotation decreases: and as denotation 'increases, connotation decreases.' This however must be rightly understood. The rule is only true when we are dealing with a number of classes arranged in hierarchical subordination. Thus in the series, *Animals—vertebrates—mammals—felidae—lions*, we have such a series. The connotation of 'vertebrate' is greater than that of 'animal': the denotation is less. The connotation of 'lion' is the greatest of all: its denotation is least.

Except where we are dealing with such classes, the rule is not verified. Thus I may increase the connotation by adding some attribute, which is found in every member of the class, as for instance *crow—black crow*. Here no change takes place. Again the denotation of a term is increased by the birth of new individuals: but this makes no change in the connotation. It is, moreover, inaccurate to state the rule in mathematical language, thus: "Connotation and denotation vary 'in inverse ratio." There is no mathematical relation between the two. One alteration in connotation may make an immense change in denotation, as e.g. *zebra—tame zebra*.

(2) *Non-connotative Terms*. The first kind of Non-connotative terms mentioned by Mill are those which denote a subject only, not connoting any attribute. This group consists solely of proper names. These are merely distinguishing marks of the individuals, and are not held conditionally on their retaining certain definite qualities. Connotative names on the other hand are changed when the quality changes: the man we once called 'thin,' we afterwards call 'stout.' For Connotative names primarily signify the characteristic or quality, and secondarily the individual object to which they are applied. It is entitled to the name, because it possesses the quality. Proper names have no meaning in this sense. A man cannot claim a proper name because he possesses certain qualities. However zealous a

philanthropist he may be, he cannot ask us to call him Lord Shaftesbury.

This point has been contested by Mr. Bradley and others, on the ground that names grow to acquire a connotation, and that the name of a friend recalls his qualities to my mind. It is true that it may recall them. But they do not therefore constitute the connotation of the name. The proper name designating an individual recalls his qualities by *association*. So far are they from forming its connotation that though the individual may lose them, he will not thereby lose his title to his name.

It has been further urged that many proper names do in fact signify definite attributes. Thus 'John Smith' is said to signify 'man' and 'Teuton.' Has then every male Teuton a right to term himself John Smith? And is not 'John Smith' in all probability the proper name of not a few negroes? It can scarcely be urged that this name is really significative of attributes. Again, it is true that many proper names of places and families originally indicated the possession of certain attributes, e.g., 'Norfolk,' 'Edinburgh,' 'Southampton,' 'Grosvenor.' But these have long lost all connexion with the meaning they once had. Nor can any argument be drawn from the fact that it is possible to use such terms as 'a Don Quixote' as connotative. Such a use indicates that the word has ceased to be a proper name.

The other kind of Non-connotative terms are Abstract names. These signify an attribute only. The question has been raised whether these terms are rightly reckoned as non-connotative. Several logicians maintain that there is no term without denotation. They hold that Abstract terms denote an object, namely the quality which they connote; that their denotation and connotation coincide. A consideration of what the denotation of a term truly is, will shew that this view cannot be admitted. The denotation consists of the *real objects* expressed by the concepts. Now there is nothing in the real order corresponding to the Abstract term. In

that term our concept represents the attribute as though it were an independent entity. That is to say, it represents it in a way in which no attribute can exist in the real order. These terms therefore have no denotation.

* The meaning attached to this division of terms by the Scholastic logicians was altogether different from that which we have just discussed. They distinguish them as follows :—

A Connotative term is one which expresses an attribute as qualifying a subject.

A Non-connotative term is one which expresses a nature or attribute as an independent entity.

All adjectives are connotative terms : for each adjective signifies some special attribute as qualifying some person or thing. Thus 'courageous' signifies the attribute of 'courage' as determining some subject. 'Prudent' similarly signifies the attribute of 'prudence' as qualifying a person.[1] On the other hand all substantives, as 'man,' 'father,' 'humanity,' 'paternity,' signify some entity that is conceived as independent and not as the qualification of a subject.

The logical value of the distinction depends on the difference that there is in the real order between Substances and Accidents. Substances are expressed by nouns-substantive. Accidents on the other hand are expressed by adjectives : for an accident is not an independent entity, but a mere qualification of a substance. Hence a distinction between them is necessary in the conceptual representation of the real order : and this distinction is expressed by the Non-connotative and the Connotative term. Yet, as we have already had occasion to notice, the power of the intellect enables it to conceive things otherwise than as they actually exist. It can conceive the accident as though it were an independent entity.[2] When it does so it employs the abstract term, and that term is a substantive and is non-connotative.

The Scholastic distinction is therefore philosophically justified. It corresponds to a fundamental difference in our mental conceptions, and the logic of the concept would be incomplete without it.

Mill, as an adherent of the Empiricist school, rejected the distinction between substance and accident. There was there-

[1] These Connotative terms when considered in relation to the characteristic, the presence of which they signify, and from which they are etymologically derived, were called **Denominatives** (*παρωνυμα. Categ.* c. 1, § 5, c. 8, § 27). The abstract attribute was termed the **Denominant.**

[2] The abstract term does not represent the accident as a substance, but simply as an independent determination. Only concrete terms can represent substance.

fore no place in his logic for a division of terms which involved its recognition. He solved the difficulty that thus presented itself, by putting all general terms signifying substances on a par with adjectives, and transferred them *en bloc* to the class of connotative names. The resulting division is devoid of philosophical value. A division of terms in the science of Logic must express different ways of conceiving the real order: and when a class of terms is designated by a common name, this should indicate that all these terms are conceived in a similar manner. This is certainly not the case in regard to Mill's Non-connotative terms. Proper names and Abstract names have conceptually nothing in common. Proper names are not significant of any concept at all: they simply denominate individual objects of sense-perception.

§ 10. Positive and Negative Terms.

A Positive Term is one which signifies the presence of some attribute.

A Negative Term is one which signifies the absence of some attribute.

Thus as examples of Positive terms, we may take 'living,' 'present,' 'equal'; and as examples of Negative terms 'lifeless,' 'absent,' 'unequal,' 'not-man,' 'nonentity.' A special class of Negative terms is constituted by what are called **Privative Terms.** These express the absence of the attribute in an object *in which it might have been expected to exist*, as for instance 'blind,' 'dumb.' It would be correct to speak of some animalculae as 'sightless' or 'eyeless,' but not as 'blind': for the term 'blind' implies the absence of sight where it is normally to be found.[1]

It should be noted that very many Negative terms, such as, e.g., 'impatient,' 'careless,' 'inhospitable,' are understood to imply the presence of positive qualities, opposite to those designated by the corresponding Positive terms. There is, however, one class of Negative terms to which no possible positive signification can adhere, and which have as Mr. Keynes says, 'a thorough-going negative character. These are terms of the form 'not-man,' 'not-white.' They denote everything which does not possess the

[1] On Privative terms see St. Thomas, *Summa Theol.* 1. Q. 33, Art. 4, ad. 2.

positive quality to which the negation is attached. They are called **Infinite (i.e. Indeterminate) terms** (*nomen infinitum*, ὄνομα ἀόριστον). The name was given them by Aristotle, because they do not fulfil the natural purpose of the name, which is to designate some determinate character or some definite individual. They are wholly indeterminate in their signification: the term 'not-man' is equally applicable to what is real and to the unreal. I can say 'A horse is not-man,' and 'A griffin is not-man.'

Certain recent logicians have denied that it lies within our power to form a concept of 'not-man.' They assert that we cannot hold together in a single thought things which have nothing in common. It is quite true that we cannot hold together things which have nothing in common, by means of a positive concept. We can however do so by a negative concept. The logical significance of the Negative term lies in this very fact, that it witnesses to our power to conceive the absence of some quality as though it were a positive reality. We conceive negations as though they were real things: we give them conceptual realization. The very fact that these terms can stand as the subject or predicate of a sentence is a proof that there is a thought corresponding to them.[1] Were it not for this power of conceiving by negations, we should have no thought corresponding to the word 'nothing.' It is true we cannot *imagine* 'nothing': but we can conceive it. The proposition 'Created Being was once nothing' gives an intelligible sense. Similarly we can form concepts of not-man,' 'not-white.'

§ 11. Absolute and Relative Terms.

An Absolute term is a name, which in its meaning implies no reference to anything else.

A Relative term is one, which, over and above the

[1] 'Quia tamen [nomen infinitum] significat per modum nominis quod potest subjici et praedicari, requiritur ad minus suppositum in apprehensione." St. Thomas *in Periherm.* lect. 4, ¶. Excludit quaedam.

object it denotes, implies in its signification another object also receiving a name from the same fact which is the ground of the first name.

We are all familiar with certain names which are given in pairs; so that if an object exists, to which one of the two may be applied, we know that there must also be an object to which the other may be applied. Such are 'parent, child,' 'master, servant,' 'king, subject,' 'husband, wife,' 'equal, equal,' etc., etc. If it can be truly affirmed of any one that he is a parent, then there is also some one of whom the term 'child' is predicated. If there is a master, there must also be a servant. Such terms are known as Relative terms: and each is said to be the **correlative** of the other. When a term does not involve a correlative, it is known as Absolute.

It should be carefully observed that the mere fact of a relation between two objects, does not make their names Relative names. Peter and Andrew are not Relative names, though the two men be brothers. For a name to be Relative, it must be such as to express a concept in which the relation is the object of thought. Now I can conceive Peter, without thinking of him as a brother: his brotherhood may be altogether outside my mind's field of vision. But if I think of him *as* brother, then in my thought I necessarily *refer* him to the person whose brother he is, and my thought is a *relative* concept. The Relative term for the case in question must be one that signifies such a relative concept. It must express Peter precisely under the aspect of his connexion with Andrew: it must be the term 'brother' (*Categ.* c. 7, § 8).

A Relative term may be concrete or abstract. If the name signifies the related object, we have the concrete relative, e.g., 'master,' 'father.' If we express the relation in separation from the object to which it belongs, it is abstract, e.g., 'dominion,' 'paternity.' Abstract relatives have their correlative terms. Thus 'paternity' corresponds to 'filiation,' 'dominion' to

'subjection,' the 'friendship' of the one friend to the 'friendship' of the other, the 'equality' which is predicated of one of two equals, to that which is predicated of the other.

The logical significance of this division of terms will not have escaped notice. We are here dealing with a special way in which the mind can represent the real order. It is capable on the one hand of representing its object in isolation : and on the other it can represent it in the light of the connexion by which it is related to something else. Further, it can fashion relations, even when they do not exist in the real order. Thus it can conceive a thing, which is viewed under one aspect, as 'identical' with itself viewed under another aspect. Yet in the real order there can be no such thing as a relation of 'identity.' To have a real, as distinguished from a merely conceptual relation, there must be two things, not one only.

§ 12. **Terms of First and Second Intention.** This distinction has been omitted by many of the recent English logicians.[1] It is, however, of the highest importance ; and the student who has thoroughly grasped its significance, will find his labour in the study of Logic much lightened.

A term of First Intention is one which is applicable to the object, as it exists in the real order.

A term of Second Intention is one which is applicable to the object, only as it exists in the conceptual order.[2]

Of the terms which may be predicated of an object as signifying its attributes, not all belong to it as it is in the real order. Some belong to it only in so far as it is represented in the mind. Thus I may not only say, 'The oak is a forest-tree,' 'The oak is deciduous' ; but I may go on to say, 'The oak is the subject of my

[1] See however an accurate account by Sir William Hamilton in *Edinburgh Review*, vol. 57, p. 210. "The distinction," he there says, "is necessary to be 'known, not only on its own account as a highly philosophical determination, 'but as the condition of any understanding of the Scholastic Philosophy."

[2] Cf. St. Thomas, *Opusc.* 39, *De Natura Generis*, c. 12.

judgment,' 'The oak is a universal concept.' Here, as is manifest, I have two totally different orders of predicates. One sort belongs to the object in its own natural mode of existence: the other belongs to it in so far as it is represented conceptually, in so far that is, as it is realized in the logical order. To this class of terms belong many with which we shall have to deal in subsequent chapters, such as *genus, species, major, minor and middle term*, etc. These two orders of predicates are called respectively terms of First Intention and terms of Second Intention. The terminology will be understood, when it is remembered that *intentio* is a word used by the mediæval logicians to signify 'an act of the mind.' The first act of the mind is that by which the mind conceives and knows the thing as it is in the real order: the second act of the mind is that by which it knows the thing as it is in the conceptual order.

The reason why we claim that the distinction is of primary importance, is that Logic is wholly concerned with the consideration of things as they are in the conceptual order,—with things as they are mentally represented, and hence as they are subjects, predicates, universal terms, etc. It is not concerned with the real order as such, but with the manner in which the mind represents that real order. Hence Logic is wholly concerned with Second Intentions: and it was not without cause that the mediæval logicians defined it simply as the Science of Second Intentions.

Thus St. Thomas, *Opusc. de Universalibus*, c. 2, 'Logica principaliter est de secundis intentionibus.' Scotus, *Super Universalia Porphyrii*, Q. 3, quotes the saying that 'Logic deals with Second Intentions as applied to First,' which he attributes to Boethius. It is however said not to occur in that author's works. Goudin (1640–1695) says, 'Omnes Thomistae assignant ens rationis seu secundas intentiones pro objecto formali Logicae.' *Logica Maj.* Quaest. Prae. Art. 1. This distinction first appears in the works of the Arabian commentators on Aristotle.

§ 13. Univocal, Equivocal and Analogous Terms.

A Univocal Term is one which is always employed with

the same intension. An Equivocal term, on the other hand, is one which can be used to express two entirely different meanings, as e.g. the word 'bit' is used to signify either a morsel or part of a horse's harness. Equivocal terms are of no logical importance. Properly speaking an equivocal term is not one but two terms. **An Analogous Term is one which is employed to express meanings partly, but not wholly, the same.** These terms are of two kinds. (1) One class do not, for our purpose, differ from Equivocal terms. The word 'healthy' may serve as an example. As applied to a man, it means that his physical condition is satisfactory. As applied to food, it signifies, not that the food is physically sound, but that it is calculated to produce health in man. The two meanings here, are as distinct as those of an Equivocal term. (2) The other class of Analogous terms must be carefully noticed. These terms do not, it is true, like Univocal terms, convey precisely the same meaning wherever they are employed. When an Analogous term is applied to objects between which the analogy exists, its meaning in the two cases is in so far different, that the characteristic signified is present *in different grades*. Yet we can express both forms of the characteristic in question by a single concept, because there exists between them a likeness of proportion (ἀναλογία). When for instance we say that God is the 'cause' of the world, and that the sculptor is the 'cause' of the statue, the word 'cause' is analogous. God causes the world in a different sense from that in which the sculptor causes the statue. Yet owing to the proportional resemblance between the two cases, we can form an Analogous concept, and employ an Analogous term. Similarly the word 'thing' is analogous. We say that a man is a 'thing,' and that a thought is a 'thing'; but a man and a thought are not things in the same sense.

§ 14. **Opposition of Terms.** Terms can be opposed to each other in the following ways :—

(1) **Contradictory Opposition is the opposition between**

a term and its negation, e.g. 'man, not-man,' 'white, not-white.' It is characteristic of this opposition that the two terms are not merely mutually exclusive, but they are exhaustive of all possible things. Everything, no matter what it be, whether it be matter or spirit, real or unreal, is either white or not white.

The opposition which exists between **Repugnant terms** (§ 2), e.g. between 'red' and 'white,' is a special case of Contradictory Opposition. The reason they exclude each other, lies in the fact that while both are colours, they are colours of *different species*. What is red cannot be also white. It is not-white.

(2) **Contrary Opposition is the opposition between two classes, which are furthest removed from each other among those which belong to the same genus.** Such are, for instance, 'white, black,' 'pious, impious,' 'kind, cruel.' This opposition arises because our concepts of certain series of qualities represent them as passing by gradation from one extreme to the other: there is no abrupt transition such as is found to exist between contradictories. Where this gradation of qualities is found, the extremes are known as contraries.

Many logicians speak of Contradictory opposition as formal, because it can be symbolically represented as 'A, not-A': Contrary opposition they term material, because we are unable to say whether two terms are contraries, unless we know the actual things which they signify. On the view of Logic which we are defending, there is no room for this distinction. The mental processes which can be symbolically represented, do not differ in any essential from those which cannot. The mind recognizes Contrary opposition between the concepts 'pious' and 'impious,' 'white' and 'black,' precisely in the same manner as it recognizes the Contradictory opposition between 'A' and 'not-A.'

§ 15. **The 'Suppositio' of the Term.** Even where we are dealing with one and the same univocal term, there are various ways in which it may be construed.

The same term may stand for something different. These various uses of the term were termed by the Latin logicians its *suppositiones*, (from *supponere*, 'to stand for'). Although a little repetition may be involved, it will be well to distinguish here the principal among the different ways in which a term may be used.

(1) *Collective and Distributive use.* When anything is affirmed or denied of a plural subject, the predicate may apply either to the individuals, who constitute the subject, taken separately, or to them taken as a group. The former is known as the distributive use (*suppositio distributiva*) of the term; the latter as the collective use (*suppositio collectiva*). The propositions, 'The citizens raised a monument to the dead statesman,' and 'The citizens voted in the election,' will sufficiently illustrate the two cases.

(2) *Real and Logical use.* This distinction depends on whether the speaker refers to the object as it is in the real order, or as it is in his concept. Thus, if I say "The King of England is at Windsor," the use is real (*suppositio realis*). If I say, "'The King of England' is the subject of my sentence," the use is logical (*suppositio logica*).

(3) '*Suppositio materialis.*' When a word is taken to signify simply the spoken sound, or the written symbol, it is said to be used in its *suppositio materialis*, e.g. "'To run' is a verb." "'Run' is a word of three letters."

CHAPTER III.

THE JUDGMENT AND THE PROPOSITION.

§ 1. **The Proposition.** As the Term is the external expression of the Concept, so the Proposition is the expression of the Judgment. The Proposition may be defined as **a verbal expression in which we affirm or deny an attribute of a subject** (*de Interp.* c. 6, § 1). It is also sometimes defined as **a verbal expression enunciating a truth or falsity** (*de Interp.* c. 1) : for it is characteristic of every proposition that it must be either true or false. The form of the proposition is *S is* (or *is not*) *P*, e.g. 'The lion is vertebrate,' 'Caesar is not alive.'

A proposition of the kind we have described, is commonly known as the **Categorical Proposition,** to distinguish it from Conditional propositions. In these we do not assert the attribute of the subject absolutely : we merely affirm that, given certain conditions, it belongs to it. In regard to the Categorical proposition, the following points are to be noted :—(1) It is always in the indicative mood. In the other grammatical moods, the mind does not judge that the attribute belongs to the subject, but expresses a wish that it may be so, or gives an injunction that it should be so. In the indicative alone we affirm (or deny) the attribute of the subject. Thus we have, 'The messenger is speaking,' 'May the messenger speak!' 'Speak, messenger!' So too it is only when the attribute is affirmed of the subject that the mind reaches truth or falsity. For truth is attained when the mind assigns to the subject an attribute which belongs to it in the real order. (2) The logical proposition is always stated in

the present tense. Our purpose in Logic, as we have seen, is to study the mode in which the mind represents the real order. As regards this the question of present, past or future is purely accidental. The time-determination does not affect the mental representation as such. Hence differences of tense so necessary in the use of language for practical ends have no place in Logic. (3) The logical predicate is always separated from the copula. In the language of common life, we frequently express them in one word, as for instance, 'The bird flies.' In Logic we must say, 'The bird is flying.'

This same process must be performed whenever we get mutilated expressions, such as 'Wolf!' 'Fire!' 'Rain!' For Logic demands that every sentence, whatever its grammatical form, shall be so analysed and expressed, as to represent as closely as possible the intellectual act. Sir W. Hamilton stated this in what is sometimes termed *Hamilton's Postulate*, viz.: 'Logic postulates to be allowed 'to state explicitly in language, whatever is implicitly 'contained in thought.' Our three mutilated expressions may be respectively resolved into, 'A wolf is near,' 'A fire is burning,' 'Rain is falling.'

We must carefully distinguish between the 'is' of the copula, and the same word when it means 'to exist.' The copula does not signify that the subject exists.[1] I may say 'A chiliagon is a geometrical figure,' even though no chiliagon has ever existed. The copula in affirmative sentences denotes **the objective identity of the subject and predicate**: they are expressions representing one and the same object. In negative sentences the copula 'is not' signifies the objective diversity of the terms: the predicate is not applicable to the object denoted by the subject. I may say, 'The lion is vertebrate,' because the term 'vertebrate' is rightly applied to the same object as the term 'lion.' I cannot say 'The octopus is vertebrate.' This relation between the subject and predicate of the proposition, arises immedi-

[1] The question of the 'Implication of Existence' in the copula, will be more fully dealt with in Ch. 7.

ately from the nature of the mental act which the proposition represents—the judgment. In every affirmative judgment the two terms are different mental expressions of the same object. The same object is expressed in one concept as 'lion,' in another as 'vertebrate.' But the object which I conceive as 'octopus,' I cannot conceive also as 'vertebrate.' Hence the judgment, 'The octopus is vertebrate,' is impossible.

Here we see how totally Scholastic Logic differs from Formal Logic. Strictly, Formal Logic should take no account of the content of the subject and predicate. To it every judgment is simply *S is P*. Scholastic Logic rejects any judgments in which the concepts do not represent the same objective reality, e.g. 'Men are circles,' 'Cows are lions.' In these the two notions are repugnant the one to the other. The judgment is impossible.

§ 2. **Analysis of the Judgment.** Although as we have seen, the subject and predicate of the judgment are different concepts of the same thing, it is important to bear in mind that it is the subject which directly expresses *the thing*, i.e. that to which attributes belong. The predicate expresses *the thing as qualified by a particular attribute or form*.[1] Whenever we fix our attention on a thing, our mind immediately commences to abstract the attributes from the object of thought, and affirm them of it one by one. It judges: 'The thing is hard—is black—is brittle, etc.'[2] Here the predicate in each case is the attribute or form—not indeed the attribute considered in separation from the object in which it inheres, i.e. hardness, blackness, etc., but considered as qualifying the thing. It is easy to see how such judgments develop into more complex ones. The hard,

[1] Cf. St. Thomas, *Summa Theol.* I. Q. 13, Art. 12.

[2] "C'est donc grâce à l'abstraction intellectuelle que les choses sont affirmables les unes des autres, et peuvent faire fonction de prédicat dans les propositions." Mercier, § 31. Similarly Themistius, commenting on Arist. *De Anima*, III. vi. § 2, says 'ἡ μὲν γὰρ ὡς ἓν φαντάζεται τὸν βαδίζοντα Σωκράτην, ὁ νοῦς δὲ διαιρεῖ χωρὶς μὲν τὸν Σ., χωρὶς δὲ τὸ βαδίζει' "The sense-faculty gives us the phantasm of *Socrates walking* as a single whole: the intellect abstracts, and separates *Socrates* on the one hand from *is walking* on the other." Them. 202, 10 cited in Rodier, *Traité de l'âme*, II. 471.

brittle thing will be called 'coal': and the judgment will take predicates of a less primary character.[1]

A judgment is said to be *true* when the form expressed by the predicate is really found in the object denoted by the subject. Thus, if I see some object, e.g. Socrates, and I judge 'Socrates is walking,' my judgment is a true one if the attribute 'walking,' which I affirm of Socrates in thought, does in fact belong to him in the real order. Hence truth is defined as *the conformity of the mind with its object*. For in every true affirmative judgment the mental concept expressed in the predicate is in conformity with a real attribute belonging to the external object. As regards negative judgments the case is somewhat different. In them we declare that the form expressed in the predicate is not to be found in the object to which the judgment refers. Yet in a somewhat wide sense we can say that in negative judgments also the mind is conformed to its object. In judging a form not to belong to an object which in fact does not possess it, my mind is in correspondence with reality. But negation is a secondary and subsidiary form of truth. In affirmation there is perfect correspondence between the mental form expressed in the predicate and the objective reality.[2]

Grammatically the subject does not always take the first place. It is the meaning of the proposition, not the arrangement of the words, which tells us which is the subject and which the predicate. The term which qualifies or defines the other, whether it comes first or last, is the predicate. Thus in the words, 'Blessed are the meek,' it is the meek who form the logical subject.

We have stated in the previous section that the copula expresses the objective identity of the subject and predicate. We must now enquire why this identity is expressed by the verb 'to be.'

The reason will appear if we examine what is meant by

[1] When the attribute expressed by the predicate is not something perceived by the senses, the process of judgment is not by direct abstraction. Experience and education store the mind with concepts of numerous attributes. On one ground or another we recognize the presence of one of these in the subject, and judge, e.g., 'Caius is a coward,' 'Balbus is wise.'

[2] Cf. St. Thomas, *I. Sent. XIX.* Q. 5, Art. 1, ad 1.

the ' being ' of an object. If we speak of the real order, we can at once distinguish two senses of the term. The ' being ' of an object may signify (1) its *existence*,—that in virtue of which the thing *is* : or it may signify (2) the *nature* of the thing—that in virtue of which it is *what it is*. Thus for instance we say of Socrates that he *is a man* : of Bucephalus, that he *is a horse*. And many other characteristics than these may be affirmed of each of them. For it is plain that besides the essential nature ' man ' or ' horse,' numerous other qualities go to make an individual object what it is. Size, colour, etc. etc., all go to constitute the complete entity.

In considering the ' to be ' of the copula, we are however not concerned directly with the real order, but with its representation in the mind. Our words are the manifestation of our thoughts, and when we speak of things, we speak of them as they are mentally represented. Now when our mind forms a judgment concerning an object, the function of the copula is to declare that some attribute belongs to that object,—to tell us *what the object is*. In other words the ' to be ' of the copula represents not the ' being ' of existence, but the second sense of ' being,' that namely in which it means the nature of the thing. This will appear still more plainly if we reflect that we can make a series of true judgments about an object, irrespective of whether it exists or not. Our judgment, e.g. that ' the horse is a quadruped ' would be true even were the last member of the species *equus* extinct. Indeed so little has the ' is ' of the copula to do with existence, that when we desire to affirm the fact of existence, we do not employ the subject-copula-predicate sentence at all, but simply say ' Socrates *is*.' [1]

But here we must call attention to a point of very great importance. It will be remembered that when discuss-

[1] It should be noticed that existence adds no new note or determination to the nature of a finite being. The nature is complete in all its characteristics apart from the actuality which existence confers. Hence our concept of the nature is complete without respect to the question, whether the thing exists If it were possible to have Singular concepts, the concept, say of Socrates, would be the same in all its determinations, whether he existed or not.

ing Negative terms (Ch. 2, § 9) we saw that we are able to conceive as though they were real entities, things which are in fact simply the negations of entities. And thus we find that in many of our judgments, the predicate is not a quality or determination in the real order. It is a mere negation, which in the conceptual order I conceive as though it were a positive characteristic, as e.g. in the proposition, 'The horse is riderless.' 'To be riderless' is no positive characteristic of the horse. Sometimes both subject and predicate are of this character: for instance, 'Blindness deprives men of much happiness.' Here not merely is a privation conceived as if it were a real subject; but a purely negative result is conceived as a positive action.

From this it will easily appear that we cannot strictly speaking say that the 'is' of the copula expresses that the subject is determined in some way: for the predicate may not be a real determination at all. It expresses that the subject is *conceived* as determined in some way. As we have already said, it expresses the objective identity of subject and predicate.

Mill gravely informs us that his father was the first among philosophers to notice that 'to be' in the sense of 'to exist,' has not the same signification as when it means to be some specified thing, as 'to be a man'; and adds that Aristotle and all the ancients believed it to have a common meaning wherever used. "The fog," he adds, 'which rose from this narrow spot diffused itself at 'an early period over the whole surface of Metaphysics" (*Logic*, Bk. I. c. 4, § 1). Mill frequently falls into error when criticizing the philosophy of Aristotle and his followers, with whose writings he was but imperfectly acquainted. Nowhere perhaps is he more astray than here. Not merely was the distinction carefully noted by Aristotle [1]: but the various senses of 'Being' was one of the points most canvassed in the writings of the Scholastics.

[1] Thus in *Soph. Elenchi*, c. 5, he says, οὐ γὰρ ταὐτὸν εἶναί τέ τι καὶ εἶναι ἁπλῶς ('*To be something* is not the same as *to be*.') Cf. *de Interp.* c. 11, §§ 9, 10.

It remains to be noticed that a judgment is a single act of the mind. No mistake could be greater than to represent it as three separate acts, corresponding respectively to subject, copula and predicate. Such a view might seem to be implied, when it is said that in affirmation we have the conjunction, in negation the separation of two concepts. But it is manifest that a synthesis, in which we recognize a relation of identity, and a separation in which we judge that such a relation is absent, are alike single acts.[1]

Certain points which remain to be considered in regard to the Scholastic theory of the judgment, must be dealt with later.[2]

* The following two citations will shew that the view we have taken of the 'being' expressed by the copula, as referring to the nature of the object *as conceived*, is that held by St. Thomas and his great commentator Cajetan :—

"Sciendum est quod *Esse* dicitur tripliciter. Uno modo 'dicitur *Esse* ipsa quidditas vel natura rei, sicut dicitur quod 'definitio est oratio significans *quid est esse* : definitio enim quid-'ditatem rei significat. Alio modo dicitur *Esse* ipse actus essen-'tiae. . . . Tertio modo dicitur *Esse*, quod significat veritatem 'compositionis in propositionibus, secundum quod *est* dicitur 'copula : et secundum hoc est in intellectu componente et divi-'dente." St. Thomas *in I. Sent. dist.* 33, Q. 1, Art. 1, ad. 1.

Cajetan takes as an example the proposition 'Navis est sine gubernatore,' and says : "Navis caret gubernatore nullo intellectu 'considerante : absentia tamen gubernatoris nullum esse sub-'stantiale aut accidentale ponit in navi. Unde navem esse sine 'gubernatore, extra animam non est aliquod, sed non esse guber-'natam. Acquirunt autem esse privationes et negationes ex 'hoc quod intellectus intelligens privationes per habitus, et nega-'tiones per affirmationes, format in se ipso rei carentis aliquo modo 'idolum quoddam. Verbi gratia cum intellectus format in se 'idolum quoddam navis gubernatore carentis, quod est ipsa pro-'positio mentalis, non-praesentia gubernatoris quae extra animam 'nil ponit, in anima fit ens per hoc quod intellectus fecit ipsam 'propositionis terminum aliquem." *Comment. in De Ente et Essentia*, c. 1.

The same view is implied in Aristotle's well known distinction

[1] On the unity of the act of judgment Aristotle is explicit. He calls it "a synthesis of concepts as though they were but one" (*σύνθεσίς τις νοημάτων ὥσπερ ἓν ὄντων*), *de Anima*, III. c. 6, § 41.

[2] See below, Ch. 7, § 1, Ch. 9, § 4, Ch. 10, § 5.

between 'being,' in the real order, and 'being' which signifies truth, *Met.* V. c. 4, § 1.

§ 3. **Quality of Propositions.** In every proposition *P* must be either affirmed or denied of *S*. This alternative determines the **Quality** of the proposition, which must be either (1) affirmative, or (2) negative. This division is ultimate. Some logicians have, it is true, endeavoured to reduce all propositions to the affirmative form by writing *S is not-P*. But the difference cannot be thus bridged. *S is not-P* is, of course, equivalent to *S is not P*. But they differ the one from the other : since in *S is not P* we deny the positive concept *P* of *S*, and in *S is not-P* we affirm the negative concept *not-P* of *S*. The negative and affirmative forms remain radically distinct.

Kant admits three forms, Affirmative, Negative, Infinite, *S is P, S is not P, S is not-P*. His motive in assigning the Infinite judgment to a separate class, instead of reckoning them with the Affirmatives to which they rightly belong, seems to have been the desire that his scheme of Categories should present an harmonious appearance. A triple division was required in its other portions, and a triple division must perforce be found for the Quality of judgments.

§ 4. **Quantity of Propositions.** In any affirmation or negation, *P* may be affirmed or denied, (1) of all the objects denoted by the subject-term, e.g. 'All men are mortal'; or (2) of only some of these objects, e.g. 'Some men are negroes' : or (3) there may be no sign to mark whether the predicate refers to some only or to all, e.g. 'Pleasure is not a good' : or (4) the subject may be a singular term, e.g. 'Socrates is wise,' 'The highest of the Alps has been scaled.' These various alternatives lead to the division of propositions according to quantity.

A Universal proposition is one, in which the predicate is affirmed (or denied) of a subject, taken in its whole extension and distributively.

We have already explained (Ch. 2, § 14) that when a sub-

ject is employed distributively, the predicate is affirmed of every individual denoted by the subject. When we say, 'All sparrows are winged,' we mean that every individual sparrow is possessed of wings. A proposition in which the subject is understood collectively is not universal. Thus the proposition, 'All the slates covered the roof,' is not a universal proposition. The predicate is not affirmable of each individual denoted by the subject, but of the individuals as forming one group. Hence, whenever the word *All* (and not *Every*) is employed to qualify the subject, care must be taken to observe whether it be understood collectively or distributively.

It is plain that though the Affirmative Universal is of the form *All S is P*, the Negative Universal will not be *All S is not P*, but *No S is P*. The form *All S is not P* does not exclude *P* from each and every individual *S*, as at once appears in the proposition 'All soldiers are not generals.' If, however, I say, 'No Englishmen are negroes,' I exclude the attribute from every member of the class.

The employment of the plural in a universal proposition, e.g. 'All men are mortal,' may possibly mislead the student into supposing that in the subject the intellect conceives a number of individuals. This is, of course, impossible. The mental act is more truly represented by the Latin form 'Omnis *homo* est mortalis.' The subject of the judgment is *the universal nature* 'man,' not a number of individuals. The adjective 'All' does not multiply the concept, but signifies that to whatever entity the nature 'man' belongs, to that entity the attribute 'mortal' also belongs.

A Particular proposition is one in which the predicate is affirmed (or denied) of a part only of the extension of the subject.

The form of the Particular proposition is *Some S are* (or *are not*) *P*; for instance, 'Some soldiers are brave,' 'Some rich men are not generous.' The sense, in which the word 'some' is here used, differs in certain respects from that in which it is ordinarily employed. In ordinary use, when we speak, e.g. of 'some' men, we are under-

stood to mean more than one, and also to exclude the supposition that what we say may be true of all men. 'Some' means 'several but not all.' In Logic, the 'some' of a particular proposition, may be used even where the predicate might be truly affirmed of all: and it may be used also even if there be but one individual to whom it could be applied. Thus I may say, 'Some birds have wings,' even though it be the case that all birds possess them: and 'Some men are eight feet high,' though in fact there be but one such man. 'Some' leaves the extension to which reference is made wholly *indeterminate*.[1]

The essential distinction then between Universal and Particular propositions lies in this, that Universals deal with the whole class, Particulars with an indeterminate portion of the class. And here it is well to call attention to the fact, that universal propositions are of two sorts. The majority of them cannot be attained by mere enumeration of instances. Some indeed can. I can arrive at the universal truth, that 'All the apostles were Jews,' by a process of counting. But propositions of this character are of minor moment. Enumeration will not serve me in regard to such propositions as, 'All men are mortal,' 'All birds are oviparous.' Here, I refer not merely to an incalculable number of past instances, but also to the future. All laws of nature known to science are propositions of this character. The aim and object of scientific enquiry is to establish such universal truths.

How is it that we can affirm a predicate of individuals, which have not come within our experience? The explanation lies in the fact, that in these propositions we know the predicate to be invariably connected with the universal class-notion employed in the subject. In a later part of Logic, we shall consider how we reach this

[1] The reason for this is easy to see. When the word has the significance "some only," it is really equivalent to two propositions, one affirmative, one negative. When it is used in reference to certain definite individuals, A. B. C., it is equivalent to so many singular judgments. It is only in its indeterminate reference that it is an independent and elementary thought-form.

knowledge. It is sufficient here to observe that to whatever individuals the notion 'man' is applicable, the predicate 'mortal' is applicable also. In virtue of their being men, they possess the attribute of mortality. The universality of these propositions rests not on enumeration, but on our knowledge of the constant connexion between the concepts of the subject and predicate.

It remains to consider Indesignate and Singular propositions.

Indesignate propositions are such as have no sign of quantity. As far as form is concerned, they may be universal, or they may be particular. If I say, 'Old men are melancholy,' it does not appear, whether I am speaking of all old men, or of some only. Hence indesignate propositions have no place in Logic, until a sign of quantity is affixed to them. In some cases indeed the Indesignate is used to signify that the predicate is connected necessarily with the subject, e.g. 'Man is mortal.' Here the proposition is of course equivalent to a universal. For these judgments in which the Indesignate form stands not for individuals, but for the class-nature, some authors employ the convenient term **Generic judgments.** But it should be observed that we do not know their universal character from the logical form, but from our previous acquaintance with the matter under consideration. Very often the Indesignate is used for what are termed *moral universals*, as in the example already given, 'Old men are melancholy.' A moral universal admits exceptions, and hence is logically a particular.

The **Singular proposition** is, as we have said, one whose subject is either a significant Singular term or a proper name. These propositions present some anomalies. On the one hand, the individual object is a member of a class, and it appears incongruous to treat it as though it were itself a class. On the other, the definition of a Universal proposition is applicable to them, for the predicate is affirmed of the subject in its whole extension, the extension in this case being restricted to a single individual.

Modern logicians have resolved to treat this proposition as a Universal, and it will be convenient to adhere to that arrangement.

The older logicians classify the Singular proposition separately, and assign it neither to the Universal nor to the Particular.[1] This was, it would seem, the more scientific course. For the Universal and Particular are distinguished by the manner in which the concept employed as subject is understood in regard to extension. But as we have explained above (Ch. 2, § 1), we have no singular concepts. Hence there is a fundamental difference between such a proposition as, ' All men are mortal,' and ' Socrates is a philosopher.'

Propositions whose subject is a Collective term are Singular propositions. Thus if I say, ' All the apples filled the bowl,' it is clear that I refer to this group of apples considered as a single object.

§ 5. **The Fourfold Scheme of Propositions.** The last paragraph has shown us that the two fundamental forms of the proposition are the Universal and the Particular. In one of these two, every known truth can be expressed. For the assertion made is either known to hold good of the subject in its whole extension, or not. If it is known to hold good, we use the Universal proposition. If it does not hold good as regards the whole extension of the subject, or if, though it holds good, we do not know this to be the case, we use the Particular. This distinction, combined with that based on quality, gives us the fourfold scheme, viz. : Universal Affirmative, Particular Affirmative, Universal Negative, Particular Negative. These are respectively indicated by the letters, *A.I.E.O.* These letters are the vowels of the two Latin words, *Affirmo* (I affirm) and *Nego* (I deny). The first vowel in each stands for the Universal, the second vowel for the Particular.

[1] Cf. St. Thomas, *Opusc.* 44, *Summa Totius Logicae, de Interp.* c. 6. (This Opusculum, though found among the works of St. Thomas, is from another hand.)

Another notation, which is found convenient, is *SaP*, *SiP*, *SeP*, *SoP*: this notation has symbols for the subject and predicate, as well as for quantity and quality. Hence, our four propositions may be thus expressed.

All *S* are *P*.	*A*. *SaP*.
Some *S* are *P*.	*I*. *SiP*.
No *S* are *P*.	*E*. *SeP*.
Some *S* are not *P*.	*O*. *SoP*.

§ 6. **Analytic and Synthetic Propositions.** This distinction is based on the fact that each of our Judgments is based on one or other of two very different motives. The point will best be elucidated by a few examples. If we consider the following propositions, 'The angles of every triangle are equal to two right angles,' 'The whole is greater than its part,' 'Every square has four sides,' and compare them with such propositions as 'Water freezes at 32° Fahrenheit,' 'Some cows are black,' we shall at once recognize that there is a difference between the two classes. We are, indeed, certain of the truth of all these propositions. But our certainty has a different motive in the first class, and in the second. In the case of the first class of Judgments, as soon as we consider the concepts of the subject and predicate, we see that they are necessarily bound together. A triangle must have its angles equal to two right angles; otherwise it would not be what we mean by a triangle. Were we told of any figure that its interior angles were greater or less than two right angles, we should be justified in affirming that it was not, and could not under any circumstances be a rectilinear triangle. In the same way the intension of the concepts 'whole' and 'part' excludes the supposition of a whole that is not greater than its part; for the meaning of the term 'whole,' is 'that which consists of parts.' In regard to the second class, the motive of our assent is very different. It is experience that has led to my conviction that water freezes at 32° F., and that certain cows are black. There is nothing in my notion of 'cow'

which prescribes 'blackness,' nor in my notion of 'water,' which compels me to think of it as possessing this particular freezing-point at the sea-level. In neither of these propositions are the two concepts linked together in virtue of their intension.

The former class of propositions is termed Analytic, the latter Synthetic.

The definition of Analytic and Synthetic propositions is differently given by Scholastic philosophers on the one hand, and by the greater number of Logicians since the days of Kant, on the other. The difference is of primary importance in philosophy. We place the Scholastic definitions first.

An Analytic proposition is one, in which either the predicate is contained in the intension of the subject, or the subject in the intension of the predicate.

A Synthetic proposition is one in which the connexion of subject and predicate is not involved in the intension of the terms.

It will be seen that Analytic propositions are of two kinds. The first kind consists of those in which the predicate is a term signifying either the whole intension, or part of the intension of the subject. Such is the proposition, 'Every square has four sides.' The second kind consists of those in which the predicate is an attribute which *results necessarily* from the nature of the subject.[1] For where this is the case the subject is found in the intension of the predicate. An example is furnished by the proposition, 'A triangle is a figure having its interior angles equal to two right angles.' The predicate here is not found in the intension or definition of 'triangle.' But it is an attribute which necessarily results from and is involved in the characteristics of a triangle. And if we desire to define the attribute 'having its interior angles equal to two right angles,'

[1] An attribute which is thus connected with the subject by necessary result is termed a *property* of that subject. The term will be fully discussed in Ch. 8, § 1.

we can only do so by stating that it is 'a quantitative measure proper to the angles of a triangle.' [1]

It is not however necessary that the connexion of the attribute with the subject should be evident on the first consideration. Many steps may be necessary. Every geometrical theorem gives us an Analytical proposition as its conclusion. The connexion between subject and predicate is involved in the intension of the terms: but we must often take a long series of steps before the necessity of that connexion becomes manifest to us.[2]

The modern definitions are as follows:—

An Analytic Proposition is one, in which the predicate is contained in the definition of the subject.

A Synthetic proposition is one, in which the predicate is not contained in the notion of the subject.

So prevalent have these definitions become, that in any public examination at the present day, a question, involving these terms, would certainly employ them in this latter sense.

* (*a*) We have spoken of the differences between the two definitions as of vital moment. The Kantian division of Analytic and Synthetic propositions relegates to one class propositions, our knowledge of which depends wholly on experience of the individual case, such as 'This book is bound in cloth,' and propositions such as, 'The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the remaining sides.' Neither of these, Kant tells us, can be discovered by analysis. For he considers only the case, in which the predicate is found by the analysis of the subject, and entirely

[1] Analytic Propositions were termed by the Scholastics '*Propositiones per se notae.*' Cf. Arist. *An. Post. I.* c. 4, § 3, and St. Thomas, *in An. Post. I.* lect. x. "Primus modus ejus quod est *per se* est quando praedicatur definitio de aliquo definito, vel aliquid in definitione positum: . . . secundus modus dicendi *per se* est quando subjectum ponitur in definitione praedicati, quod est proprium accidens ejus." Cf. also *De Anima*, II. lect. 14.

[2] "If without axioms it is impossible to infer," says Mr. Bradley, "I wonder where all the axioms can have come from" (*Principles of Logic*, p. 227). There is no mystery about axioms. They are Analytic propositions in which the connexion of subject and predicate is immediately evident. Cf. St. Thomas, *Summa Theol.* I. Q. 2, Art. 1. "Ex hoc aliqua propositio est per se nota, quod praedicatum includitur in ratione subjecti. . . . Si igitur notum sit omnibus de praedicato et de subjecto quid sit, propositio erit omnibus per se nota."

ignores the case, in which the subject-term is revealed by an analysis of the predicate. What account then is to be given of our conviction as to the truth of such propositions as that relating to the square on the hypotenuse? If they are not analytic, two hypotheses only are possible. Either (1) we accept them on a dictate of our understanding, of which no account can be given. They are *synthetic a priori*. This is Kant's solution. Or (2) their truth is a conclusion, at which we arrive from an examination of individual instances, but we possess no ground for saying, that they must be true, that e.g. every right-angled triangle has the property described. This is Mill's solution.

(*b*) There have been three theories as to the object of Analytic propositions. Mill (following Hobbes) holds that they are concerned with the meaning of names only, and terms them Verbal propositions. Leibnitz held that they are concerned with our concepts. The Scholastics taught that they are truths relating to things, though known through our concepts, and expressed in words. Professor Case well says, "The division of propositions 'into verbal and real is defective. A verbal is not necessarily 'opposed to a real proposition, a predicate does not cease to be 'characteristic of a thing by becoming the meaning of its name, 'and there are some propositions which are verbal and real, 'such as, all bodies are extended, the whole is greater than its 'part. . . . Sometimes the same analytical judgment is at once 'real, notional and verbal, e.g. the whole is, is conceived, and 'means that which is greater than its part" (*Physical Realism*, p. 340).

(*c*) It is sometimes said, that every synthetic judgment becomes analytic with the growth of our knowledge, that e.g. 'George III. died in 1820,' is an analytic judgment to one who knows the history of that period. The argument is quite fallacious. The facts, which occur to an individual member of a class, are not necessary notes of his nature, forming the connotation of the concept which expresses it. In the first place, of individuals as such we have no concepts: all concepts are universal (Ch. 2, § 1). And secondly, even were it possible to have a concept expressing the essential nature of the individual, the purely contingent facts relating to him would not be part of it. Of course, if I form a complex concept applicable to George III., such as e.g. 'The King of England at the beginning of the nineteenth century,' and to this add the note 'who died in 1820,' then I may form the analytic proposition, 'The King of England at the beginning of the nineteenth century, who died in 1820, died in 1820.' But the value of an analytic proposition of this kind is not great.

(*d*) Analytic propositions are also termed Essential, Explicative, *a priori*, Verbal; and correspondingly, synthetic proposi-

tions are known as Accidental, Ampliative, *a posteriori*, Real. On this whole subject, Professor Case's *Physical Realism*, pp. 334–353, may be consulted with advantage.

§ 7. **Complex Propositions.** Complex propositions are such as have a complex term for their subject or their predicate. By a complex term is understood a many-worded term, consisting of two or more distinct parts, so that it expresses, not merely the nature of the thing denoted, but also one or more qualifications belonging to it, e.g. 'the white knight,' 'the roller which is in my garden,' and the like. These qualifications are often (as in the second of the instances just given), expressed by subordinate clauses, introduced by a relative. Yet it is manifest that, even if a complex term involves two or three such clauses, the term is but one, and constitutes a single subject or predicate, as the case may be.

Two forms of complex propositions are ordinarily distinguished by logicians. The distinction is grammatical, not logical, and is given in order to put us on our guard against ambiguity.

(1) Propositions with an **explicative** qualification. In these the qualification belongs to every individual signified by the general name, to which it belongs. Thus in the proposition, 'Whales, which are mammals, are aquatic animals,' the relative clause is applicable to every individual, that is signified by the general name 'whales.'

(2) Propositions with a **restrictive** (or **determinative**) qualification. In these, the qualification restricts the signification of the general name to a certain part of its denotation. Thus in the sentence, 'All nations, that have been civilized, have cultivated philosophy,' the qualification does not belong to all the members of the class indicated. Not all nations are civilized.

The time determination involved in the use of the past and future tenses of the verb, is a special form of complexity in the proposition. This however, as we

have noted, Logic is enabled to disregard. Another constantly recurring form is that produced by the employment of transitive verbs, followed by an object, e.g. 'Brutus slew his benefactor,' which gives as the logical predicate, the complex term 'a slayer of his benefactor.'

§ 8. **Compound Categorical Propositions.** It often happens that what grammatically is a single assertion, is resolvable into two or more propositions, each with its own subject and predicate. In such cases, we have the Compound Categorical Proposition. Propositions of this kind are divided into two classes,—those whose character is apparent from their grammatical structure (*aperté compositi*), and those in which the grammatical form does not manifest their composite nature (*occulté compositi*). These latter are termed *Exponibles*.

(*a*) **Propositions compound in form.** Of these there are three classes :—

1. *Copulative propositions.* These are affirmative propositions, in which there are two or more subjects or predicates or both. Hence they are resolvable into a number of independent affirmative propositions : e.g. 'Peter and Paul ended their days at Rome.' This is equivalent to 'Peter ended his days at Rome. Paul ended his days at Rome.'

2. *Remotive propositions.* These are negations similarly united. The conjunctions employed will be such as the negative form demands. For example, 'Neither riches nor honours can banish anxiety' ; this sentence may be resolved, 'Riches cannot banish anxiety. Honours cannot banish anxiety.' 'No Mohammedan will eat swine's flesh or drink wine.' This, in its logical expression, becomes, 'No Mohammedan will eat swine's flesh. No Mohammedan will drink wine.'

3. *Discretive or Adversative propositions.* Here we have either two affirmative propositions, or an affirmative and a negative proposition, connected by an adver-

sative conjunction, such as *but, although, yet.* Thus: 'William I. was brave but not magnanimous.' This gives us the two propositions 'William I. was brave. William I. was not magnanimous.'

(*b*) **Exponible propositions.** In these, as we have said, there is nothing in the grammatical structure of the sentence to indicate that it is equivalent to more than one logical proposition. Here also, three classes are ordinarily enumerated.

(1) *Exclusive propositions.* These contain a word, such as 'alone,' attached to the subject, and thus excluding the predicate from any other subject than this one. Hence two propositions are necessary to declare the full meaning, one to affirm the predicate of this subject, and another to deny it of all others. For instance, 'God alone is omnipotent.' This is equivalent to 'God is omnipotent. No other is omnipotent.'

(2) *Exceptive propositions.* In these, the subject term is restricted in its application by a word such as *except—save*, which excludes a portion of its denotation—e.g. 'All the crew save one were drowned.' Here again, two exponent propositions are needed, the one denying the predicate of the excepted part, the other affirming it of the remainder. The example just given will become, 'One of the crew was not drowned. The remaining members were drowned.'

If the order of the terms is altered, then both Exclusives and Exceptives may be expressed by a single exponent. 'Only God is omnipotent,' will become 'All that is omnipotent is God'; and 'All the crew save one were drowned,' will be 'The portion of the crew that was not drowned was one man.' But if the original order is to be preserved, two propositions are necessary. The reasons which justify a change of order will be dealt with in Ch. 5.

(3) *Inceptive and Desitive propositions.* In these a statement is made as to the commencement or ending of something; e.g. 'Printing became customary after 'the fifteenth century,' 'Paganism ceased in England 'about the year 700 A.D.' These are resolved by two

propositions, one relating to the state of things before the time indicated, and one relating to what occurred subsequently. Thus the first example will become 'Printing was not customary before the close of the fifteenth century. Printing was customary after that date.'

§ 9. **Modal Propositions.** The Modal proposition affords us another case in which the traditional terminology differs from that in vogue since the days of Kant. Here too we shall first explain modality as understood by the Scholastic philosophers, and then deal with the Kantian account.

The characteristic of the Modal, is that the copula undergoes modification, in order to express the manner in which the predicate belongs to the subject. There are propositions, in which the attribute affirmed belongs to the subject by strict necessity. Thus 'mortality' is an attribute that is necessarily connected with the subject 'man.' In other cases the element of necessity is absent. 'To be learned' is affirmable of some men only. It is not an attribute belonging necessarily to the nature 'man.' The pure categorical draws no distinction between these cases. We employ the same copula 'is,' whether the connexion is necessary or contingent. But in the Modal proposition, the nature of the connexion between attribute and subject receives expression.

It has been frequently objected, that this whole question belongs not to Logic but to Metaphysics. Thus Sir W. Hamilton says, "Necessity, Possibility, etc. . . . 'do not relate to the connexion of the subject and predicate . . . as terms in thought, but as realities in 'existence: they are metaphysical, not logical conditions." This objection rests on a misconception as to the province of Logic. Necessity and Possibility as objective facts, belong to the real order. But as mentally expressed by us, they belong to the logical order; and a treatise on Logic would be incomplete without

some mention of the manner in which the mental judgment represents these metaphysical conditions.

The relation of the attribute to the subject is, objectively, determined by one of three modes. These are (1) the **Necessary,** in which the attribute belongs necessarily to the subject. This is expressed by a proposition of the form, 'Men are necessarily mortal,' 'Equilateral triangles are necessarily equiangular.' (2) The **Impossible:** in this case the predicate is repugnant to the subject, e.g. 'It is impossible for irrational creatures to exercise free will.' And (3) the **Possible** (or **Contingent**). In this case the predicate belongs to the subject in some instances, while in other instances it is not found with it. Thus e.g. 'It is possible for a man to be a grammarian.' The conjunction of the two attributes involves no impossibility, but on the other hand is not necessary. This relation may be asserted from two points of view. We may assert the possibility of the connexion between subject and predicate, and express the proposition as it is expressed above. Or we may declare the possibility of their separation: and in this case the proposition will take the form, 'It is possible for a man not to be a grammarian.' Hence though there are but three modes, there are four fundamental forms of Modal propositions, as there are four fundamental forms of Categorical.

A difficulty is occasioned by the fact that ambiguity attaches to the word 'possible.' 'Possible' may have the sense in which we have just explained it. It may however, include in its signification the Necessary also; for if a predicate belongs necessarily to a subject, we can say with truth that that subject is capable of receiving it.[1] If all triangles must have three angles, it is true to say that it is possible for a triangle to have three angles.

[1] *Summa Totius Logicae*, Tract. 6, c. 13. "Notandum quod *possibile* 'dupliciter potest sumi: vel in toto suo significato, et tunc comprehendit 'necessarium et contingens. . . . Alio modo, sumitur solum pro contingen-'tibus." Similarly Aristotle tells us that 'possible' when used in regard of what is necessary, is employed in a distinct sense 'τὸ γὰρ ἀνάγκαιον ὁμωνύμως ἐνδέχεσθαι λέγομεν,' *An. Prior. I.* c. 13, § 1. On this subject see also *De Interp.* c. 13, § 9.

And similarly the assertion that it is possible for a subject not to have such and such a predicate, may have a sense in which it includes the Impossible.

The Modal may be expressed in two forms. In the first of these, the mode itself constitutes the predicate, having for its subject the proposition whose copula it affects, e.g. 'That man should be mortal is necessary,' 'that a bird should have gay plumage is possible.' Modals of this form are all singular, since the subject is not a term, but a proposition taken as a whole. Nevertheless the modes of necessity and impossibility are a sure sign that the proposition in question is universal; while on the other hand the mode of possibility, in the sense of merely possible (as distinguished from the case in which possibility is predicated of a necessary judgment) is indicative of a particular proposition. In the second forms of the Modal the mode qualifies the copula itself: e.g. All triangles are necessarily three-angled. Modals of this form are not singular but take their quantity from their subject.

* Kant's division of Modals is based, not on the objective relation of the predicate to the subject, but on the subjective certainty of the thinker. He divides judgments into the **Problematic,** i.e. '*S* may be *P*,' the **Assertoric,** i.e. '*S* is *P*,' and the **Apodictic,** i.e. '*S* must be *P*.' Of the problematic judgment he says that it expresses "a free choice of admitting such a pro-'position, and a purely optional admission of it into the under-'standing." The assertoric judgment "implies logical reality or 'truth." The apodictic gives us the same judgment as the assertoric, when it is recognized as determined by the formal laws of the understanding, and therefore as subjectively necessary (see Ch. i. *note* (5)). In regard to this division it may be said in the first place that such a proposition as '*S* may be *P*' is of no value to the logician. It is a mere declaration of ignorance, and not a judgment at all. Secondly, since the apodictic judgment enunciates the same truth as the assertoric, merely involving that the speaker recognizes more clearly the subjective necessity under which he lies of thus judging, there is no reason why he should not express the assertoric in the same form as the apodictic '*S* must be *P*.' The root error of this view is the failure to see that the copula is not a mere mental act of union, but expresses the objective connexion between the subject and its attribute in the real order (see Ch. 9, § 4).

The influence of the Kantian system is to be seen in many recent logicians. We are not infrequently told that when a truth is styled 'necessary,' nothing more is meant than a 'necessity of thought,' and that the term has no reference to the real order. Mr. Bradley tells us, "a necessary truth is really an inference, and an inference is a necessary truth" (*Principles*, p. 221). Similarly Mr. Bosanquet writes, "Every necessary 'truth must, in so far as it is necessary, present itself as the 'conclusion from an antecedent" (*Logic*, II. 222). Such a view as this must needs be fatal to any hope of attaining certitude in philosophy or science. The existence of any necessary first principles is denied. But where there are no necessary principles, there can be no necessity in the conclusion derived from them.[1]

§ 10. **Reduction of Propositions to Logical Form.** The sentences employed in literature and in ordinary conversation exhibit considerable variety of form and complexity of structure. It is possible however to analyse them and express them in the shape of *A E I O* propositions. This process is styled their reduction to logical form. By submitting sentences to this analysis we reach the simple elements of thought, which are contained in them. It is plain that this is very different from grammatical analysis into parts of speech. That process is concerned not with thoughts but with words. The preceding paragraphs should have rendered the task of reduction comparatively easy. Its essential feature is to obtain propositions, consisting of (1) a subject with the sign of quantity attached; (2) a copula, which must be of the form *is* or *are* (or *is not, are not*), and (3) a predicate.

We find the subject by putting to ourselves the question, *Of what or of whom is this statement made?*

We find the quantity of the subject by asking, *Is the assertion made of the whole extension of the subject, or of but part of it?*

We find the predicate by enquiring, *What is it that is asserted of the subject?*

These three points must always be considered, whenever the analysis of a sentence is attempted. Two other cautions may be added. First, that it is well, whenever it is possible, to express the predicate as an attribute, i.e. adjectivally, in order to bring out the true meaning of the proposition: e.g. the form 'All flattery is to be avoided' is better than 'All flattery is a thing to be avoided.' Secondly, that wherever it is necessary to introduce a time determination, this must be done in the predicate as in No. (7) below. The copula must always be in the present tense.

[1] Cf. Rickaby, *General Metaphysics*, p. 180.

A few examples will illustrate the process :—

(1) 'Fools despise wisdom.'

This will become, 'All fools *are* despisers of wisdom' (*A*).

(2) 'All's well that ends well.'

This will be, 'All that ends well *is* well' (*A*).

(3) 'Firm at his dangerous post he stood.'

This in logical form is, 'He *is* standing firm at his dangerous post' (*A*).

(4) 'As a man sows, so shall he reap.'

Here we have a relative sentence. The two clauses of these propositions give us the terms of a relation. Where the words 'As . . . so' are employed to introduce the clauses, the relation is one of likeness. The analysis gives us,

'In every instance, the character of a man's harvest *is* like the character of his sowing' (*A*).

If the words 'Where . . . there' are used, we have a relation of place: if 'When . . . then,' a relation of time.

(5) 'Where thy treasure is, there will thy heart be also.'

Logically, this is, 'In every instance, the place of your treasure *is* the place of your heart' (*A*).

(6) 'Love is akin to madness.'

Here the subject is used without any sign of quantity, but clearly stands for the whole denotation of the term. The proposition becomes, 'All cases of love *are* akin to madness.'

Where we have compound or exponible propositions, they need resolving into their component parts, e.g. :—

(7) 'Lions and tigers once lived wild in Europe, but not now.'

This gives us four propositions.

'Some lions *are* animals, that once lived wild in Europe' (*I*).

'Some tigers *are* animals, that once lived wild in Europe' (*I*).

'No lions *are* living wild in Europe now' (*E*).

'No tigers *are* living wild in Europe now' (*E*).

(8) 'Only the just enjoy peace of mind.'

This is resolved into:

'Some of the just *are* enjoying peace of mind' (*I*).

'None, who are not just, *are* enjoying peace of mind' (*E*).

(9) 'All save he had fled.'

Here we have a case, where the full force of the proposition cannot be brought out in the analysis, since we have no universal term by which to designate all the remainder. The reduction gives :—

'He *is* not fleeing' (*E*).

'Some (the rest) *are* fleeing' (*I*).

(10) 'The great is not good, but the good is great.'

Notice should be taken of the 'reduplicative' use of the word

great' in the first clause. It signifies 'the great *as such*,' or the great, just in so far as it is great.' This must be expressed in the analysis :—

'The great, merely in virtue of its greatness, *is not* good' (*E*).

'The good *is* great' (*A*).

Other Signs of Quantity. It will be useful to mention a few other modes of expressing Quantity besides those we have already noticed.

A. The universal affirmative is occasionally denoted by the expressions, *Any, Whoever, He who, Always, In every case.*

I may be denoted by *A few, Certain, Often, Generally, Most.*

E may be expressed by the word *Never.*

O has equivalents in *A few . . . not, Not all . . . are, All . . . are not, Few, Certain . . . not.*

The word *Most* has been placed as one of the equivalents of *I*. The proposition 'Most *S*'s are *P*' signifies that 'Some (more than half) *S*'s are *P*,' but does not necessarily imply in addition that 'Some *S*'s are not *P*.' It merely signifies that the majority of instances have been examined, and found to possess the attribute *P*. Thus we might say, 'Most English flowering-plants are dicotyledonous,' without desiring to commit ourselves to any opinion as to the whole flora : or again, after looking at seven cards out of a hand at whist, we might say, 'Most of the cards in this hand are court-cards,' knowing that it was possible they might all prove to be so. Similarly we might say, 'Few English flowering-plants are monocotyledonous,' even if we were ignorant whether there were any of that character. Hence *Few* is commonly reckoned as merely a sign of the proposition *O*.[1] The words *Hardly any, Scarcely any* are also regarded as equivalent to *O*. The use of *All* with a negative to signify *O* should be carefully noticed. 'Not all the crew were lost,' will be expressed 'Some of the crew were not lost.'

Special note should be taken as to whether the terms, to which words such as *All, A few,* etc. etc. are attached, are used distributively or collectively (Ch. 2, § 14). Wherever the use is collective the proposition is singular. 'All the men built a raft' is a case in point. The proposition may be expressed, 'The whole body of men *is* building a raft.'

§ 11. **Hypothetical Propositions.** Besides the Categorical propositions which we have hitherto been considering, there is another class of judgments called

[1] The difference between *A few* and *Few* is to be observed. *A few* is equivalent to *some*. "*Few*," says Mr. Keynes, "has a negative force. And 'Few *S*'s are *P*' may be regarded as equivalent to 'Most *S*'s are not *P*.'"

Conditional. These are distinguished from Categoricals by the fact that in them the predicate is not asserted absolutely of the subject. They are divided into two classes, termed Hypothetical and Disjunctive. In the present section we are concerned with the Hypothetical.

A Hypothetical Proposition is one in which the predication made in one proposition, is asserted as a consequence from that made in another. The proposition on which the truth of the other depends, is called the **Antecedent:** that which follows on its admission, is called the **Consequent.** Thus in the proposition, 'If the shepherd be negligent, the sheep go astray,' the antecedent is 'If the shepherd be negligent'; the consequent is 'the sheep go astray.' It will be seen that neither part of the proposition is independently asserted as true. We do not affirm that 'the shepherd is negligent,' nor yet that 'the sheep go astray.' It is the *nexus* between the two, the dependence of consequent on antecedent, which is affirmed.

There are two forms in which the hypothetical sentence may be expressed. These are (1) If A is B, C is D, and (2) If S is M, it is P. Judgments constructed according to the first formula, may usually by a little manipulation be expressed in the second form also. But it is incorrect to say that the latter is a more fundamental type than the former.

Hypotheticals of the second form, can be expressed categorically, by substituting in the place of 'If S is M, it is P,' the form 'All S that is M is P,' or 'All SM is P.' Similarly for the categorical 'All S is P,' we may write, **'If anything is S, it is P.'**

Some writers on Logic have maintained that the categorical and hypothetical propositions are in fact equivalent. There can be no doubt that this opinion is erroneous. In the categorical we state unconditionally that S is P. In the hypothetical we state that S is P, *if certain conditions are fulfilled.* The constituent parts of the categorical are related as subject and attribute: the parts of a hypothetical are related as reason and consequent. Nor is it only the mental forms that are different. The fact to be expressed positively demands one form to the

exclusion of the other. Such propositions as 'Gold is yellow,' and 'If the King comes, a salute will be fired,' are distorted when they are expressed as 'If anything is gold, it is yellow,' and 'The case of the King's arrival is a case of firing a salute.' In regard to the employment of the one form in place of the other, Professor Case has well said: "Taking the carelessly ex-'pressed propositions of ordinary life [logicians] do not perceive 'that similar propositions are often differently expressed, e.g. ''I being a man am mortal,' and 'If I am a man I am mortal': 'and conversely that different judgments are often similarly 'expressed. In ordinary life we may say 'All men are mortal,' '. . . 'All candidates arriving five minutes late are fined.' . . . 'But of these universal propositions, the first expresses a cate-'gorical belief . . . the other is a slipshod expression of the 'hypothetical belief, 'If any candidates arrive late, they are 'fined.'" *Encycl. Brit.* (10th ed.), vol. 30, p. 333, Art. Logic.

Quantity and Quality of Hypotheticals. All hypothetical propositions are affirmative. If we desire to meet a hypothetical with its negation, we must deny what it affirms. That is to say we must deny the nexus between the antecedent and consequent. This is done by the form 'Although S is M, it need not be P.' The negative of 'If he is poor, he is uneducated,' is 'Although he is poor, he may not be uneducated.' These negative forms, however, are not themselves hypotheticals: for they do not assert the dependence of consequent on antecedent.

There can be no differences of quantity in hypotheticals, because there is no question of extension. The affirmation, as we have seen, relates solely to the nexus between the two members of the proposition. Hence every hypothetical is singular.

§ 12. **Disjunctive Propositions.**

A Disjunctive Proposition is one which makes an alternative predication.

Disjunctives like Hypotheticals are of two forms: (1) Either A is B, or C is D; and (2) S is either P or Q, e.g. 'Either the general was incompetent or his subordinates were disobedient,' 'Religions are either false or true.'

It has been much disputed whether the alternatives in a disjunctive are mutually exclusive or not : in other words, whether we not only know that one must be true, but also that if the one is true, the other is certainly false. Thus supposing we are aware that '*S* is either *P* or *Q*,' and are then informed that '*S* is *P*,' can we conclude that *S* is not *Q*? We shall consider this point in a subsequent chapter.[1]

The Disjunctive can be expressed by means of Hypothetical propositions. If it be maintained that the disjunction is exclusive, we need two hypothetical propositions to represent a disjunctive, viz., (1) If *S* is *P*, it is not *Q*. (2) If *S* is not *P*, it is *Q*. If the mutual exclusiveness be denied, a single hypothetical will suffice. viz., 'If *S* is not *P*, it is *Q*.'

Quantity and Quality of Disjunctives. By virtue of their form all disjunctives are affirmative. The alternative is necessarily asserted. However, a difference in quantity is possible. The proposition may be of the form 'All *S* are *P* or *Q*'; or it may be particular, as, 'Some *S* are *P* or *Q*.'

A form of proposition termed by the Scholastics *Conjunctive* gives us what is practically the negative form of the Disjunctive. Its formula is '*S* is not both *P* and *Q*,' 'The King is not both at London and Windsor.'

The whole terminology of Conditionals is in confusion. We have followed that preferred by Hamilton (*Logic*, I. 236) and subsequently by several other authors. Some logicians make hypothetical the genus, and give the name conditional to those we have called hypothetical. This division is found in Whately and is accepted by Mill (I. 91). Perhaps the most satisfactory division is that of Boethius. He terms the genus *conditionalis* or *hypothetica* indifferently, and calls the species respectively *conjuncta* (*connexa*) and *disjuncta*.

[1] See below, Ch. 14, § 4.

CHAPTER IV.

THE LAWS OF THOUGHT.

§ 1. The Laws of Thought. In each science there are certain principles or laws, which are recognized as fundamental within that science. Every conclusion which it claims to have demonstrated, depends for its validity on the truth of those principles. Such for instance are the definitions of Euclid in regard to Geometry (the science of abstract spatial extension), and the laws of motion in regard to the science of Mechanics. In each case the principles have their own sphere of application. They are principles of this or that science, and beyond it they are not operative. There are, however, certain laws, which are not confined within the limits of any one of the special sciences, but which apply to all that *is*, to all that has a right to the name of Being or Thing. For instance the law of causality which lays down that *every event must have a cause*, is such a principle as this. It is not a law of one of the special sciences, but is true of all things. It belongs to that universal science of Metaphysics or Ontology, of which something has been said in Ch. 1, § 3.

Just as there are laws which apply to the whole realm of Being,—to the real order in its full extent,—so too there are laws which govern the whole of the conceptual order, and on which depends the validity of every judgment, whatever it may be. These are the Laws of Thought, which form the subject of the present chapter. They are three in number :—

(1) **The Law of Contradiction,** viz. : **Contradictory judgments** (e.g. *A* is *B*, *A* is not *B*) **cannot both be true.**

(2) **The Law of Identity,** viz. : **Everything is what it is.**

(3) **The Law of Excluded Middle,** viz. : **Of two contradictory judgments** (*A* is *B*, *A* is not *B*) **the one must be true, the other false.**

These three laws we shall proceed to consider in detail. But first, it will be well to ask ourselves in what sense they are termed laws. For the word 'law' is used in various senses. In its primary signification it means an ordinance imposed by a legitimate superior on the body politic, and carrying with it an obligation of obedience. But it is also employed to signify a uniform mode of acting observed by some natural agent. In this sense we use the term 'laws of nature,' e.g. the law of gravitation, the law that water under a certain pressure freezes at 32° F., etc. Laws of nature are only called laws by analogy : there is of course, no question here of the obedience which one will ought to yield to another. The law is simply our description of the way in which the agent does in fact act. It tells us what *is*, not what *ought to be*. In yet another meaning we use it to denote a norm or standard, to which we must conform in order to achieve some end. Thus we may speak of the laws of perspective. If we wish our drawing to be accurate, we must observe them. Otherwise, we shall not attain our object.

It is in this last sense that we employ the word, when we speak about the laws of thought. It is certainly the case that we are unable to judge a pair of contradictory propositions to be true, if we are conscious of the contradiction. But it not infrequently happens that men unconsciously hold opinions, which are really contradictory the one of the other, though because they are expressed in different words, or from some confusion of mind, their mutual opposition is not recognized. Hence the laws of thought cannot strictly be termed laws in the second of the senses we have noticed above. But since in all our mental judgments our end and object is to attain truth, they are rightly termed laws in the last

sense mentioned : for if they are not observed, our judgments are not true but false.

§ 2. **The Law of Contradiction.** The form in which we have given the principle of Contradiction, 'Contradictory judgments cannot both be true,' is that in which, with various slight modifications it is several times enunciated by Aristotle.[1] He, moreover, is careful to point out that where judgments are contradictory to each other, the predicate must be referred to the subject in the same way in each, and the point of time must be identical. "A refutation," he says, "occurs when something is both affirmed and denied 'of one and the same subject . . . and when it is denied 'in the identical respect, relation, manner and time, 'in which it has been affirmed." [2] It might be true to say both that 'the prime minister is capable,' and that 'the prime minister is not capable,' if the capacity referred to was in the one case capacity for government, in the other capacity for writing Greek verse : or if we were speaking of different periods in his life.

Mill adopts a more cumbrous phraseology. He words the law as follows : 'The affirmation of an assertion and the denial of its contradictory are logical equivalents, which it is allowable and indispensable to make use of as logically convertible" (*Exam. of Hamilton*, p. 414).

This law, as we have said, is a ruling principle of the whole conceptual order. It applies to all that *is thought*. But the order of thought—of conceptual Being—is essentially a representative order. It manifests the order of *things*. And this law of thought is the conceptual expression of a fundamental necessity of the real order : to the logical principle corresponds a metaphysical principle. This metaphysical law may be stated : "The same attribute cannot at one and the same time both belong and not belong to the same thing" (Arist. *Met.* III., c. 3, § 10).

[1] *Met.*, III., c. 6, § 10. ἀδύνατον τὴν ἀντίφασιν ἀληθεύεσθαι ἅμα κατὰ τοῦ αὐτοῦ.

[2] *Soph. Elenchi.* c. 5, § 5. ἔλεγχος μὲν γὰρ ἀντίφασις τοῦ αὐτοῦ καὶ ἑνος . . κατὰ ταὐτὸ καὶ πρὸς ταὐτὸ καὶ ὡσαύτως καὶ ἐν τῷ αὐτῷ χρόνῳ.

Another form in which it is frequently expressed, is : "It is impossible for the same thing both to be and not to be, at the same time." [1] How closely the logical principle represents the metaphysical will at once be seen, if we express the former as : "The same attribute cannot at one and the same time be both affirmed and denied of the same thing." But the student should be careful to distinguish the various expressions of the law, and when dealing with logical questions not to state the principle in a metaphysical form, nor vice versâ.

This law Aristotle declares to be the first of all axioms, and the most certain of all principles (*Met.* X., c. 5, § 1).

* The question will doubtless suggest itself, on what grounds this is asserted to be the first of all axioms. A brief examination will show us that the principle of Contradiction is the first Analytic proposition, which we attain through an analysis of our most primary notion—the notion of 'Being' or 'thing.'

This notion, which we apply equally to all entities whatever, calls for a brief consideration.

We are accustomed to name objects from their various determinations and perfections. We term one man a 'runner,' because the perfection denoted by the word 'to run,' characterizes him, and we call another a 'painter' for a similar reason. Further, we apply these denominatives to them, even though the perfection is not at the moment in a state of actualization. The man is called a 'runner' or a 'painter,' not because he is actually running or painting, but because he has the capacity to elicit these acts. 'Being' is a denominative of this type. It is applied to objects in virtue of that primary perfection signified by the verb 'to be,' as understood in the first of the senses mentioned in Ch. 3, § 2, namely 'to exist.' The notion which expresses this primary characteristic of 'Being' or 'actuality,' is clear to us from the dawn of our intelligence. It is absolutely simple. We cannot explain it by any that is simpler : for its simplicity is ultimate. Indeed were there not primary notions of this kind, it would be impossible to explain anything. The mind would be lost in an infinite regress, as it endeavoured to find some idea which did not itself need elucidation.

What then is the Analytic proposition which unfolds the intension of this term, which is the first principle to emerge from the consideration of our primary concept ? Its very simplicity

[1] It is to be observed that the principle of Contradiction is a modal proposition *de impossibili*, and the principle of Excluded Middle a modal *de necessario* (Ch. 3, § 9).

prohibits our explaining it otherwise than by declaring its difference from its opposite, viz. that it is essentially opposed to non-existence.[1] Yet we cannot state the principle as 'A Being is that which is not non-existent,' for as we have noticed, 'Being' is applied not merely to that which does at present exist, but to such objects of thought as we see *can* exist. A chiliagon may be termed a 'thing' or a 'Being.' Our proposition must be expressed, 'A Being *which is*, cannot at the same time not be'; or as otherwise phrased, 'It is impossible for the same thing both to be and not to be at the same time.' Here then we have the principle of Contradiction, as the first of principles derived by analysis from the primary notion.

In regard of each Being, however, we must consider not merely its existence, but its nature: that which makes it *what it is*. The principle may be enunciated not merely in reference to the former, but to the latter: for the nature of an entity determines the mode of its existence. As thus expressed, we get the form; 'The same attribute cannot at the same time both belong and not belong to the same thing.' The logical expression, as we have seen, is identical with this, save that it refers to the mental act by which we judge about the thing: 'The same attribute cannot at the same time be both affirmed and denied of the same thing.'

§ 3. **The Law of Identity.** This principle is often stated in the form $A = A$. This, however, is manifestly a formula, and not the enunciation of a philosophic principle. Locke (*Essay*, Bk. 4, c. 7) enunciates it as 'Whatever is, is,' and this form appears to be philosophically correct. Like the principle of Contradiction, this law is an Analytic proposition explicative of the concept of Being. Its connexion with that principle will appear plainly if we express it as 'A Being *which is*, is.' In this form we see that the only difference between the two is that in the one case we affirm that things which exist, exist: in the other, that things which exist, cannot not exist.

Like the principle of Contradiction also, it may be enunciated in reference to the nature, which determines the existence. Leibniz has given expression to the law

[1] On Being and Not-being as the primary concepts of the understanding, see St. Thomas, *Opusc.* 44, *Summa Totius Logicae*, Tract 3, c. 1, ¶ Ad videndum. Cf. *Summa Theol.* I., Q. 11, Art. 2. ad 4.

in this form. He words it 'Everything is what it is.' Leibniz's form will serve us also for the logical order, if it be understood as signifying that every subject of predication is what it is, i.e. that whatever attribute is affirmed of any subject, is in fact an attribute of that subject.

Mill somewhat unnecessarily introduces the question of verbal expression. He enunciates the law as : "Whatever is true in one form of words, is true in every other form of words, which conveys the same meaning" (*Exam. of Hamilton*, p. 409).

It is the universal practice at present to treat the principle of Identity separately from the principle of Contradiction. Scholastic authors, however, do not admit its claim to rank as a really independent principle. At most they admit that it is a rudimentary form of the principle of Contradiction.[1] They urge that the predicate of an Analytic proposition must in some way explicate the notion of the subject. This principle does not do so. The predicate and the subject are the same concept. It is mere tautology.

There is, it may be owned, some force in this objection. The principle tells us nothing. Yet we must remember that Being is a concept which does not admit of analysis properly so called. Hence perhaps justification may be found for a tautologous principle here, which could not be adduced in any other case. The form is permissible, because it is indicative of the fact, that we have arrived at the limits of all explanation. But in order for the principle to convey any information, and to be of any service, it must be developed into the law of Contradiction.

* The separate treatment of the two principles first became usual after the time of Leibniz. It is true that Parmenides the Eleatic (circa B.C. 490) had enunciated the principle *Being is* (ἐὸν ἔμμεναι) as the foundation of his philosophy. But Aristotle

[1] Cf. Pesch. *Instit. Logicae*, vol. 3, § 1230. Ad usum principii identitatis quod attinet, illud a Peripateticis nunquam in sua propria forma adhibitum videmus. Est enim vagum et indeterminatum, et principiorum potius radicem continet et germen imperfectum.

emphatically affirms that the law of Contradiction is the first of all principles: and his decision for long went undisputed. Among mediæval authors the Spanish Scotist Antonius Andreæ (ob. 1320) argues that the first place should belong to the principle 'Every Being is a Being' (*Omne Ens est Ens, Qq. in Met.* IV., Q. 4). But the authority both of St. Thomas (*Met.* IV., lect. 6) and of Scotus (*Quaest. sup. Met.* IV., Q. 3) was against him: and he is expressly refuted by Suarez (*Disp. Met.* III., § 3). Leibniz however makes the principle of Identity, which he gives as 'Everything is what it is,' the first of the primitive truths of reason which are affirmative, and the principle of Contradiction, 'A proposition is either true or false' the first of the negative truths (*Nouv. Ess.* IV., 2, § 1). He further says, "the statement that a thing is what it is, is prior to the statement that it is not another thing" (*Nouv. Ess.* IV., 7, § 9). Here as it would seem, is the real ground for the introduction of the principle of Identity as distinct from that of Contradiction. It appeared impossible that the primary analytic principle should be negative. If however, the view taken in the last section is accurate, the negative form is the necessary consequence of the primary character of the principle. We can only explain the perfectly simple by distinguishing it from that which it is not.

§ 4. **The Law of Excluded Middle.** Aristotle enunciates this principle in the form given above, "Of two contradictory judgments, the one must be true and the other false" (*Met.* III., c. 8, §§ 3, 4). He says also, "Between the two members of a contradiction, there is no middle term" (*Met.* III., c. 7, § 1).[1]

As a metaphysical principle, it is stated, 'A thing must either be or not be.' The truth of this is evident from the immediacy of the opposition between *being* and *not-being*. The truth of the logical principle is capable of demonstration as follows. Where we have two contradictories, we have affirmation and negation, *is* and *is not*. If the member which constitutes the mental judgment corresponds with the reality, whether it be in affirmation or negation, then the mind has attained truth. Should it, however, not be in conformity with its object, the judgment is false. That is to say, the mind has either

[1] ἀνάγκη γὰρ τῆς ἀντιφάσεως θάτερον εἶναι τὸ μόριον ἀληθές . . . θάτερον γὰρ μέρος τῆς ἀντιφάσεως ψεῦδός ἐστιν (III., c. 8). Ἀνάγκη μὲν οὖν μεταξὺ ἀντιφάσεως ἐνδέχεται εἶναι οὐδέν (III., c. 7): and see *Cat.*, c. 10, §§ 31–40.

judged that what is, is not, or that what is not, is. Wherever, therefore, the judgment is false, the contradictory judgment, whether it be the affirmative *is*, or the negative *is not*, will be true. Hence of two contradictories, the one must be true, the other false.[1]

The close connexion between the logical principle and the metaphysical at once appears, when we reflect, that in affirmation, we are attributing a certain conceptual being to the subject; in negation, we assert that it does not possess this being (Ch. 3, § 2). All contradictories therefore present the alternative between *being* and *not being*.

The way in which the principle is expressed by certain logicians, "Of any two contradictory predicates, one 'must belong to every subject," is unsatisfactory. It supposes that the *predicates*, and not the *propositions*, are contradictory to each other, and is represented by the formula, 'A is either B or not-B.' But, as we have seen, the primary form of negation is the negative judgment, not an affirmative judgment with a negative predicate; and in the expression of a fundamental law, it is the primary form that we need. Mill employs the following formula:—"It is allowable to substitute for the denial of either of two contradictory propositions, the assertion of the other" (*Exam. of Hamilton*, p. 416).

It should be carefully noted that the law of Excluded Middle is in no way concerned with Contrary terms.[2] We have explained Contrary terms as those which express the widest possible difference among classes belonging to the same genus, e.g. 'white, black,' 'convex, concave,' 'love, hatred.' There is, of course, a mean between terms such as these. Objects possessed of any

[1] Arist. *Met.* III., 7, § 1; St. Thomas, *in Met.* IV., lect. 16.

[2] Mr. Bosanquet's views as to negation lead him into this error. He holds that negation *quâ* negation is void of all significance, and that the true value of a negative judgment is to be sought in its positive content. Hence he concludes that the principle of Excluded Middle relates to contraries (*Logic*, II. 210). A full treatment of the import of the negative judgment and of the relation between contradictory and contrary propositions, will be found in St. Thomas, *Opusc.* 33, *De Quatuor Oppositis*, cc. 1, 2.

other variety of colour are neither white nor black; a plane surface is neither concave nor convex; and indifference is neither love nor hatred. It is manifest, however, that an object must be either white or not white, convex or not convex; and that in regard to any particular individual, it is either true that we do or that we do not feel love towards him.

The principle has not passed unchallenged. Mill, in the interests of the empiricist philosophy, declared the law to be a mere generalization from experience. We have no grounds, he thought, for regarding it as necessary. Indeed he goes further, and maintains that "it is not even true except with a large 'qualification. . . . 'Abracadabra is a second intention,' is neither 'true nor false. Between the true and false there is a third possi-'bility, the Unmeaning." Such an argument can scarcely be treated as serious. An unmeaning proposition is not a judgment at all. Of more moment perhaps is the Hegelian objection. The very basis of the Hegelian philosophy is the reconciliation of opposites. Becoming is supposed to owe its origin to the union of Being and Not-Being, and the whole of Nature is regarded as constituted by this dialectic development. Hegel himself argues against the principle of Excluded Middle by pointing out that between $+A$ and $-A$ lies A. As against this view, it is urged that in Hegel's system the opposites are in fact contraries not contradictories, and that the individual does not owe its origin to them, but that they are obtained by abstraction from the individual. Thus if it be urged that at dawn we can say with equal truth 'It is day' and 'It is not day,' and that the state of dawn is constituted by these opposites, it is answered that the two moments are not, as alleged, contradictory opposites, but the contraries 'dark' and 'light': and that dawn is not in any sense constituted by a dialectic development out of darkness and light, though we can mentally abstract these concepts from the state of dawn.

* § 5. **Other Views as to the Source of the Laws of Thought.** In other schools of philosophy, very various accounts are given as to the nature and origin of these laws. It seems well to notice here three theories differing widely from that set forth in the preceding sections. These views respectively regard the laws of thought (1) as subjective laws of the understanding, of whose objective validity, however, we can have no rational guarantee, (2) as principles determining the growth of that 'experience,' which men erroneously distinguish into thought on

the one hand, and things on the other hand, (3) as mere generalizations from experience.

The first view is that of Kant. Among English logicians it is explicitly taught by Mansel. He tells us that the principles of thought are "laws under which the mind is compelled to 'think, and which it cannot transgress, otherwise than negatively by ceasing to think at all." "It may be," he adds, "that the conditions of possible thought correspond to conditions of possible being, that what is to us inconceivable is 'in itself non-existent. But of this, from the nature of the 'case, it is impossible to have any evidence" (*Prolog. Logica*, 71, 72). It is needless to point out that such a view as this leads directly to philosophic scepticism.

The second view is represented by Mr. Bosanquet. We have already (Ch. 1, *note* (7)) called attention to the theory held by the neo-hegelian school of logicians, according to which the operations of the mind are vital functions by which the so-called 'real' world has been constituted. It is under this aspect Mr. Bradley regards the laws of thought. He holds that we cannot say that these principles are merely laws of thought, if by that we mean thought as distinguished from things: "Since a separation between intelligence and experience is purely fictitious, 'there is nothing to be gained by cutting down the content of 'these principles to a minimum in the hope of restricting their 'reference to thought as opposed to things." They are "the animating principles of growth" which govern the development of experience: and if we recognize them as "postulates of knowledge," this is because "on analysis of experience, they are 'found to be active factors in it from the first" (*Logic*, II. 205–207).

Mill (*Logic*, I., p. 308. *Exam. of Hamilton*, p. 417) explicitly repudiates the view that these principles are subjective laws of the thinking faculty. He holds that they are conclusions derived from a constant experience of their truth. We have never as a matter of fact known any case where two contradictories have been simultaneously true. Hence we rightly lay down a general but empirically discovered principle to that effect. As for the law of Excluded Middle, we have already seen that he holds that it needs qualification before it can be admitted as universally true. It must, he admits, be owned that "we cannot 'now conceive the opposite of these laws to be true. But this 'inconceivability is of little value as a criterion of truth to 'those who know how artificial, modifiable, the creatures of 'circumstances and alterable by circumstances, most of the sup'posed necessities of thought are." Constant experience of one character is, in his view, sufficient to lead us to regard its opposite as inconceivable.

CHAPTER V.

DIAGRAMMATIC REPRESENTATION OF PROPOSITIONS: OPPOSITION OF PROPOSITIONS.

§ 1. **Diagrammatic Representation of Propositions: Euler's Circles.** It might well seem that the task of representing by a diagram the mental act of judgment, necessarily involved an impossibility. Take, for instance, a simple proposition such as, 'The orange is round.' How, it may be asked, can there be any material representation of so essentially a psychical act as the abstraction of 'roundness' from the concrete object? Or how can any figure express a universal concept? If indeed we proposed to represent the proposition as it is mentally conceived, these objections would be unanswerable. But we may, if we will, consider not the mental judgment, but, what is a very different thing, the extension of the terms constituting the judgment. In extension we have to do not with the conceptual order, but with the real: and hence diagrammatic representation ceases to be intrinsically impossible. If the terms be thus understood, the proposition is viewed not as expressing a relation of attribute to subject, but as expressing a relation between two classes. The proposition 'All men are mortal,' will be looked on as a statement of the relation that exists between the class 'man' and the class 'mortal things.' This is, of course, a purely artificial method of treating the proposition. But the student will find it, in many respects, helpful, provided that he bears in mind that the diagrams exhibiting the relations of the classes, represent *things* and not thoughts.

The most usual method employed, is that entitled

Euler's Circles. It was devised by the Swiss logician Euler (1707–1803). In it *S* and *P* are represented each by a circle, the circle standing for the collection of objects denoted by the term. By the aid of these circles, the various relations compatible with each of the four propositions *A.E.I.O* are all of them capable of diagrammatic expression.

FIG. 1.

FIG. 2.

A propositions. The proposition *S a P* will be represented either by Fig. 1 or by Fig. 2. The class signified by the subject may fall within the extension of the predicate term, as in 'All men are mortal,' 'All triangles are plane figures.' In this case, Fig. 1 accurately represents the relation of the two classes. Or the extension of the two terms may be identical, as in 'All men are rational animals'; 'All equilateral triangles are equiangular.' In this case, the two circles must be coincident, and Fig. 2 is representative of the proposition.

FIG. 3.

E propositions. In every proposition of the form *S e P*, we deny that the two classes have any members

in common. The one class is declared to fall altogether outside the other, e.g. 'No fishes have lungs,' 'No scalene triangles are equiangular.' Hence these propositions are represented by Fig. 3.

I propositions. Figs. 4 and 5 show us the two cases of the proposition *S i P*, in which it is some and not all

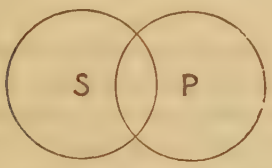


FIG. 4.

FIG. 5.

of the class *S*, which fall within the class *P*. Fig. 4 represents the case in which there exist other members of the class *P* besides those which are found within the extension of *S*. 'Some men are black' will serve as an example. There are many objects belonging to the class 'black,' which are not men. Fig. 5 stands for the case in which there are no members of the class *P*, save those which are also *S*. 'Some elements are metals,' 'Some men are Aryans,' are instances of such propositions.

But *S i P* does not exclude the truth of *S a P*: for, as will be remembered, 'some' in a particular proposition is understood to mean, 'some, it may be all.' It is true to say, 'Some equilateral triangles are equiangular,' even though every equilateral triangle be such. The mere form of the proposition does not tell us which relation in fact exists between the two classes denoted by the terms. Hence not merely Fig. 4 and Fig. 5, but also Fig. 1 and Fig. 2 may be the relation to which the **I** proposition refers. When this proposition is used, there are four possible cases.

O propositions. In propositions of the form *S o P*, we assert that some of the objects denoted by the object

term, fall outside the extension of the predicate. When it is the case that while some are outside, some also fall within the extension of P, then the $S\ o\ P$ proposition will, like $S\ i\ P$, be represented by Figs. 4 and 5. Fig. 4 represents the case, in which P not only has some members within the class S, but also some outside it, e.g. 'Some men are not black.' Fig. 5 gives us the class in which P is a class falling entirely within S, e.g. 'Some men are not Aryans.'

But here too, the form of the proposition is compatible with the case when the classes are mutually exclusive, and when they have no members in common, e.g. "Some scalene triangles are not equiangular." Where it is thus employed, the classes will stand in the relation represented by Fig. 3. For the $S\ o\ P$ proposition there are, therefore, three possible cases.

For a diagrammatic scheme to be thoroughly satisfactory, it is usually stated that three conditions must be fulfilled :—

(1) The relation expressed by the diagram must be *evident*, as soon as the principle of representation is understood.

(2) Each diagram must represent *one relation* and only one.

(3) Each proposition in the schedule of propositions, must be represented by *one diagram* alone.

Euler's circles have been found fault with on the ground that they fail to fulfil this condition. The reason of this failure is easy to understand. Diagrammatic representation is only possible in regard to extension. The true function of the proposition is not however to express the extension of the terms, but the inherence of an attribute in a subject. There are five possible relations between the extension of two classes, and there are but four fundamental propositions. Hence it would be idle to look for a perfect system of representation.

Other systems of representation have been suggested by various writers on Logic : but by the nature of the case none of them is really satisfactory. Two of these

methods we shall describe. But first it is necessary to deal with the distribution of terms in a proposition,—a point that will be found to have important bearings on several questions in Logic.

§ 2. **Distribution of Terms in a Proposition.** A term is said to be **distributed** in a proposition, when from the form of the proposition we know that it must be taken in its whole extension. When the form of the proposition is not such as to indicate that the whole extension is referred to, it is said to be **undistributed.**

If we consider the affirmative propositions *A* and *I*, we shall see that neither of them distributes the predicate. When we say, ' All equilateral triangles are equiangular,' it is true that the predicate ' equiangular ' refers here to all equiangular triangles ; for there are no equiangular triangles besides those that are equilateral. But we do not know this from the form of the proposition. The judgment ' All crocodiles are amphibious,' is of precisely the same form, and here the predicate does not refer to *all* amphibious creatures, but only to those which are crocodiles.

The same reasoning holds good in the case of the proposition *I*. We cannot tell from the form of the proposition, whether the whole extension of the term employed in the predicate is referred to or not. In the proposition ' Some flowers are fragrant,' the predicate is not taken in its full extension. In the proposition ' Some mammals are whales,' every member of the class ' whale ' falls within the reference of the term : for whales are a small class contained within mammals. But we could not discover this from the form of the proposition.

In the negative propositions *E* and *O*, the reverse is the case. The subject of a proposition is excluded from the whole extension of the class signified by the predicate term. Thus if I say, ' No fishes are amphibious,' I exclude the subject ' fishes,' from the whole class of amphibious creatures. If I say, ' Some European nations are not Aryan,' I exclude the European races of which I

speak in the subject from the whole extension of the class denominated 'Aryan.'

As regards the subject it is plain that it is distributed in all universal propositions, and left undistributed in all particular propositions. Universal propositions, of their very nature, refer to the whole class signified by the subject term.

Hence we have the following rules, which should be carefully noted :—

Universal propositions (*A.E*) **distribute their subject.**

Negative propositions (*E.O*) **distribute their predicate.**

It will be noticed that *E* distributes both its terms, while on the other hand *I* distributes neither subject nor predicate.

* § 3. **Other Methods of Diagrammatic Representation.** Other ways of representing the proposition by means of diagrams have been suggested by various logicians. To two of these systems we call attention here. They are (1) that proposed by the German logician Lambert (1728–1777), and (2) that of which Dr. Venn makes use in his *Symbolic Logic*.

(1) *Lambert's method.* Lambert employed a system in which two horizontal straight lines, one above the other, represent the terms of the proposition. The lower represents the subject, the higher the predicate. Should the term be *undistributed*, the line is partly continuous and partly dotted. If it be *distributed*, there is no dotted portion. When the proposition is affirmative, the continuous part of the predicate line is immediately above the continuous part of the subject line : when it is negative the continuous portions are not one above the other, but are separated by a small interval. The forms of the four propositions, will thus be :—

The figure here given for I is not that suggested by Lambert himself, but one proposed in lieu of it by Dr. Venn. Lambert's own figure is which, as Dr. Venn observes, "might consistently be interpreted to cover the case of 'No *S* is *P*,' as well as suggesting the possibility of there being no *S* at all" (*Symbolic Logic*, p. 431).

In regard to this system, it may be said that it certainly ful-

fils the requirements of having but one diagram for each proposition, and of being based on a simple and clear principle of representation. Yet all will feel that it gives but little assistance. The value of diagrams lies in the fact that they are able to represent to us the relation between the extension of the classes denoted by the terms, a concrete point of view that often assists the beginner. The same form of proposition, however, is, as we have seen, significative of a variety of such relations. Hence if we employ but one figure, it must necessarily be ambiguous. As soon as we seek to represent the undistributed term, we have to confess our ignorance of what the relation in extension really is. We do not know where the line should stop. The chief utility of the figure is thus lost.

(2) *Dr. Venn's method.* The system employed by Dr. Venn is only available for the representation of universal propositions. Circles are employed in this scheme, but the use made of them is quite different to what we have seen in the Eulerian diagrams. The primary diagram consists of two intersecting circles x and y. This provides us with four compartments, viz., x which is y (xy), x which is not y $(x\bar{y})$, y which is not x $(\bar{x}y)$, and things which are neither x nor y $(\bar{x}\bar{y})$. This last compartment is sufficiently represented by the blank space outside the circles.

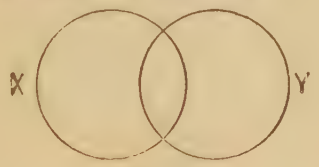


FIG. 1.

The method now proceeds on Dr. Venn's view that all universal propositions may be adequately represented by denying the existence of certain classes. Thus, 'All x is y' is adequately represented by the denial that there is any x which is not y: 'No x is y' denies that there is any x which is y: 'All x is all y' denies (*a*) that there is any x not y, and (*b*) that there is any y not x. The representation of the propositions is effected by shading out the compartment, whose existence is denied. The following diagrams represent the propositions just mentioned :—

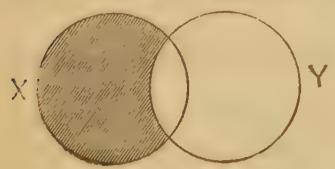


FIG. 2.

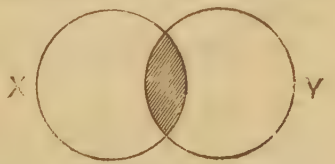


FIG. 3.

Fig. 2. 'All x is y' excludes $x\bar{y}$. Fig. 3. 'No x is y' excludes xy. Fig. 4. 'All x is all y' excludes $x\bar{y}$ and $\bar{x}y$. The system

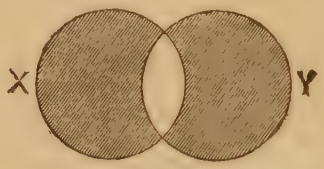


Fig. 4.

lends itself to the representation of universal propositions with more than 2 terms, e.g. "All x is y or z." In this case, we have to introduce a new circle intersecting all the compartments already existing, and thus doubling their number: for the use of 3 terms will entail a need of eight compartments, viz., xyz, $xy\bar{z}$ $x\bar{y}z$, $\bar{x}yz$, $x\bar{y}\bar{z}$. $\bar{x}\bar{y}z$, $\bar{x}y\bar{z}$, $\bar{x}\bar{y}\bar{z}$. 'All x is y or z' denies the existence of the compartment $x\bar{y}\bar{z}$ (Fig. 5).

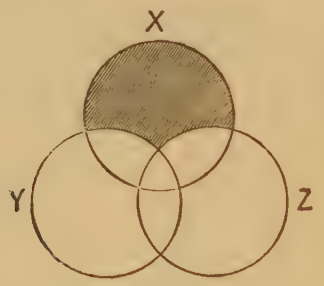


Fig. 5.

The utility of the system is materially diminished by its being applicable solely to universal propositions. Moreover, the view that we can obtain an adequate representation of affirmative universals by denying the existence of classes, is one to which, as we shall show later (Ch. 7, § 5), serious objections may be urged.

§ 4. The Opposition of Propositions.

The Opposition of Propositions is the relation which exists between propositions having the same subject and predicate, but differing in quantity, or in quality, or both.

The various differences give rise to four kinds of opposition.

The difference may be in both quality and quantity. In this case it must be either between an *A* proposition and an *O* proposition, or else between an *E* proposition

and an *I* proposition. Opposition of this character is styled **contradictory.**

The difference may be in quality alone. Here two cases arise; for the propositions in question may be either universal or particular. The opposition between two universal propositions differing in quality (*A* and *E*), is termed **contrary**. That between two particular propositions, which differ in quality (*I* and *O*), is termed **subcontrary.**

Where two propositions differ in quantity only, the opposition between them is said to be **subaltern.** Thus there is said to be subaltern opposition between *A* and *I* and between *E* and *O*. Of these the universal proposition is termed the *subalternant,* and the particular the *subalternate.*

It is plain that this last relation is merely termed opposition by a conventional usage of that word. A universal and a particular of the same quality, can hardly be said to be opposed in the natural sense of the term, for the truth of the universal carries with it the truth of the particular. When *A* is true, *I* must be true. Indeed even in regard to sub-contrary opposition, Aristotle says that it is merely verbal: for the truth of one is compatible with the truth of the other, e.g. 'Some men are just,' 'Some men are not just.'

The relations just explained are set out in the figure commonly called the *square of opposition.*

We shall now proceed to examine the various kinds of opposition in detail :—

(1) *Contradictory Opposition.* Of two contradictory propositions, **the one must be true, the other must be false.** For example, the two propositions, ' All crows are black,' ' Some crows are not black ', can neither be both true, nor both false. This will be easily seen by the aid of Euler's circles. The *A* proposition is represented by Fig. 1 or Fig. 2 in which no part of the circle *S* falls outside the limits of the circle *P*. In the *O* proposition, either a part or the whole of *S* falls outside *P*. It is plain that *S* must either be such that no part is outside *P*, or such that part is outside. Hence either *A* or *O* must be true. On the other hand it cannot be such that it both has no portion outside *P*, and at the same time, some portion outside *P*. Hence either *A* or *O* must be false.

Contradictory opposition is the only form, of which it is the case that if one of the propositions is true, the other is false, and if one is false the other is true. Hence it is said to be the most perfect form of opposition. The characteristic to which we have just adverted, gives to this kind of opposition a special importance in controversy, to which we shall refer in the following section.

The rule of contradictory opposition is easily proved from the laws of thought. In every case of contradiction, we have judgments which are opposed as *A is B*, *A is not B*. Thus in the judgments, ' All crows are black,' ' Some crows are not black,' the negative proposition denies that the attribute ' black ' belongs to each and all of those individuals, of which it has been asserted : *B* is asserted and denied of one and the same *A*. But the law of Contradiction has shewn us that where we have two propositions of this form, one must be false : and the law of Excluded Middle has taught us that one must be true.

(2) *Contrary Opposition.* Propositions which are opposed as contraries, **cannot both be true, but may both be false.** Thus it cannot be at the same time true that ' All metals are heavier than water,' and that ' No metals are heavier than water.' For if it be true that there are

no metals heavier than water, then we may most certainly assert the proposition: 'Some metals are not heavier than water.' This, however, is the contradictory of 'All metals are heavier than water.' Hence the judgment, 'No metals are heavier than water,' excludes the truth of the judgment, 'All metals are heavier than water.'

It is possible for each of two contraries to be false. For to assert that both can be false together, is simply to assert that their two contradictories can be simultaneously true: that is, that the propositions *I* and *O* can be true together. But we have already seen that this can be the case. It may be alike true that, 'Some men are black,' and that, 'Some men are not black.'

A consideration of Euler's diagrams will confirm the accuracy of these conclusions. The *A* proposition (Figs. 1 or 2) shews us the circle *S* with no part of it outside *P*: the *E* proposition shews us the two circles entirely separated. These cannot both be the case at the same time. But it may happen that neither is the case. For *S* may be partly within, and partly without *P*.

(3) *Sub-contrary Opposition.* Propositions, which are opposed as sub-contraries, **cannot both be false, but may both be true.** They cannot both be false; for in that case, their two contradictories, viz., *A* and *E*, would both be true. And this we have shewn to be impossible. They may both be true. For, as has just been said, there is no impossibility involved in asserting both, 'Some men are black,' and, 'Some men are not black.' For this reason the older logicians refused the name of opposition both to subcontrary and subaltern propositions (*Summa Totius Logicae*, Tract 6, c. 8).

Here again Euler's circles serve us as illustrations. The *I* proposition is represented by Figs. 1, 2, 4, and 5: the *O* proposition by Figs. 3, 4 and 5. Hence Figs. 4 and 5 represent cases where *I* and *O* are simultaneously true. But if *I* is false, the contradictory proposition *E* must be true: and this being the case, Fig. 3 alone will represent the relation of the terms. Similarly if *O* is false, the contradictory *A* must be true: and the relation of

the terms must be represented by Figs. 1 or 2. But Fig. 3 is incompatible with either of these.

(4) *Subaltern Opposition.* In regard to subaltern propositions, **the truth of the universal involves the truth of the particular, but not vice versa; and the falsity of the particular involves the falsity of the universal, but not vice versa.** The truth of these rules is easily seen. If *A* is true, then *I* is certainly true. It is commonly said that the truth of *I* follows from that of *A* by the law of Identity. It would perhaps be more accurate to say that it needs no proof, as the same attribute is asserted of individuals, of which it has already been affirmed.

It is, however, clear that the truth of *I* does not involve the truth of *A*. Some men may be black, and yet it may not be the case that all men are so.

The falsity of the particular involves the truth of its contradictory, e.g. If *I* is false, *E* is true. But the contradictory of the subaltern is the contrary of the subalternant. If then it be admitted as true, the universal subalternant proposition is false, e.g., *E* being admitted as true, *A* must be false. But if it be the universal which be false, and the contradictory particular be true, it does not follow that the subaltern of the universal is false. For two sub-contraries may be true together.

The explanations already given of the use, which may be made of Euler's circles, have probably been sufficient to enable the student to notice how these rules also are illustrated by them.

§ 5. **Opposition as a Means of Inference.** We have hitherto been treating of opposition as a relation existing between two propositions. But it will not have escaped notice that it also provides us with a means, by which we may pass from a statement as to the truth or the falsity of a given proposition to a statement as to the truth or falsity of other propositions related to the first by the various kinds of opposition. The following table from Mansel (*Aldrich*, p. 52) enumerates the inferences, which we may thus draw :—

1. If *A* is true ; *O* is false, *E* false, *I* true.
2. If *A* is false ; *O* is true, *E* unknown, *I* unknown.
3. If *E* is true ; *I* is false, *A* false, *O* true.
4. If *E* is false ; *I* is true, *A* unknown, *O* unknown.
5. If *I* is true ; *E* is false, *A* unknown, *O* unknown.
6. If *I* is false ; *E* is true, *O* true, *A* false.
7. If *O* is true ; *A* is false, *E* unknown, *I* unknown.
8. If *O* is false ; *A* is true, *I* true, *E* false.

Thus from the truth of a universal (Nos. 1 and 3), or from the falsehood of a particular (Nos. 6 and 8) we may infer the quality of all the opposed propositions : but from the falsehood of a universal (Nos. 2 and 4) and from the truth of a particular, we can only infer the quality of its contradictory.

Since we argue directly from the truth of *O* to the falsehood of *A*, contradictory opposition acquires primary importance in controversy. By the production of a single negative instance, a whole general statement is at once confuted. The generalization is destroyed, not by showing that all its parts, but that one alone, is untrue.

§ 6. **Contradictory Opposition outside the Fourfold Scheme.** We have seen that it is characteristic of contradictory propositions that *the one must be false, the other true.* This is often assumed as the general definition of contradictories : and in that case it is possible to find a contradictory to every proposition, and not simply to those which can be placed in a square of opposition.

Exponible propositions are contradicted by a disjunctive asserting that at least one or other of the exponents is false. Thus the contradictory of ' All the crew save one were drowned,' is ' Either the remainder of the crew were not drowned, or the one in question was not saved.' It is plain that unless a disjunctive is used, it is possible for both propositions to be false. Thus, supposing the proposition to have been denied in the form, ' All the crew save one were not drowned,' both the original assertion and the negation would be untrue, if the facts were that not even one of the crew was saved.

The contradiction of propositions containing a numerical assertion, depends on how they are to be understood. When they are understood as conveying an exact computation, a disjunctive must be used. Thus the contradictory of the proposition, 'Fifteen candidates are presenting themselves,' will take the form, 'Either more or fewer than fifteen candidates are presenting themselves.'

Sometimes numerical statements must be understood in the sense to signify that the proposition is true of *at least* that amount: and then the disjunctive form is not required. For instance the assertion 'The age of Henry Jenkins of Swaledale was 169 years,' is contradicted by, The age of Henry Jenkins of Swaledale was less than 169 years.'

Any proposition, however complicated, may be contradicted by prefixing 'It is false that.' This method of contradiction may, for instance, be usefully employed if we are called on to deal with such a sentence as the following: "This policy, being foolish and ill-considered, could not, under any circumstances, have succeeded."

Opposition of Singular Propositions. In Ch. 3 we called attention to the fact that although the Singular proposition is frequently reckoned as a special type of the Universal judgment, yet strictly speaking it stands by itself and is neither universal nor particular. This appears plainly when it is considered in relation to our present subject. In the square of opposition it has no place. If the original terms be retained, it admits of one kind of opposition alone, viz., a contradictory. 'Caesar was killed on the Ides of March,' 'Caesar was not killed on the Ides of March,' are contradictories in the sense explained in the present section. One of the two propositions must be true, the other false.

§ 7. **Contrary Opposition outside the Fourfold Scheme.** The extended signification given to the term 'contradictory' necessitates a corresponding extension in the employment of the term 'contrary.' A contradictory, as just explained, is a proposition asserting just sufficient

to render the original statement false. A contrary is accordingly understood as a negative which goes further than this, denying more than is absolutely necessary to destroy the proposition it rebuts. Contraries thus explained, possess the property which characterizes the contraries of the square of opposition, *that both may be false, but both cannot be true.* But it will be seen that the sense in which the term is used, has been very materially altered. Where the term ' contrary ' signifies the opposition between *E* and *A*, a contrary proposition is one which denies the truth of every case embraced by the opposed judgment. In the secondary signification with which we are now concerned, this sense is altogether absent. Any form of denial, which goes further than what is barely sufficient, is reckoned as a contrary. Thus, where a disjunctive is needed as a contradictory, the denial of either one of the members taken singly, is counted as a contrary. For instance the proposition, " All householders, and they alone, are voters," receives contrary denial either by the proposition, ' Some householders are not voters,' or by ' Some others besides householders are voters.'

CHAPTER VI.

IMMEDIATE INFERENCE.

§ 1. **Immediate Inference.** Immediate Inference may be defined as **a process by which from a single given judgment, we derive another, whose truth is implied in the former.** It thus differs essentially from syllogistic inference (Ch. 1, § 2), in which the mind passes from *two* truths already known to a third truth distinct from either of them. In the syllogism there is a real advance in knowledge. The conclusion is not formally contained in either premiss taken separately. Here however, we have no advance in knowledge. The conclusion of an immediate inference declares some fact already formally asserted in the premiss. If we have understood the premiss, we already know the conclusion. On this ground the name of Inference is sometimes denied to Immediate Inferences. Mr. Bradley even says: "As long as you keep to categorical affirmatives, 'your conversion or opposition is not rational but 'simply grammatical" (*Principles*, p. 392). The question as to whether these processes have a right to the name of Inference, must depend on the precise meaning we attach to that term. If it is understood to signify an advance in knowledge,—and this would appear to be the more correct sense,—it is wrongly applied to them. If however it includes any *mental transition* from a datum to something involved in that datum, its use here is justified. For it is certainly erroneous to say that the change is purely grammatical. Take, for example, so simple an Immediate Inference as that by which we pass from 'Some men are not wise,' to 'Some men are not-wise.' The change is

not a mere question of grammar, but a question of conceptual expression. In the predicate of the consequent, in lieu of the positive concept ' wise,' we employ a negative concept, i.e. a totally different mental form (Ch. 2, § 9) ; and the new judgment, instead of separating subject and predicate by negation, conjoins them by affirmation. In all Immediate Inferences the changes are of a similar character : the same truth is enunciated, but the conceptual expression undergoes transformation. Thus we are justified in asserting in our definition that it is a process by which we pass from one judgment to another.

By some authors, Opposition is treated under Immediate Inference, since, as was said in Ch. 5, § 5, a knowledge of what is involved in Opposition enables us to draw certain immediate inferences. Here we have followed the more usual plan of giving separate treatment to the subject of Opposition, as being concerned primarily not with inference, but with the system of relations which hold between the four propositions of the square.

The inferences of which we are about to treat are (1) Conversion, (2) Equipollence or Obversion, (3) Contraposition, (4) Inversion, (5) Some forms of minor importance. In several recent English works these five kinds of Immediate Inference are contrasted with Opposition under the name of *Eductions.*

§ 2. **Conversion.**

Conversion is a process of immediate inference in which from a given judgment we infer another, having the predicate of the given judgment for its subject, and its subject for predicate. Thus we pass by conversion from a proposition of the form *S is P* to one of the form *P is S.* The original proposition is known as the *Convertend* : the inferred proposition as the *Converse.*

The philosophical justification of this transposition is to be found in the significance of the logical judgment, as explained in Ch. 3, §§ 1, 2. We there saw that the copula denotes the identity of the subject and predicate, as being two mental ex-

pressions of one and the same reality. Of these two mental expressions, the subject is understood as signifying the thing, the predicate as the thing qualified by some determining characteristic—some 'form of being',—which we assert of the thing. Manifestly when the subject is a significant term, it, no less than the predicate, contains some determining characteristic. If e.g. we say, 'This bronze object is spherical,' the term 'bronze,' no less than the term 'spherical,' expresses an attribute belonging to the thing. Hence we may say that the proposition declares the coinherence in the same subject of the two 'forms of being' expressed by the terms, though the form signified by the subject is assumed, that signified by the predicate is asserted. This coinherence of forms may, as regards truth, be expressed in either order indifferently. In Logic we are concerned with things *as known to us*, and our knowledge in regard to the forms may take either order. It is as true to say, 'That running thing is a dog,' as it is to say, 'That dog is running.' It follows that we may legitimately transpose the order of the terms, and may replace a proposition of the type *S is P*, by one of the type *P i S*, provided we are careful that the converse should not assert the coinherence of *S* and *P* in relation to any entity which was outside the scope of the original proposition.

The following rules are given to ensure the validity of conversion :—

(1) The converse must be of the same quality—affirmative or negative—as the convertend.

(2) No term may be distributed in the converse, which was not distributed in the convertend.

The reason of Rule 1 is obvious. In every affirmative proposition the two terms are different mental expressions of the same object or objects. The inferred proposition in a conversion, refers to the identical object to which the premiss referred, and employs the same terms. It is then manifestly impossible that it should be negative: for a negative proposition asserts that one of the terms is inadmissible as a mental expression of the thing in question. In the same way it is equally impossible that a negative proposition should, after conversion, appear as an affirmative.

As regards Rule 2, we have already pointed out (Ch. 5, § 2) that where a term is undistributed, there is nothing in the form of the proposition to show that it is to be

understood of the whole class of objects belonging to the extension of the term. If then, after conversion, a term previously undistributed is distributed, more is asserted in the converse than was contained in the convertend. If we add anything, we are going beyond the point to which our warrant extends. Even if what we say be true, we shall not have drawn it from the original proposition. It will not be an inference. Examples of this will appear in the case of the *A* proposition, which we are now about to discuss.

(a) **Conversion of A propositions.** In *A* propositions the predicate is undistributed. Hence we cannot convert *All S is P* into the form *All P is S*. To do so would be to violate the rule which we have just explained. A glance at Fig. 1 of Euler's circles, will shew us the fallacy which would be contained in this. Such a conversion might well involve the assertion

that the larger class lay within the smaller. Because we can say, 'All men are animals,' it does not follow that, 'All animals are men.' We must therefore be careful to keep *P* undistributed in the converse, which will thus appear as *Some P is S*, 'Some animals are men.' This kind of conversion is known as *Conversio per accidens* or sometimes as *conversion by limitation.* It is true that in certain cases the conversion of *All S is P* to the form *All P is S* may give us a true result. Thus it is not only true that "All right-angled triangles 'have the square on the hypotenuse equal to the sum 'of the squares on the two sides containing the right 'angle," but the converse proposition may be asserted in the form *All P is S*. This however does not appear from the form of the original proposition, and cannot be inferred from it. We learn it from other sources.

(b) **Conversion of E and I propositions.** In *E* and *I* propositions, there is no need to change the quantity of the proposition, which we convert. *No S is P* becomes *No P is S*, and *Some S is P* is converted to the form *Some P is S*. Conversion of this character is known as *simple conversion*. In converting these propositions, in this way no rule is violated. The proposition *S e P* distributes both its terms. Hence the converse *P e S* does not distribute what was undistributed in the convertend. Similarly *S i P* has both subject and predicate undistributed; and the same is true of its converse, *P i S*. Euler's circles will assure us as to the correctness of the process. *S e P* is represented by Fig. 3. Thus if we say 'No plant

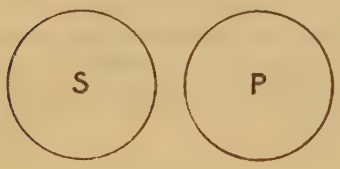


Fig. 3.

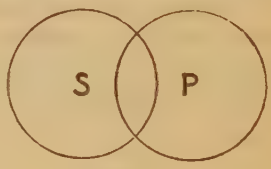


Fig. 4.

Fig. 5.

is capable of sensation,' we exclude one class altogether from the other. Hence the converse also must deny any union between them. It will be 'No being capable of sensation is a plant.' *S i P* is represented by Figs. 4 and 5. It is manifest that in either case we may with perfect truth frame a proposition of the form *P i S*. If we have affirmed, e.g. 'Some Europeans are Englishmen' (Fig. 5), we may also say, 'Some Englishmen are Europeans.' Should, however, *S i P* be employed where *S a P* might have been used, there too, as we saw in discussing the *A* proposition, we may legitimately convert to *P i S*.

(c) **The O proposition and conversion.** The *O* proposition does not admit of conversion consistently with the observance of the rules. The proposition *S o P* is particular negative : as such its subject is undistributed, its predicate distributed. Conversion would necessitate that the proposition should be of the form *P o S*. But in *P o S*, the predicate *S* must be distributed. This is impossible, as it is undistributed in the convertend. Fig. 5 will render this clear. Because we have said 'Some Europeans are not Englishmen,' we cannot conclude, 'Some Englishmen are not Europeans.' The place of conversion in regard to this proposition, is filled, as will be seen, by Contraposition.

(d) **Conversion of Singular propositions.** Only those singular propositions in which the predicate as well as the subject is a singular term, are capable of conversion. In all others the transposition of the terms alters the grammatical form, but not the logical. We may express the sentence 'Plato was a philosopher,' as 'One of the philosophers is (*was*) Plato' ; but Plato is still the subject —that of which something is said. On the other hand where both terms are singular, either of the two may legitimately take the place of subject, e.g. 'The man coming towards us is Socrates.' Such propositions are converted simply.

It will not have escaped notice that when it is desired to convert a proposition, it is absolutely necessary first to express it in full logical form, with subject, copula and predicate, and with the sign of quantity prefixed. Otherwise mistakes are bound to occur : and a proposition, such as, e.g. 'The industrious usually become wealthy,' will be converted to the form 'The wealthy usually become industrious.' The proposition should first be expressed as 'Some (*most*) of the industrious are men who become wealthy' ; and the converse will then appear correctly as : 'Some men who become wealthy are industrious.'

* § 3. **Aristotle's Proof of Conversion.** The following proofs to establish the validity of the process of conversion, are given by Aristotle in *An. Prior I.*, c. 2, §§ 2, 3.

(1) *E propositions.* If *No S is P*, then it will follow that *No*

P is S : for if not, let it be supposed that something of which *P* can be predicated, say *C*, is *S*. Then since *C* is both *S* and *P*, it follows that *Some S is P*. This however is the direct contradictory of our datum; therefore it, and the supposition on which it is based, are false.

(2) *A propositions.* If *All S is P*, it will follow that *Some P is S*. For if it be supposed that this is false, then *No P is S*. But we have already shewn that when *No P is S*, it follows that *No S is P*. But this result is incompatible with our datum that *All S is P*.

(3) *I propositions.* If *Some S is P*, it will follow that *Some P is S*. For if not, then *No P is S*. But this involves that *No S is P*.

It is manifest that every step in this proof rests upon the Laws of Thought.

§ 4. Equipollence or Obversion.[1]

Equipollence or Obversion is a process of immediate inference in which the inferred judgment, while retaining the original subject, has for its predicate the contradictory of the original predicate. The original proposition is termed the Obvertend, the inferred proposition the Obverse. It is evident that as in the Obverse we have substituted for the original predicate its contradictory term, we must also change the quality of the proposition : thus, we pass from *S is P* to *S is not not-P*. The rule for the process may be stated :—

Change the quality of the proposition, and substitute for the predicate its contradictory term.

The process is applicable to all the four fundamental propositions. The subjoined table will shew the equipollent forms of each :—

Original proposition.	*S a P*	*S i P*	*S e P*	*S P.*
Obverse.	*S e* $\bar{P}$	*S o* $\bar{P}$	*S a* $\bar{P}$	*S i* $\bar{P}$.

The following will serve as examples :—

[1] Equipollence is the term employed by the majority of the Latin writers, and among English logicians has the authority both of Mill and Mansel. The use of the term Obversion in this sense, appears to be due to Bain. By Hamilton it is used as synonymous with Conversion (*Logic*, I. 262). The process is briefly touched on by Aristotle, *de Interp.*, c. 10, § 15.

Original proposition.	Obverse.
All men are mortal.	No men are not-mortal.
No philosophers are practical.	All philosophers are not-practical.
Some judges are just.	Some judges are not not-just.
Some ministers are not wise.	Some ministers are not-wise.

Where it is possible to find a single word, which accurately expresses the sense of the contradictory term, such a word should be employed in place of it. Thus in these examples, we may with advantage use the words 'immortal,' 'unpractical,' 'unjust,' 'unwise' in lieu of 'not-mortal,' etc.

The validity of the process is proved by the Laws of Thought. The law of Contradiction shews us that if *All S is P*, then *No S is not-P*: for according to this principle, if anything is *P*, it is thereby proved not to be *not-P*. The Obversion of the negative judgment, follows from the Law of Excluded Middle. *S* must be either *P* or *not-P*: and since we know *No S is P*, we conclude that *All S is not-P*.

A consideration of the diagrams will confirm these conclusions. Thus, if we assert that 'All men are mortal,' the circle representing 'men' falls within the limit of the circle representing 'mortal.' It is thus apparent that no part of the circle 'men' is to be found in the space which denotes 'non-mortals.'

Obverted Converse. Among the immediate inferences which can be derived from the four fundamental propositions, must be reckoned not only the Obverse forms of the propositions themselves, but the Obverted Converse. This will be as follows :—

Converse.	$P\,i\,S$	$P\,i\,S$	$P\,e\,S$	none.
Obverted Converse.	$P\,o\,\overline{S}$	$P\,o\,\overline{S}$	$P\,a\,\overline{S}$	none.

§ 5. **Contraposition.**

Contraposition is a process of immediate inference

in which from a given judgment another judgment is inferred, having for its subject the contradictory of the original predicate. The rule to obtain the contraposition of a proposition may be shortly stated :—

Convert the Obverse of the proposition.

The proposition, 'All reptiles are vertebrates' has as its Obverse, 'No reptiles are non-vertebrates': the Contrapositive will therefore be 'No non-vertebrates are reptiles.' Similarly, the proposition 'Some right actions are not agreeable,' will become after Contraposition, 'Some not-agreeable things are right actions.'

The following table will shew the forms assumed by the various propositions :—

Original proposition.	$S\ a\ P$	$S\ i\ P$	$S\ e\ P$	$S\ o\ P.$
Contrapositive.	$\overline{P}\ e\ S$	none	$\overline{P}\ i\ S$	$\overline{P}\ i\ S.$

It will be seen that the I proposition has no Contrapositive. For after Obversion it becomes $S\ o\ \overline{P}$, and this proposition does not admit of Conversion.

The E proposition becomes by Obversion $S\ a\ \overline{P}$. This on Conversion, must change its quantity to particular. Hence here we have what has been termed *Contrapositio per accidens.*

Obverted Contrapositive.[1] It is plain that each of the Contrapositives is itself capable of Obversion. We can in each case alter the quality of the proposition, and change the predicate to its contradictory term. This will give us :—

Contrapositive.	$\overline{P}\ e\ S$	none	$\overline{P}\ i\ S$	$\overline{P}\ i\ S.$
Obverted Contrap.	$\overline{P}\ a\ \overline{S}$	none	$\overline{P}\ o\ \overline{S}$	$\overline{P}\ o\ \overline{S}.$

[1] It will be observed that the Obverted Contrapositive retains the quality of the original proposition. The older Latin logicians considered Contraposition merely as a method for obtaining the conversion of the terms, and as deriving its importance from the fact that in this way it was possible to convert the O proposition. They called it not *Contrapositio*, but *Conversio per Contrapositionem.* Hence they reserved the name for the obverted-contrapositive, since it alone follows the rule of Conversion that the quality remains unchanged. Cf. e.g. Sylv. Maurus (*Quaest. Phil.* I., p. 65). In this they followed Boethius. *De Syll. Cat.* 1. (P. L. vol. 64, c. 807). Contraposition is recognized by Aristotle in *Topics*, II. 8.

§ 6. **Inversion.**

Inversion is a process of immediate inference, in which from a given judgment another judgment is inferred, having for its subject the contradictory of the original subject. We have seen how to pass from a proposition having S for its subject, to propositions having P or $\overline{P}$ for their subjects. We now proceed to enquire whether it be possible to pass to one having $\overline{S}$ for its subject.

The process of obtaining such a proposition must be similar to that employed in the other methods of immediate inference, viz.: Conversion and Obversion. The mode in which we arrive at the Inverse of $S\ a\ P$ is shown in the left hand column below. It will be observed that the first step taken is to obvert. In the right hand column we have a similar process, but commencing with Conversion: and it is seen that after two steps we find ourselves with the proposition $P\ o\ S$ to be converted. This however is impossible; and hence it appears that we cannot find the Inverse by that route.

	$S\ a\ P.$		$S\ a\ P.$
By Obversion	$S\ e\ \overline{P}.$	By Conversion	$P\ i\ S.$
By Conversion	$\overline{P}\ e\ S.$	By Obversion	$P\ o\ \overline{S}.$
By Obversion	$\overline{P}\ a\ \overline{S}.$	By Conversion	——
By Conversion	$\overline{S}\ i\ \overline{P}.$		
By Obversion	$\overline{S}\ o\ P.$		

In the case of the proposition $S\ e\ P$, it will be found on experiment that if we commence with Obversion, we are again met with the difficulty of an O proposition demanding Conversion. If however we commence with Conversion, we pass through the series $S\ e\ P, P\ e\ S, P\ a\ \overline{S}, \overline{S}\ i\ P$, which gives us the required proposition.

In the case of I and O propositions, experiment will shew that in no case is it possible to get an Inverse form.

Our results may be tabulated thus:—

Original proposition.	$S\ a\ P$	$S\ i\ P$	$S\ e\ P$	$S\ o\ P.$
Inverse.	$\overline{S}\ o\ P$	none	$\overline{S}\ i\ P$	none.

Obverted Inverse. Just as we have seen that it is possible to obtain obverted forms of the Converse and Contrapositive, so we may obvert the Inverse forms which we have just reached. The Obverse of $\overline{S}\ o\ P$ will be $\overline{S}\ i\ \overline{P}$; that of $\overline{S}\ i\ P$ will be $\overline{S}\ o\ \overline{P}$.

§ 7. **Table of Results.** We may summarize the results at which we have arrived as follows.

	A	I	E	O
Original proposition.	$S\,a\,P$	$S\,i\,P$	$S\,e\,P$	$S\,o\,P.$
Obverse.	$S\,e\,\overline{P}$	$S\,o\,\overline{P}$	$S\,a\,\overline{P}$	$S\,i\,\overline{P}.$
Converse.	$P\,i\,S$	$P\,i\,S$	$P\,e\,S$	
Obverted Converse.	$P\,o\,\overline{S}$	$P\,o\,\overline{S}$	$P\,a\,\overline{S}$	
Contrapositive.	$\overline{P}\,e\,S$		$\overline{P}\,i\,S$	$\overline{P}\,i\,S.$
Obverted Contrapositive.	$\overline{P}\,a\,\overline{S}$		$\overline{P}\,o\,\overline{S}$	$\overline{P}\,o\,\overline{S}.$
Inverse.	$\overline{S}\,o\,P$		$\overline{S}\,i\,P$	
Obverted Inverse.	$\overline{S}\,i\,\overline{P}$		$\overline{S}\,o\,\overline{P}$	

§ 8. **Other Varieties of Immediate Inference.**

(1) **Immediate inference by added determinants** is a process of immediate inference, which consists in limiting both the subject and predicate by the same determinant. The formula for the process will be *S is P*, therefore *Sa is Pa*. For instance, 'A statue is a work of art'; therefore 'A beautiful statue is a beautiful work of art,' 'A statue by Canova is a work of art by Canova.' This form of immediate inference is only possible, when the determinant qualifies the terms under precisely the same aspect. Thus, I cannot argue that because 'A prizefighter is a man,' therefore 'A good prizefighter is a good man.' In this case 'good' as qualifying 'man,' signifies that the man realizes in himself the ideal of man; that is, he lives in accordance with the prescriptions of his rational nature. As qualifying 'prizefighter,' the word 'good' merely means that he realizes in himself the ideal of a prizefighter; that is, he possesses in a high degree the art of knocking other men down.

(2) **Inference by omitted determinants.** Here we have an inference, in which from a proposition affirming of

a given subject an attribute qualified by a determinant, we infer a proposition affirming of the same subject the attribute without qualification. Thus, from 'Men are rational mortals,' we conclude 'Men are mortals.' A fallacy may arise here, if the determinant is such as to alter the meaning of the predicate. Thus, we cannot argue from 'Spiritualistic manifestations are pretended facts,' to 'Spiritualistic manifestations are facts.' [1]

(3) **Immediate inference by complex conception.** This process is closely analogous to inference by added determinants. It consists in employing both the subject and predicate of the original as parts of a more complex conception. 'Bronze is a metal,' therefore 'A statue in bronze is a statue in metal.' 'A negro is a man,' therefore 'The death of a negro is the death of a man.' In these cases, the subject and predicate of the original proposition are not determined by what is introduced, but are themselves employed as determinants. Fallacies may, of course, arise here, similar to those we have already examined. Thus, we cannot infer from 'All judges are lawyers,' that therefore 'A majority of the judges is a majority of lawyers.'

(4) **Immediate inference by converse relation** is a process of inference, by which from a proposition stating the relation in which Q stands to S, we infer another proposition stating the converse relation in which S stands to Q. The terms are transposed, and the relative name which was attributed to the previous subject, is consequently replaced by the correlative, which is attributable to the new subject. Thus, from 'Socrates was the husband of Xantippe,' we conclude 'Xantippe

[1] Venn, *Symbolic Logic*, p. 286. This form of immediate inference is two or three times discussed by Aristotle, *de Interp.*, c. 11, § 8, *Topics*, II., c. 11, § 4 *Soph. Elenchi*, c. 25.

The principle governing its application was by the Latin logicians stated in the form: *A dicto secundum-quid valet illatio ad dictum simpliciter, quando determinatio non est diminuens.* St. Thomas, *Opusc.* 35, *de Fallaciis*, c. 11, *de Potentia*, Q. 9, Art. 5, obj. 2. See also Ch. 17, § 10 below.

Mr. Bradley's criticisms (*Principles*, p. 394) would appear to indicate that he has overlooked Aristotle's discussion of the point.

was the wife of Socrates': from 'Onesimus was the slave of Philemon,' to 'Philemon was the master of Onesimus.'

Some of the Formal logicians have denied that this mode of immediate inference falls within the province of Logic. There is, they urge, no necessary law of thought, by which we pass from '*A* is the father of *B*' to '*B* is the child of *A*.' To do so, it is necessary to be acquainted with the meaning of the terms. The process cannot be symbolically represented.

From the point of view of the Formal logician the objection has much force. But to the Scholastic logician, who holds that the object of Logic is the conceptual expression of the real, it presents no difficulty. Whoever employs the relative concept 'father', must have in his mind the correlative 'child,' and hence can pass by immediate inference from '*A* is the father of *B*,' to '*B* is the child of *A*.' The inference is expressly recognized by Aristotle (*Categ.*, c. 7, § 6).

Immediate inference by modal consequence. By this method of inference, we conclude from the fact that something is necessary, to the fact that it is possible. For instance from the proposition 'It is necessary for an equilateral triangle to be equiangular,' to 'It is possible for an equilateral triangle to be equiangular.' At first sight there appears a difficulty in concluding from necessity to possibility. But if it is not the case that what is necessary is possible, then by the principle of Excluded Middle, the contradictory is true, and we must say that it is impossible (*de Interp.*, c. 13, § 9). As this conclusion is false, we recognize that what is necessary is also possible, and similarly that whenever we can assert that something is impossible, we can also assert that it is *possible* for this *not to be*. In this case, we use the words in the second of the two senses noted in Ch. 3, § 9.

CHAPTER VII.

THE IMPORT OF PROPOSITIONS.

§ 1. **Import of Propositions—Predicative View.** In this chapter we shall be occupied in considering various theories as to the precise nature of the relation expressed in the mental act of predication. We have already in Ch. 3, §§ 1, 2, Ch. 6, § 2 explained our own view on this subject in some detail. For it was not possible to treat of the proposition, or of the process of conversion, without first indicating how the relation between subject and predicate should be understood. Briefly to recapitulate what we have said, both the subject and predicate express the object of the judgment under some aspect, the subject however being construed to signify the thing, the predicate some attribute which we affirm (or deny) of the thing. The copula declares that the object expressed by the subject, and that expressed by the predicate are identical. Further, inasmuch as the judgment deals with the object as it is known, and as in regard to our knowledge there is no fixed order as to which aspect of an object we know first, a judgment may reverse the natural order of predication. What is naturally a mere attribute, may stand as the subject; what is naturally the subject, may be affirmed as though it were the attribute. Thus we may have a case in which a proper name, a name whose special office it is to denote the individual concrete thing, stands in the place of the attribute, as when we say, 'That object coming this way, is Socrates.' Logically, this transposition of the natural order is perfectly legitimate. For the function of the predicate is simply to tell us what the subject is; and when we define the subject to be

this or that particular individual—to be the man Socrates, —this is most certainly done.

That there is a natural order of predication will easily be seen on recalling the distinction between substance and accident (Ch. 2, § 7). Clearly it is the substance which supports the accidents. They determine it, and characterize it: the substance is not something which determines and characterizes them. It possesses independent existence. They exist as its determinations, and not in their own right. Hence Aristotle rightly says that though we can express our proposition in such a form as, 'The object coming this way is Callias,' such an order of the terms is not natural but *per accidens*.[1]

The same is true where the subject is not an accident, but a substantial term of wider generality, e.g. 'That man is Socrates.'[2]

This view of the proposition according to which the subject is understood as the thing and the predicate as the attribute,—or as it is sometimes put, in which the subject is construed in extension, and the predicate in intension,—is known as the **Predicative View.**

During the past century the most diverse theories on this point have been held by logicians. The principal of these we shall proceed to examine. In the course of the discussion, we shall be brought across another problem, which of recent years has afforded matter for debate,—the question namely, whether a categorical proposition implies that things corresponding to its terms actually exist.

§ 2. **The Class-inclusion View.** Those who interpret the proposition on the class-inclusion view, hold that both subject and predicate are conceived in exten-

[1] *An. Prior I.*, c. 27, § 3. See also *An. Post. I.*, c. 22, § 2, where he points out that such a proposition as 'That white object is a stick' is predication *per accidens*, since 'the white' did not become a stick, but *vice versa*. On this St. Thomas comments as follows. "Subjectum *fit* hoc quod praedicatur 'de ipso sicut de subjecto. . . . Cum ergo non sit verum dicere quod *Album* '*fiat lignum*, manifestum est quod album non *est* lignum proprie et per se 'loquendo. Sed si hoc concedatur *Album est lignum*, intelligitur per accidens, 'quia scilicet illud particulare subjectum, cui accidit album, est lignum. Iste 'ergo est sensus hujusmodi praedicationis in qua subjectum praedicatur de 'accidente," *in An. Post. I.*, lect. 33.

Jevons (*Principles of Science*, p. 39) has completely misunderstood Aristotle's meaning on this subject.

[2] See below, Ch. 8, § 1.

sion. They believe the true significance of a proposition is to assert that the objects denoted by the subject are included in, or excluded from the class signified by the predicate, e.g. 'Men are mortal' asserts that all men fall within the class 'mortal.' The predicate on this interpretation must necessarily be read collectively—as signifying the whole class considered as a unit. A distributive use would be impossible. All men are included within mortal things, as these constitute a whole: they are not included within them, each taken separately. The subject on the other hand may be either collectively or distributively used.

We have already pointed out that in regard of the real order, inclusion in a class is *involved in* affirmative propositions, and exclusion from a class in negations: and that it is due to this that we are able to frame a scheme of diagrams corresponding to the four fundamental propositions. But the question before us, is not whether the proposition does or does not indicate the existence of classes in the objective order: but whether this is the relation conceived by the mind, and verbally expressed by the subject copula and predicate.

A fatal objection to the theory is the fact, that were it true, we should not express our propositions in the form 'All men are mortal.' They would assume some such form as 'All men are included among mortals.' If 'mortals' signifies the class of mortal things, it is inaccurate to say 'All men are mortal.' Every man is not the class 'mortal.' On the other hand, on the predicative view, the subject, copula and predicate are the natural mode of statement: for 'mortal' is a true expression of what men *are*. It is sometimes indeed urged that there are a certain number of propositions, which are naturally understood in this way, namely those which are sometimes termed 'judgments of classification,' e.g. 'Lions are Felidae,' 'Daisies are Compositae.' It is, however, quite inaccurate to say that here we affirm a relation between two classes. By 'Lions are Felidae,' we signify that every lion has the

attributes that mark a Felis. The copula here has no inclusive signification. It declares the identity between each of the things denoted by the subject, and the same things differently conceived in the predicate. No argument can be drawn from these propositions in support of the class-inclusion view.

* *Quantification of the Predicate.* This is a special form of the Class-inclusion view, which we owe to Sir W. Hamilton. At one time it was accepted by several logicians of note, as e.g. Dr. Thomson, and Dr. Baynes. It has long since been recognized to be quite untenable, and possesses only historic interest. Hamilton held that the essential function of a proposition is to compare two notions in respect of their quantity (*Logic*, II., p. 257). If this be so, it follows as a matter of course that the quantity of the predicate, even though not expressed, must be known. This, he maintained, is in fact the case, so that mentally at least the predicate, like the subject, is always either universal or singular. The scheme of fundamental propositions becomes consequently eightfold, as follows :—

All *S* is all *P*.	*U*. e.g. All triangles are all trilaterals.
All *S* is some *P*.	*A*. e.g. All triangles are some figures.
Some *S* is all *P*.	*Y*. e.g. Some figures are all triangles.
Some *S* is some *P*.	*I*. e.g. Some triangles are some irregular figures.
No *S* is any *P*.	*E*. e.g. No triangles are any squares.
No *S* is some *P*.	η. e.g. No triangles are some figures.
Some *S* is not any *P*.	*O*. e.g. Some figures are not any triangles.
Some *S* is not some *P*.	ω. e.g. Some triangles are not some figures.

The theory has nothing to recommend it. In thought we do not quantify the predicate. We may e.g. affirm that 'All rhinoceroses have a single horn,' without reflecting in any manner on the extent of the class, whose members are single-horned. Further, a list of eight propositions each expressive of a different relation between the classes, is impossible. There are but five such relations possible, as we saw in our consideration of Euler's circles. If the essential function of the proposition is to express a quantitative relation there must be five forms, and no more. The redundancy of the eightfold scheme appears as soon as it is examined. *A* and η express the same relation. *Y* and *O* correspond in a similar manner. Finally the proposition ω conveys no information whatever about the relation between the classes. It is compatible with any of the other forms, even with *U*. 'All

dogs are all latrants' (*U*); but it is equally true that 'Some dogs (terriers) are not some latrants (mastiffs).'

Equational View. Professor Jevons held that the proposition signified not inclusion, but an equation between the extension of the subject and that of the predicate. There is, he tells us, exact analogy between mathematical equality, and the relation of subject and predicate (*Principles of Science*, p. 68). He would express the proposition in this form :—

Common Salt=Sodium Chloride.

Chlorophyll=green colouring matter of plants.

A difficulty however arises when the predicate denotes a wider class than the subject. Here if the equation is to be correct, we cannot write 'Men=mortals.' Nor is it more satisfactory to write 'Men=some mortals.' For by 'some mortals,' we might indicate quite another part of the extension of the class 'mortal,' than that to which 'man' is equated. Hence Jevons employs the form 'Men=men mortals.'

This view is open to the objections we have urged against the Class-inclusion view. We do not think of the predicate in quantity. Hence as an explanation of the mental act of judgment, the theory is valueless. Moreover the representation of the proposition as an equation, does away with the fundamental distinction between subject and predicate. And if the sign (=) is to be understood in its usual sense, then all predication is reduced to the form *S is equal to P*. Here the words 'equal to' form a constituent part of the predicate. It is manifest however, that in the great majority of our judgments we are concerned not with a relation of equality but of identity, and that our predicate is not 'equal to *P*,' but *P*.

§ 3. **The Attributive View.** On this view both subject and predicate are read in intension : and the import of the proposition is held to be that the attributes signified by the subject are accompanied by those signified by the predicate. Mill is ordinarily regarded as the principal supporter of this view, but, as will be seen, he supports it under certain important qualifications. He points out, that where we have a general proposition such as 'All men are mortal,' we do not mean that the attribute signified by the predicate is possessed by a certain number of individuals, whose possession of it we have verified. Most of the individuals signified by such a subject are not known to us. From this he concludes that the import of the judgment is simply that the attributes of the

predicate *constantly accompany* the attributes of the subject, and that the proposition just quoted should be interpreted, ' Mortality constantly accompanies the attributes of man ' (*Logic*, I., c. 5, § 4).

Mill is correct when he says that we can only form things into classes on the ground that they possess common attributes. Yet it by no means follows from this, that the true import of a proposition is simply that the attributes of the predicate always (or sometimes) accompany those of the subject. The subject in such a judgment as ' All men are mortal,' is conceived as *concrete* : we do not conceive the abstract notion ' humanity,' but ' man.' Of this concrete subject, we affirm that since it is qualified by mortality, it *is* mortal. Not only therefore is he wrong in his explanation of the subject, but he is in error too as regards the copula. The copula does not express concomitance, it tells us what the subject *is*.

Mill in fact makes admissions, which in great measure deprive his argument of its force. ' It is true,' he says ' that we usually construe the subject of a proposition in extension.' And in regard to propositions, in which the subject is non-connotative, he owns that their import is that the individual thing denoted by the subject, has the attributes connoted by the predicate. Moreover, when he interprets the proposition ' All men are mortal,' in the form ' Whatever has the attributes connoted by the subject, has also those connoted by the predicate,' he practically concedes all that is required. We have here for subject, not the mere intension, viz.: the abstract attributes, but an indefinite number of concrete things, ' Whatever entity has the attributes.'

These admissions should not however lead us to suppose that Mill had abandoned the Attributive view for the traditional interpretation. A theory which reduced general propositions to statements about the coexistence and sequence of attributes, was in harmony with his general philosophic position.

§ 4. **Implication of Existence.** It has been a matter of much discussion among recent logicians, whether in a categorical proposition, the existence of objects corresponding to the subject and the predicate, is implied. The question seems to have first been raised explicitly by the German philosopher Herbart (1776–1841). Very various views have been taken. Ueberweg (*System of Logic*, § 68) teaches that all propositions imply the existence of the subject, save those in which the predicate is such as to abolish the subject-notion, e.g. "An absolutely greatest number is impossible." Mill (*Logic*, I., p. 124) holds that analytic propositions do not involve the existence of their subjects, but that synthetic propositions do so. Mr. Keynes and Dr. Venn alike teach that universal propositions have no such implication, but that where a particular proposition is used, something corresponding to the subject must exist. This may appear strange in the face of such a judgment as 'Some griffins have long claws.' But we are told that the existence need not be in the physical universe; it is in the 'universe of discourse.' It will be necessary briefly to examine this notion of the 'universe of discourse.'

(1) *Universe of discourse.* As first employed, the phrase signified no more than that by a convention our propositions, as verbally expressed, are restricted in their reference.[1] Thus if I say, 'No one now wears chain-armour,' I refer only to soldiers. Actors are not under consideration; and hence soldiers might be said to constitute my universe of discourse.

As used now however, the phrase has taken a different signification. We are told that things, which do not exist in the actual physical universe, can exist in 'the universe of mythology,' 'the universe of folk-lore,' 'the universe of the imaginable,' 'the universe of heraldry.' These various forms of existence are called 'empirical' to distinguish them from logical existence, which belongs to mere objects of thought.[2]

[1] De Morgan, *Formal Logic*, c. iv.

[2] Keynes, *Formal Logic*, Part II., c. 7. Venn, *Symbolic Logic*, p. 127. Cf. Bradley, *Appearance and Reality*, p. 367.

It is scarcely necessary to bring serious arguments against this view. It is rightly termed by Mr. Wolf "an extravagant conception, which does not really 'mean what it seems to mean" (*Studies on Logic*, p. 71). It may however be well to indicate to what these various universes really amount. Now in the first place it is evident that if a thing possesses *actual existence* at all, it exists in this physical universe. The griffins of heraldry exist. At the present day, they are usually made of paint and pasteboard. We need not expect to find them in any other world than this. Many indeed of the objects, concerning which we form judgments have no *actual existence* at all. They are mere objects of thought. The idea by which they are represented, exists as a physical fact: but the object thought of belongs not to the real but to the conceptual order. A distinction should however be drawn between two classes of such ideal entities. Some of the objects of thought about which judgments are framed, and propositions enunciated, are mere creatures of the imagination; these, we imagine as existing in the physical universe conditioned by place and time. Thus I may say 'The wrath of the Homeric gods is terrible,' 'Hamlet was Prince of Denmark,' 'The upas-tree is the tree, the mere approach to which involves death.' But in all these cases the subject as verbally expressed is elliptical. The full form of the judgment would be 'The wrath of the gods imagined and described by Homer, is terrible.' The second class of objects possessing merely conceptual existence is of far more moment. Here we are not concerned with individuals pictured by the imagination. This class consists of entities, which though they do not exist, are not repugnant to the constitution of the physical universe; and which are expressed in universal concepts. Such for instance are those mathematical figures, which have never, so far as we know, existed, but whose nature and properties can be accurately determined. Here our proposition needs no amplification: it is true as it stands. We

may affirm that, ' Some geometrical figures are chiliagons,' and the accuracy of the statement is indisputable. This point will appear clearly in what follows.

(2) *Implication of existence.* Having now determined that if an entity exists at all, whether as matter or as spirit, it exists in this physical or actual universe and not in another, we may the more securely face the question, whether or not existence is implied in categorical propositions.

There can be no doubt, that the categorical form as such, contains no such implication. We have already pointed out (Ch. 4, § 2) that our concepts, while they accurately represent the nature of the thing conceived, abstract altogether from the question of its existence. In regard to the subject they tell us *what it is,* they do not tell us *that it is.* If then our concept represents the nature of some entity, and we abstract one of the notes which determine its nature, and affirm this note of the entity, our judgment is true even should the thing in question be a mere ' possible.' Even should every animal perish from the face of the earth, we cannot hold that it would be untrue to affirm mortality of animal, and that the assertion would become true once more if animals were again created.[1] It is this that explains and justifies propositions, such as ' All chiliagons have a thousand angles.' The proposition is true even though the chiliagon has never existed : for the predicate affirmed is one that necessarily belongs to the subject which my concept represents.

It is plain that such propositions may be formed both as universals and particulars. I may say, ' All chiliagons have a thousand angles,' or I may say ' Some plane figures are chiliagons.'

We do not, however, mean to assert that there are no categorical propositions, which imply the existence of their subject terms. We have merely asserted that the categorical form as such does not imply it. There are certain classes of categoricals, which by reason of

[1] Bradley, *Principles,* p. 47.

what is affirmed in them, do contain such an implication. Thus (1) all propositions, in which existence is actually predicated, of course, signify the existence of the subject, e.g. God exists. (2) Wherever the predicate affirms that the subject has acted, or has been the recipient of an action, the proposition implies the existence of the subject. We may define an entity, and assert of it the properties, which flow from its nature, even though it has never existed. But we cannot truthfully affirm that it has acted, unless it has existed. The mere nature cannot act. Propositions such as 'Brutus slew his benefactor,' 'The meteor fell to the earth,' certainly imply the existence of their subjects. And (3) the judgments, in which a demonstrative pronoun is attached to the subject, may be said to fall within the same category. The pronoun is a sign that we are concerned with a subject, which is not merely existing but under observation.[1]

* It has been maintained by several logicians of eminence, that the view which we have been defending, reduces the categorical to a hypothetical, and that on our showing 'All S is P,' should be read 'All S (if S exists) is P,' and 'Some S is P,' should be read 'Some S (if S exists) is P.'[2] To this we may reply, 'All S (if S exists) is P' is a quite inaccurate representation of the proposition as we understand it. The existence of S is wholly outside the scope of the categorical as such. The proposition 'All S is P' may indeed have been reached through experience of existing things, e.g. 'All crows are black': but it may have been reached by a process of purely intellectual conception, e.g. 'All chiliagons have a thousand angles.' The information which it communicates, concerns the nature, and

[1] It is a mistake to suppose that this question was not raised by the Scholastics. The following citation deals with it succinctly. "Ab *est* tertio 'adjacente, ad *est* secundum adjacens, in enunciationibus contingentibus recte 'concluditur: ut *Si mortui sunt miseri, ergo mortui sunt.* Atqui vero in pro-'nunciatis necessariis non item. Non enim necessario infertur *Homo est ani-'mal, ergo homo est.*" Sanderson, *Logica*, p. 103 (ed. Antwerp, 1589). The author of this work, John Sanderson, Canon of Cambray, had been Logic-reader at Cambridge, but became one of the exiles for religion under Elizabeth. The book is dedicated to Cardinal Allen, and contains prefatory verses by Gregory Martin, B. Edmund Campion and other members of that notable group of scholars.

[2] Venn, *Symbolic Logic*, pp. 135–137.

the nature alone : and the proposition is true, if it accurately expresses *what* the nature is. Chiliagons are possible entities, not actually existing things. Similarly, when we say, 'Some geometrical figures are chiliagons,' the last thing that we intend to assert, is that 'If any geometrical figures exist, some of these figures are chiliagons.'

Implication of Existence and Immediate Inference. The bearing of the various views as to the implication of existence, upon the processes of immediate inference, has been discussed with customary thoroughness by Mr. Keynes. It will be sufficient to summarize the conclusions to which he is brought, as to the effect respectively of the traditional doctrine here defended, and of his own doctrine.

In regard to his discussion of the traditional doctrine, it is to be noted that he starts from the presupposition that the propositions are to be interpreted according to the explanation we have just rejected, viz.: 'All S is P' as equivalent to 'All S (if S exists) is P.' Starting from this erroneous hypothesis he argues (*a*) that the conversion of A is invalid. From 'All S (if there be any S) is P,' we cannot conclude that 'Some P (if there be any P) is S.' Similarly the conversion of I is invalid. The invalidity of these conversions involves (*b*) that the contraposition of E and O, and (*c*) that the inversion of A and E are also invalid. In regard to the doctrine of Opposition, (*d*) Contradiction does not hold good. For 'All S is P' merely denies that there are any S not P; and 'Some S is not P' asserts that, 'If there be any S, some are not P.' Where S does not exist in 'the universe of discourse,' both these propositions may be true. Finally (*e*) Contrariety does not hold good. 'All S is P,' and 'No S is P' may both be true : for it is alike the case that there are no SP's and no S not-P's, when S is not found in the 'universe of discourse.'

These conclusions are all based on the erroneous interpretation of the proposition. Interpreted as we have understood it, all the processes of immediate inference are valid.

In regard to his own view, viz. : that universal propositions have no existential implication, but that particular propositions imply the existence of their subjects, he arrives at the following results :—

(*a*) The conversion of A is invalid, for the same reason as was given above : and similarly (*b*) the contraposition of E, and (*c*) the inversion of A and E are invalid. The conversions of E and I are valid. These results may be summarized by saying that we may infer a universal from a universal, and a particular from a particular, but not a particular from a universal. In regard to Opposition (*d*) Subalternation manifestly does not hold good, and (*e*) Contrariety for the reasons alleged above is

invalid. (*f*) Sub-contrariety is invalid also: for where S is not found, both sub-contraries may be false.

Mr. Keynes holds that these results, notwithstanding the havoc they make in the doctrine of immediate inference, are more satisfactory than those of the traditional view, since they secure the validity of the two important processes of (1) contradiction, (2) simple conversion. But, as we have seen, his estimate of the traditional view is vitiated by the initial mistake as to the interpretation of the proposition.

* § 5. **The Compartmental View.** Our discussion on the implication of existence in propositions will, we trust, throw some light on the theories as to their import, which we have yet to discuss. The view we propose to consider in the present section, owes its origin to Dr. Venn. Mr. Keynes also has accepted it. Dr. Venn, when dealing with the import of propositions, calls attention to the fact that, though it is quite impossible to admit that universal propositions should be understood as signifying the existence of their subjects, yet all of them do imply that certain things do *not* exist. If I affirm 'All x is y,' the assertion certainly contains the information that no such things as $x\bar{y}$ exist. Thus 'All men are mortal' has the *non-existential* implication, 'No non-mortal men exist.' Hence he urges that "in respect of what such a proposition affirms, it can 'only be regarded as conditional, but in respect of what it denies, 'it may be regarded as absolute. The proposition 'All x is y' '. . . declares the non-existence of a certain combination, viz.: 'of things which are both x and not-y. But it does not tell us 'whether there is any y at all; or if there be, whether there is 'also any x."

This view has been called the Compartmental view, because its characteristic feature is that the essential import of the universal proposition is regarded as consisting in the fact that it empties a definite compartment. 'All x is y' empties the compartment $x\bar{y}$; 'All y is x,' the compartment $\bar{x}y$; 'No x is y,' the compartment xy. The affirmative import, 'If there are such things as x, then all the x's are y,' is regarded as a secondary and derivative implication.

Those who have found no reason to disagree with what was said in our last section, will not be prepared to accept this view. We maintain in the first place, that the categorical proposition, 'All x is y,' is in no sense a conditional proposition, but the direct affirmation of an attribute in relation to a known subject. Secondly, we maintain that it is not the case that the negative form is the primary import, and the so-called conditional affirmative form the secondary; but on the contrary, the negative form is a mere inference from the affirmative form.

A further objection may be raised on the ground that the non-existential interpretation is quite inadequate as representing the true import of the proposition. When discussing Dr. Venn's system of diagrams (Ch. 5, § 3) we promised to say something on this point. The negative, 'There are no $x\bar{y}$,' merely tells us that at the present time there happen to be no x's that are not y. It does not in any way imply that whenever and wherever an x is found, that x is always y. It is this surely, which is the point of real importance. Yet the theory tells us that not this, but the comparatively unimportant 'There are no $x\bar{y}$,' is the true and primary significance of the proposition.

The treatment of the particular proposition is equally unsatisfactory. In contrast with the universal, it is asserted it does imply the existence of its subject. "If it did not do so," says Dr. Venn, "it would have absolutely nothing certain to tell 'us . . . it can extinguish no class, and establish no class, and 'has therefore no categorical information to give us." It is quite true that *existentially* the particular proposition can extinguish no class, and establish no class; but a judgment such as 'Some figures are chiliagons,' is most assuredly categorical, and informs us that there is a class of possible entities, which have the definite characteristic of having a thousand sides. It is urged on behalf of Dr. Venn's interpretation, that only thus do we get perfect contradiction between the universal and the particular. 'All x is y' and 'Some x is not y' are true contradictories, if they respectively mean 'No $x\bar{y}$ exists' and 'Some $x\bar{y}$ exists.' Of these one must be false; the other must be true. But to this it may be replied that the ordinary interpretation of the two judgments provides us with contradiction, when rightly understood, and that the contradiction here is obtained by making the universal mean far less, and the particular far more, than rightly understood they do mean.

The German logician, F. Brentano, held the same view in a more extreme form. He urged that the ordinary mode of stating the *A.E.I.O* propositions was logically indefensible, and that the correct forms should be :—

A. There is not an immortal man (=All men are mortal);
E. There is not a live stone (=No stones are living);
I. There is a sick man (=Some men are sick);
O. There is an unlearned man (=Some men are not learned).

The existential significance of the copula would, he claimed, thus appear plainly. Such a theory carries its own refutation with it. It is manifest to any one who reflects, that as a matter of fact we do not think in these forms.

* § 6. **Mr. Bradley on the Proposition.** Mr. Bradley defines judgment as 'the reference of an ideal content to reality.' The

true subject of a judgment is, he tells us, not the grammatical subject, but the reality itself. The whole judgment is of the nature of a predicate representing attributes which are referred to the real world. Thus, "in '*A* precedes *B*,' this whole relation '*A*–*B* is the predicate, and in saying this is true, we treat it as 'an adjective of the real world. It is a quality of something 'beyond mere *A*–*B*. But if this is so, the reality to which the 'adjective *A*–*B* is referred, is the subject of *A*–*B*" (*Principles*, Bk. I., c. 1, § 17). If this view be correct, and the two terms of the judgment form in fact a single predicate, it follows, of course, that the copula is meaningless. Mr. Bradley accepts the conclusion. In its ordinary acceptation, he tells us, the copula is 'a superstition.' The traditional account of the judgment, that it is a synthesis of ideas, he rejects as manifestly false. Not only is it the case, he urges, that one idea is sufficient for a judgment, as when I point to an animal in motion, and say 'Running'; but further, did judgment consist in a synthesis of ideas, it would be confined to a sphere of unreal universals. "When 'I say 'Gold is yellow,' then certainly some fact is present to 'my mind. But universal gold and universal yellow are not 'realities" (*Principles*, Bk. I., c. 2, § 5). Hence he concludes that the true account of judgment is to be found in saying that we refer the ideal content of our assertion to the real order.

Mr. Bradley's arguments are of little weight. In the proposition 'Gold is yellow,' those who hold the term 'Gold' to be the true subject, are not bound to interpret the judgment as meaning no more than that the universal idea of gold is qualified by the universal idea of yellowness. On their showing, it means that that which the concept of gold represents, namely the real entities for which it stands, are qualified by the real determination 'yellow.' But the real entity is not the subject of the intellectual judgment, any more than the real determination is a predicate. 'Subject' and 'predicate' belong to the thing as conceived. The concepts are, it is true, universal: they omit the individualizing notes of the several pieces of gold. But they do not deny their presence: they merely abstract from them. Thus the term 'Gold' can be used as representative of every piece of that metal. Further, since we possess knowledge, not merely through the concepts of the intellect, but through the senses which perceive the individual, we may employ the universal concept to designate a particular individual, by using the demonstrative pronoun and saying 'This gold is yellow.'

The contention that we can express a judgment by a single word without either subject or copula, has already been criticized (Ch. 3, § 1). An expression such as 'Running!' is a mere mutilated proposition. It is an abbreviated sentence, and represents the mental judgment 'That object is running.'

§ 7. **Import of the Hypothetical Proposition.** In Ch. 3, § 2, we saw that the explanation of the Categorical judgment was to be found in the comparison of 'being' in the conceptual order, with 'being' in the real order. A somewhat similar comparison will afford us the clue to the meaning of the hypothetical proposition. The latter expresses the *dependence* of the consequent on the antecedent in the order of conceptual being. If the mind admits the antecedent, it must perforce admit the consequent. In the real order, effects depend on causes, properties on the natures from which they flow. The conceptual order is not thus limited. The mind argues from effect to cause, from property to substance, as well as from cause to effect and from substance to property. It not only judges that 'If the bomb explodes, the king will be killed,' but that 'If the flags are half-mast high, the king is dead.' What is a very remote effect in the real order, is often the immediate antecedent in the conceptual order. The Scholastics accurately distinguished between what is anterior in the order of being (*prius in ordine reali*), and what is anterior in the order of thought (*prius in ordine logico*). Thus if we argue from the existence of a finite world to the existence of God, in the logical order God is posterior to the world, though in the real order the world depends on Him. It is of the utmost importance that the distinction should be borne in mind, for in the study of Logic we are explicitly concerned with the mode in which the order of thought deals with the order of reality. The terms *Ground* and *Reason* are sometimes employed to express that on which something depends in the order of thought, in contrast with the term *Cause* which signifies that on which something depends in the real order. To the followers of the Idealist school the manifest difference between conceptual dependence and real dependence presents a problem for which they are unable to offer any satisfactory solution. They reduce the whole world to relations expressed by thought: and their philosophical principles would appear to involve that

the death of the king depends just as much and just as little on the position of the flag, as on the explosion of the bomb. Thus Mr. Bosanquet tells us that apart from temporal succession which is "the natural differentia of causation" he "cannot see how the relation of conditioning differs from that of being conditioned" (*Logic II.* 262, 264).

* *Truth of the Hypothetical Propositions.* In recent works on Logic somewhat lengthy discussions have been devoted to the question as to how a hypothetical judgment can be said to be true.[1] It may well be the case that neither the antecedent nor the consequent taken separately are true. Yet somehow we attribute truth to the assertion in its entirety. In what sense can it be said to correspond with reality?

To solve this question we must distinguish two classes of hypotheticals. In regard to the first of these classes, its truth depends on the fact that the real order does not consist solely of what is *actual*. Besides what is actually real, there are real *potentialities*. An acorn is actually an acorn and nothing more: but potentially it is an oak tree. Hence it is true to say, 'If this acorn is planted, it will become an oak.' For its nature is such, that given suitable conditions, it must necessarily develop according to definite laws. Similarly we find systems such as law and social custom, which determine what would happen, under definite conditions, even though those conditions have never occurred, e.g. 'If the king were to arrive, a salute would be fired.' Our judgments are true, not because they correspond to what *is*,—about which we make no assertion—but because they correspond to what *would be*.

The second class may be illustrated by the judgment, 'If the souls of brutes are spiritual, they are immortal.' Here the judgment depends for its validity, not on a potentiality, but on the generic categorical proposition, 'The spiritual soul *as such* is immortal.' This principle, being universal, abstracts from time and place. It is therefore capable of *application* in regard to any time and any place. If the judgment '*B as such* is *C*,' is known, then we may assert regarding any given object, 'If that object is *B*, it is *C*.' Such a judgment must be true, for it is merely an application of a principle already acknowledged.

[1] Cf. e.g. Bradley, *Principles*, Bk. I., c. 2, §§ 49, 50.

CHAPTER VIII.

THE PREDICABLES.

§ 1. **The Predicables.**

The Predicables are the various relations in which universal terms may stand to the subjects of which they are predicated. The account of the Predicables given by the Scholastic philosophers, is derived from the *Isagogé* of Porphyry (Ch. 1, § 5). Porphyry's treatment of the subject differs from that of Aristotle in one or two details, which we shall explain in a subsequent section. It is however, as it explicitly professes to be, in full accord with the principles of his philosophy.

The *Isagogé* enumerates five possible relations, which a universal term holding the position of predicate, may bear to the subject of which it is predicated. It may stand as the *Genus*, or the *Species*, or the *Differentia*, or as a *Property*, or an *Accident* of the entity, of which it is affirmed.

Before explaining these terms it is important to point out that the Porphyrian account of the Predicables, depends on a fact strongly emphasized by Aristotle (*Categ.*, cc. 2, 5) that the ultimate subject of all predication is the individual. We may of course form propositions such as 'Man is mortal,' in which the subject is not an individual, but a general term. But 'man' can only stand as the subject of this proposition, because it is itself capable of being predicated of the singulars, 'Socrates,' 'Plato,' etc., etc. 'Man' has no existence save in so far as individual men exist; and it is because it can be affirmed of the individual, that the general term is able

to become a subject of attribution. This distinction between the singular term, denoting the individual substance, e.g. Socrates, and the universal term 'man,' which expresses the class-nature of Socrates, Aristotle marked by the names *Primary* and *Secondary Substances*. The individual is the *Primary Substance*: the class-nature he called *Secondary Substance*.

It is evident therefore, that when we are considering the case of some general term employed as the predicate of a proposition, e.g. the term 'animal' in 'Man is an animal,' we may consider the relation of 'animal,' not merely to its subject 'man,' but to the ultimate subject of predication, namely individual men such as Socrates, Plato, etc.

Similar considerations will be of force in regard to those abstract terms which signify a quality. Amongst these also there are general terms such as 'colour,' 'virtue,' and the like, which are sometimes found as the subject of propositions. But here too, the right of a general term to stand as a subject, is due to the fact that it is itself used as a predicate of the individual quality, e.g. of 'whiteness,' as exhibited in the individual object.[1] So that both in the case of substances, and in the case of abstract qualities, we are able to consider all general terms in reference to their ultimate subject.

We may now proceed to consider the five predicables we have enumerated above.

(i) **Species.** If we take an individual substance, e.g. Socrates, we shall see that among the predicates which can be affirmed of him, there is one which expresses his class-nature, viz.: man. What now is the comprehension of the term which signifies this class-nature? It consists of what we may call the constitutive notes or characteristics of that nature. By the word 'constitu-

[1] Arist. *Categ.*, c. 2, § 3. "In short, individuals and whatever is numerically 'one, cannot be predicated of a subject. Nothing however prevents them 'in some cases, from inhering in a subject. Thus, an individual instance of 'the science of grammar is one of those entities that inhere in a subject, 'but it is not predicated of any subject."

tive,' we signify those notes which are requisite to make the entity the kind of thing it is, which make it representative of the type to which it belongs. Thus the comprehension of the term man is 'rational animal.' For man not only possesses all the notes of the wider class animal, but also the distinctive note of rationality. An entity in which these notes are found is a man. If one of these were absent he would be no longer such. The concept which thus expresses the constitutive notes by which the individual is what he is, is said to represent the essence (*οὐσία*) of the individual. Two other terms were also employed to signify the essence. It was called the quiddity (*τὸ τί ἦν εἶναι*), because it is by the term which declares the essential nature, that we reply to the question, What is it? (*quid sit*), in regard to any object. It was also called the **species** (*εἶδος*), i.e. characterizing form. We may therefore define the species as **the sum of the essential attributes of an entity.** Or, since we are here considering it as constituting one of the relations which may exist between predicate and subject, we may put the matter thus: *Whenever the predicate expresses the sum-total of the essential attributes of the subject, it is said to be the species of the subject.* Thus, if I affirm the proposition 'Bucephalus is a horse,' 'horse' is said to be the species of Bucephalus, since the concept of 'horse,' has for its comprehension the full class-nature of its subject.

When it is said that the notes expressed by the species are those which make the thing what it is, we must not be supposed to signify that these concepts give us some peculiar insight into the inner constitution of class-natures. It is only in the case of mathematical figures, as e.g. of the triangle or circle, by reason of their extreme simplicity, that our minds can represent the actual constitutive principle of their object. Here indeed our concepts are *adequate* (Ch. II, § 3). In regard to natural classes all we can do is to form concepts representing certain characteristic notes that distinguish one class from another. We must be content that our concepts shall be *clear*.

Though the primary signification of the word 'species'

is as we have just explained, that name is often applied to a term, not in reference to the essential nature which it expresses, but in reference to the individual objects which possess that nature : in other words, it is taken not in intension, but in extension, and denotes the group of objects of which it can be predicated. In this sense it is almost synonymous with the word 'class.' And we shall see that though the definition given above is the more philosophically accurate, another based on the extensive signification is frequently employed.

(ii) **Genus.** Among the notes constitutive of the specific notion, there are some which are common both to the class in question and to other classes. Thus the notes indicated by the term 'animal' are common alike to man, and to the various tribes of brutes. A term significative of this part of the essence, provides us with a predicate, which stands to the subject in a different relation from that occupied by the species. Such a predicate is said to express the **genus** of its subject. A genus may thus be defined as **that part of the essence of the subject which it has in common with other species.** Thus 'animal' is a generic concept in relation to the various species 'man,' 'lion,' 'horse,' etc. ; 'triangle' is generic in relation to 'equiangular,' 'isosceles,' and 'scalene.'

When it is said that the genus expresses the notes common to several species, we must not be understood to signify that an essence is a collection of attributes, that are mutually independent, and separable one from another. Such a view would be quite foreign to the truth. The species differs from the genus because it is the nature as completely determined, whereas the genus is incompletely determined. Thus the notion 'triangle' is less completely determined than that of 'isosceles triangle.' But 'isosceles' and 'triangle' are not two different attributes in the sense that it is possible to have the note 'isosceles' existing apart from a triangle. Nor can we have a triangle which is not either isosceles, equilateral, or scalene. But we can form a generic concept of 'triangle,' because our intellect can abstract from the determining

characteristics, and merely represent what is common to the three species.

Genus, like species, may be understood in extension. And so understood, it will merely be a wider class, of which the various species are sub-classes. The two terms may in consequence be thus defined :—

A genus is a wider class consisting of narrower classes.

A species is a narrow class included in a genus.

It is evident that just as ' animal ' is a genus, under which ' man ' is a species, so we may find higher genera above ' animal.' ' Man '—' animal '—' living Being '—' corporeal substance,' form an ascending scale of Genera. It is this fact that frequently renders it convenient to understand the terms in the meaning expressed by these extensive definitions : for this definition of species makes the term applicable to each sub-class, as referred to the class above it. As ' man ' is a species in relation to ' animal,' so ' animal ' is a species in relation to ' living Being,' and so on. When the intensive definition is employed, ' man ' alone of all the series could be called a species. The higher classes are none of them predicated of any subject as its whole essence. It is only in reference to individuals that such a predicate is found (*Isag.*, c. 2, §§ 21, 22). But if we accept the extensive definition, then in the hierarchy of classes, all save the highest are species. This highest class is termed a *summum genus*. And all save the lowest class, which is known as an *infima species*, are genera.

(iii) **Differentia.**

The differentia is that part of a specific essence by which it is distinguished from other species of the same genus. Just as the mind can abstract what is common to various species, and express it in the concept of the genus, so it can by abstraction form a concept of that portion of the specific essence which is distinctive of it. This part of the nature is termed the differentia. Thus, ' rational ' is the differentia, which together with the genus ' animal,' constitutes the specific nature ' man.' Similarly it is by the respective differentiae that ' triangle '

becomes 'equilateral triangle,' 'isosceles triangle,' 'scalene triangle.' What we have said as to the genus, will show that in the real order the genus and the differentia are not two different parts of the nature. We cannot in the real order separate between a triangle and its equiangularity. It is the mind alone which can distinguish the differentiating form from the indeterminate generic nature, and can view the triangle and its form as equiangular or isosceles, as distinct. Thus there is a great difference between the essence of a thing in the real order, and the logical essence of which we are here treating. In the real order the parts which make the thing to be what it is are real parts. Socrates is a man because he possesses a human soul and body. The parts of the logical essence on the other hand are not real parts but conceptual parts—the incompletely determined genus and the determining differentia.

Each species in the ascending scale, is marked off from the other classes contained under its genus by a differentia. The differentia of the *infima species* is termed a *specific differentia*. The others are known as *generic differentiae*.

(iv) **Property.** Besides the notes which go to make the essence, which we have hitherto been considering, many other attributes may be predicated of a subject. Some of these are directly and necessarily connected with the essential characteristics. These are termed properties (*propria*—ἴδια), and are thus defined :—

A property is an attribute which does not form part of the essence of its subject, but results necessarily from that essence. These properties are not reckoned as forming part of the essence, because they are derivative qualities, not primary. But the fact that they follow from it by resultancy, ensures that they are found wherever that nature is present, that they are found nowhere else, and that they are constant. Hence the extension of the specific property is precisely the same as the extension of the species. The classical example is the faculty of laughter in man. This is immediately dependent alike on the intelligence and the corporeal structure of human nature. Similarly, his power of intelligent speech, and his practice

of cooking his food, have been assigned as properties of man. Besides the specific properties, there will be others, which result from the generic notes of the nature. These will, of course, not be limited to the species, but will be coextensive with the genus to which they belong.

(v) **Accident.** The fifth of the predicables embraces all those attributes, which are unconnected with the essence. It is thus defined :—

An accident is an attribute, which neither is part of the essence, nor necessarily results from the essence of the subject.[1] Accidents are *separable* or *inseparable* according as the subject is ever found without them or not. Where we are speaking of accidents in relation not to a single individual, but to a whole species, the inseparable accident is one which is found present in every individual of the class; the separable is one found only in certain members. It may well be asked what it then is, which distinguishes an inseparable accident from a property. If an attribute is found to belong to all the members of a species, should we not rank it as a property, not as an accident? Porphyry replies that the inseparable accident is found present in other species besides the one in question. The property on the contrary is peculiar to its own species (*Isag.*, c. 16, § 4). Thus the black colour of the crow is an inseparable accident. If an entity should be found lacking an attribute of this character, it would not on that account be placed in a different species (*ibid.* c. 5, § 2). This was in fact what occurred when the black swans of Australia were discovered. No one regarded them as entitled to a different specific name, though they were without what had heretofore been regarded as an absolutely constant characteristic.

In the case of accidents, not of the species but of the

[1] The term *Accident* as signifying the fifth predicable has a different sense from that in which it was employed in Ch. 2, § 7, Ch. 7, § 1. There we used it to denote a real entity whose connatural mode of existence is by inherence in substance. Such entities are termed *Real Accidents*. As a predicable it signifies a predicate which stands in a certain relation to the subject of the proposition. These are termed *Logical Accidents*. The relation belongs to the logical order.

individual, separable and inseparable accidents are otherwise distinguished. An inseparable accident is one which belongs to the subject at all times, e.g. 'To be a Mantuan' is an inseparable accident in regard to Virgil. A separable accident is one, which can only be predicated of its subject at certain times. Thus if it be affirmed, 'Virgil is writing poetry,' this is a separable accident.

Besides the five Predicables we have mentioned, Porphyry (*Isag.*, c. 2, § 37) notices the case in which a singular term is predicated of a subject, e.g. 'Socrates is the son of Sophroniscus.' This, however, has no claim to constitute a separate Predicable. The primary *praedicabilia*—terms which can be predicated of something—are necessarily universal. They express some attribute which can be affirmed of a subject : and the attribute or nature is always universally conceived. A singular term that is predicated, is not one of the primary predicable terms. We form the singular term by limiting some universal notion in regard of place and time.

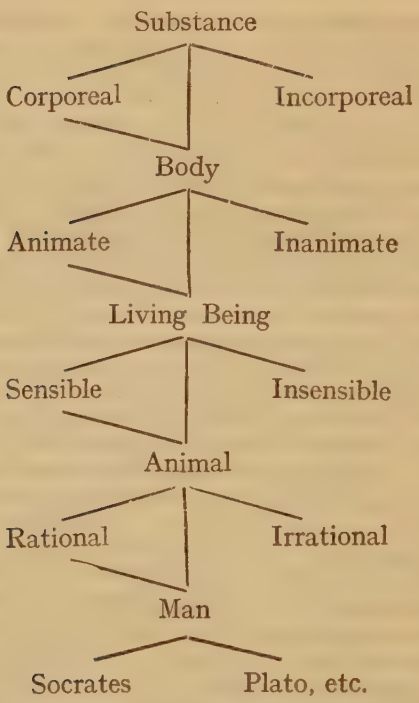
It may be easily shewn, that setting aside the case of the singular predicate, the list is exhaustive. For when an attribute is predicated of a subject, it must either belong to that subject by strict necessity or not. If it is not one of the necessary attributes, it is an accident. If, on the other hand, it is a necessary attribute, it either belongs to the constitutives of the essence, or is an attribute resulting necessarily from the essence. In the latter case it is a property. Should it belong to the constitutives of the essence, it must either be the whole essence, in which case it is a species : or it must be that part of the essence which is common to other classes—the genus : or finally the part of the essence peculiar to the species, viz. : the differentia.

* We have pointed out above that the separation of the genus and differentia is the work of the mind : and that in the thing itself we cannot distinguish two parts, one of which is 'triangle,' the other 'isosceles.' From this it would seem to follow that the differentia must implicitly involve the presence of the genus, that, e.g. the form of the figure as isosceles, necessarily supposes

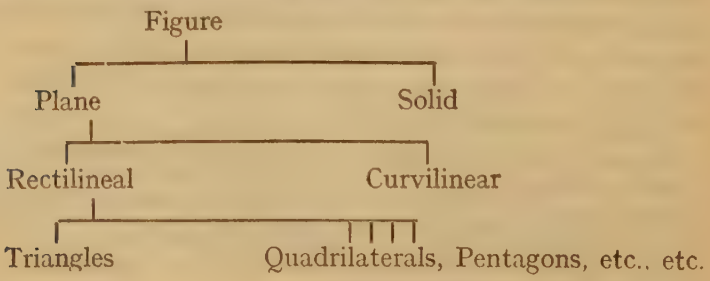
that the figure determined is a triangle. Such in fact, is the teaching of Aristotle. The specific differentia, he tells us, gives us the essence.[1] Yet a difficulty may here suggest itself. 'Rational,' it may be urged, does not involve 'man.' Do we not speak of 'rational desires,' 'rational actions' etc., etc.? To this it is replied that the term 'rational,' as predicated on the one hand of man, and on the other of actions and desires, is analogous and not univocal. As applied to man it signifies that his substantial nature itself is distinguished from that of all other animals by the faculty of reason. As used of desires and actions it merely signifies such desires and actions as are directed in accordance with what reason prescribes as right. In this sense the attribute is purely accidental, and in no sense constitutive of a natural class. Eating with a view to preserve health is a rational action, because directed by reason; but as a natural act it does not differ from the same act as performed by the lower animals. It may be further noted that the term 'rational' as signifying the differentia of man, must not be understood as synonymous with 'intelligent.' Not man alone, but God and the angels possess intelligence. 'Rational' denotes an intelligence of a particular kind, viz. one that is dependent on sense perception and arrives at its conclusions by discursive reasoning (Cf. St. Thomas, *De Pot.* Q. ix., art. 2, ad. 10). The case is somewhat similar as regards the term 'living.' It has been objected that the attribute 'living' belongs not merely to certain corporeal substances but to all incorporeal beings. It is replied that the life *animating* a body and rendering it capable of immanent action, and the life of a pure spirit are totally dissimilar. The differentia 'animate' is found in corporeal substances alone. Yet the fact that many terms, which in their primary signification denote a differentia, have in their various analogous senses a wider reference, explains how it is that in several passages Aristotle tells us that the term signifying the differentia is not convertible with the species since it may be common to more species than one (Cf. e.g. *Top.* I. c. 8, § 3).

§ 2. **The Tree of Porphyry.** After what has been said in the last section, the figure commonly termed the tree of Porphyry needs no explanation. It shews us the series of species and genera from the highest genus down to the lowest species, together with the differentiae, which go to constitute each species.

[1] *Met.* VI., c. 12, § 9. *εἰ δὴ οὕτως ἔχει, φάνερον δὴ ὅτι ἡ τελευταία διαφορὰ ἡ οὐσία τοῦ πράγματος ἔσται καὶ ὁ ὁρισμός.* Cf. St. Thomas, *Opusc.* 39, *De Natura Generis*, c. 8. "Tota namque constituitur definitio ex ultima differentia." But with these passages, compare *Topics*, VI. c. 6, § 10, and St. Thomas, *Opusc.* 26, *De Ente et Essentia*, c. 3. The differentia *implies* the genus as the subject of which, and of which alone it is a determining principle: but it is not a logical whole containing the genus.



It is evident that schemes may be drawn up to represent the genera and species, not merely of substances as in the tree of Porphyry, but also of those attributes, which qualify substance, but which can be conceived in abstraction from it. The following scheme gives us the divisions of abstract figure. It will be observed that a separate line is not given for the differentiae, since in this case our language does not possess different terms for the differentia and the species.



* § 3. **Aristotle's Predicables.** The fivefold division of Porphyry is in accord with the principles of the Aristotelian philosophy. Yet Aristotle himself gives us a scheme arranged on a different system. The Porphyrian scheme is, as we saw, based on the principle that the ultimate subject of predication is the individual, and that it is in consequence, possible to frame an ascending scale of genera and species corresponding to the order of nature. Though this is part of the teaching of Aristotle, yet he does not base his division of predicables upon it, but solely on the relation between *S* and *P*, considered without regard to their position in the scale of being. It is thus that he explains his system :—

" Whenever a predicate is affirmed of any subject, the predi-
' cate must either be coextensive with the subject or not. If
' it be coextensive, the predicate is either the definition[1] of the
' subject or a property. If it gives us the essence, it is the
' definition ; if not, a property. For we have explained the
' property, as that which is coextensive with the entity to which
' it belongs, while not forming part of its essence. If however,
' the predicate be not coextensive, it must either form part of
' the attributes comprised in the definition of the subject or not.
' If it be one of those attributes, it is either a genus or a dif-
' ferentia, since the definition is composed of the genus and the dif-
' ferentiae. If it is not one of the attributes contained in the
' definition, it is clear that it is an accident " (*Topics*, I., c. 8, §§ 2, 3).

This gives us the following scheme.

(1) Definition.	Coextensive with subject.	Containing the essential attributes.
(2) Propria.		Containing attributes not belonging to essence.
(3) Genera and Differentiae.	Not coextensive with subject.	Containing some portion of essential attribute.
(4) Accidents.		Containing attributes not connected with the essence.

In this system of Predicables, there are two points which call for notice.

(1) The omission of the species. This is readily explained if the principle on which the system is constructed be kept in view. The only case in which a predicate can express the species is,

[1] The definition is ordinarily expressed by the proximate genus and its differentia. Thus the definition of man is ' rational animal ' ; that of animal is ' sensitive living-being.'

as we saw, when the subject is an individual, e.g. 'Socrates is a man.' But as regards the relative extension of the terms, the species does not differ from the genus : neither of them is co-extensive with the subject. Hence Aristotle reckons the species here under the head of genus. Nor does the fact that he declares the genus only to express part of the essential attributes, militate against this explanation. For the species, though expressing all the constitutive notes of the *type*, the whole class-essence, cannot give us the essence of the *individual* with the differentia which separates him from others of the same species.

(2) The ranking of the differentiae with the genus as differing in extension from the subject. In *Topics*, IV., c. 2, § 11 he says that the differentia is 'either coextensive with the species or exceeds it in extension.' This is the more accurate statement. Occasionally we have a term which expresses the determining principle of a particular species as such, e.g. 'isosceles' in regard to 'triangle.' More often our term is common to various species. Thus the term 'intelligent' or 'rational' may be employed as the differentia of man, and may also be predicated of spiritual beings.

* § 4. **The Controversy on Universals.** The full discussion of the Predicables renders it necessary that we should deal with the question as to what it is that the universal term really signifies. If for instance we consider the terms 'man,' 'white,' 'round,' we see that each of these has a perfectly stable meaning, which it retains wherever it is employed. Yet though the meaning be thus invariable, the term 'man' is predicated, and rightly predicated, of different individuals—of Socrates, and of Plato, of Mill, and of Kant. What is this human nature which is one, and yet stands in the same relation to every member of the class,—which though it is one, belongs at the same time to many individuals ? [1] Various answers have been given to this question. We may hold (1) that this common nature is something real. Those who give this answer are termed **Realists.** We may say (2) that the common nature is merely a thought in the mind without objective counterpart in the real order. The adherents of this doctrine are known as **Conceptualists.**[2] Or we may say (3) that the only common element is the name, given to a variety of

[1] *Met.*, VI. c. 13, § 2. τοῦτο γὰρ λέγεται καθόλου ὃ πλείοσιν ὑπάρχειν πέφυκεν. 'We call Universal that whose nature it is to belong at once to many.'

[2] Among modern logicians Hamilton and Mansel were Conceptualists. As such also may be reckoned the Idealist logicians, whose theory of knowledge is based on the doctrines of Kant and Hegel. Among the mediæval philosophers, this was the teaching of William of Ockham (1280–1349) and his school. They were at that period known as Nominalists. But their tenets on this point agreed with those of the modern Conceptualists.

objects because of some real or fancied resemblance. This view is that of the **Nominalists.**[1]

That this is a philosophical problem of the very first importance will appear at once, when it is observed that every scientific principle relates to the universal. A merely particular fact is of no value in science. Facts attain their value, when they can be related to some law. But a general law is a law affirmed about the universal nature. If then we hold that in the real order there is nothing corresponding to the universal term : that for instance, when we assert that water at the sea-level freezes at 32° F., the assertion relates merely to a concept in the mind, or to the meaning of the word ' water ' : our philosophy, in that case, discredits the assured results of science.

The only satisfactory answer that the problem has received, is that afforded by the doctrine known as **Moderate Realism.**

We have already (Ch. 2, § 1) called attention to the manner in which the concepts of the intellect abstract from the individualizing conditions of the object of thought. Our senses distinguish individual entities, even though they be precisely similar to each other. But the mind seizes not on what is individual, but on what is characteristic : and it recognizes that the characteristics which it represents in thought, may be reproduced in an indefinite number of individuals. Take, for instance, the concepts formed by the mind if a sovereign is presented to sense. It conceives it as *gold*, as *yellow*, as *lustrous*. It is true that when these notes are first conceived by the mind, they are not at once viewed as universal : that is to say they are not referred to a number of similar objects. But the mind has only to reflect on the nature of the concept, and it immediately sees that the idea, e.g. of yellow, would serve to express not only the gold under observation, but any number of other things, provided they were similar in that particular respect. Whenever we are brought into contact with several objects presenting similar features, this universality is forced upon our notice. We frame a concept representing the notes they have in common, and abstracting from all others. We see that it is one and the same concept which represents them all, and that the name expressive of these notes may be affirmed of each. Thus my concept ' man ' represents the characteristics common to all men ; and the nature represented by the concept may be affirmed of every one.

It may however be asked, whether it is not my concept of man, rather than the nature of man, which I affirm of the individual. It is manifest that I affirm the nature, not the con-

[1] Nominalism has been the traditional doctrine of the English sensationalist school from the days of Hobbes. It finds its most notable representative in Mill.

cept. I could not say 'Socrates is a man,' if the predicate signified my concept. It signifies the nature represented in my concept.

Nor can it be urged that on this explanation, we affirm the identity of Socrates with what at most is only a part of his being, abstracted from the remainder: that on our showing we assert 'Socrates is human nature.' The concept 'man' while expressly representing the notes only that are common to all men alike, does not exclude the remaining notes. It does not signify 'human nature,' but 'a being possessed of human nature.' Implicitly it includes all the other notes of the individual of whom it is affirmed.

Our conclusions now enable us to answer the question, which we proposed to ourselves at the beginning of the section, viz.: What is the human nature, which we think of as at one and the same time one and many? We reply that this characteristic of multiplicity in unity, belongs to the nature as and only as it is conceived in the intellect; and that it consists simply in the fact that one and the same concept represents all the members of the class in question. But though we know the nature in a universal concept, yet the nature itself, which my mind makes known to me, and which I affirm regarding the subject of my judgment, is in the real order, and it is found in the objects of which it is affirmed.[1] Hence the laws of science, and all our general statements are affirmed, not (as the Nominalists tell us) merely about the common name which stands as subject, nor yet (as the Conceptualists would have it) about our concepts, but about the objects themselves, in which the common characteristics are found. It is as maintaining this position that the adherents of the traditional Scholastic doctrine are rightly termed **Realists.**

It is therefore of prime importance to notice the distinction between what is termed the *direct* and the *reflex* universal. The direct universal is simply the nature abstracted from individualizing conditions, and as thus abstracted affirmable of a subject, e.g. the nature 'man' which we can affirm of Socrates. But when by an act of reflection, we consider this nature *as it is mentally conceived*, we see that it is affirmable of each and every

[1] St. Thomas tells us that that which the mind contemplates is the nature: the concept is that *in which* it knows the nature. "Substantia ergo rei est id quod intellectus intelligit." *Opusc.*, 40, *De Potentiis Animae.* "Istud ergo sic 'formatum et expressum in anima dicitur verbum interius, et ideo comparatur ad intellectum non sicut id quo intellectus intelligit, sed sicut *in quo* in 'telligit, quia in isto sic expresso et formato videt naturam rei intellectae." *Opusc.* 12, *De Differentia Verbi Divini et Humani.* "[Verbum interius] est 'tanquam speculum *in quo* res cernitur." *Opusc.* 13, *De Natura Verbi Intellectus.*

man—that it belongs to all,—that it is one and many. This is the reflex universal.[1]

It will now be apparent how necessary is the discussion of this question to the understanding of the Predicables. For the five Predicables are simply the five possible divisions of the universal thus mentally conceived as being one and many. They give us the five relations in which a class-notion can stand to the things within the class, or as we may otherwise put it, in which a predicate can stand to the subject of which it is affirmed. A universal term in the predicate, must, as we have seen, be Species, Genus, Differentia, Property or Accident. These five therefore are Second Intentions (Ch. 2, § 11). They are terms which can only be affirmed of the nature as it is in the conceptual order : they do not belong to the object in the real order. We cannot argue because Socrates is a man, and man is a species, that therefore Socrates is a species. For Socrates is a man in the real order, while man is a species in the conceptual order alone.

The above account has shewn that this Moderate Realism—the prevalent view among the Scholastic philosophers, bore no resemblance to **Exaggerated Realism,** the extravagant doctrine according to which the universal has objective existence in the real order as a universal, and the species and the genus are held to be real things beyond the individuals of which they are asserted.[2] St. Thomas again and again rejects such a view in express terms, and tells us that his own teaching is identical with that of the two great Arabian Aristotelians, Avicenna and Averroes.[3] Nevertheless English writers on Logic, as a rule fail even to mention this view, and represent the Scholastics as ordinarily teaching the objective reality of universals. An error on so fundamental a point as this, should render the student very cautious in accepting any criticism which such writers may pass on the Aristotelians of the middle ages.

* § 5. **The Universal in Modern Logic.** The view of the universal taken by Mr. Bradley and Mr. Bosanquet is radically different from any of the theories as to its nature, which we have hitherto considered. According to this view it consists in "a persistent identity in difference"

[1] On Direct and Reflex Universals, see Maher, *Psychology*, Ch. 14, pp. 294 and *sqq.* (ed. 6).

[2] Exaggerated Realism is famous as the doctrine of Plato. A somewhat similar view was maintained by William of Champeaux (d. 1121).

[3] The following will serve as a typical passage. *Opusc.* 39, *De Natura Generis*, c. 5. "In re igitur nihil est commune multis, quia quidquid est in re ‹ est singulare, uni soli communicabile. Quod autem commune est, dicitur per ‹ intellectum. Intellectus enim facit universalitatem in rebus, ut dicit Com- ‹ mentator [i.e. Averroes] supra librum de Anima."

(Bosanquet, II., p. 92). This identity is not understood as a conceptual identity expressing things, which are in the real order not the same but similar, but as a real identity. "If we say 'that *A* and *B* are alike," says Mr. Bradley, "we must be taken 'to mean that they are so far the same. . . . If *A* and *B* for 'instance both have lungs or gills, they are so far the same" (*Principles*, Bk. II., Pt. 1, c. 6, § 3). "What seems the same, *is* the 'same and cannot be made different by any diversity" (*ibid.* § 5). The belief, he urges, that there is no reality except exclusive particulars, is not proved. It is "a mere inherited pre-'conception which has got to think itself a real fact" (*ibid.* § 8). Similarly Mr. Bosanquet tells us that "Mortality is affirmed of 'all individual men in virtue of a oneness of nature running through 'them all : and therefore we must take individual unity to be a 'matter of degree, and to be wholly absent in no content that 'can be presented to thought as designating a subject of judg-'ment" (*Logic*, I., p. 148). This view, that universality involves identity is a natural result of the Hegelian philosophy, which, as we have seen, denies the existence of any distinction between thought and things. A further development of the same doctrine is to regard the concrete individual as a universal. The finite individual is universal, says Mr. Bradley, because it is "the identity of its own internal diversity" (*Principles*, Bk. I. c. 6, § 38). (Cf. Bosanquet, I., p. 210.)

There are thus two fundamental errors in the theory. First, distinct things are asserted to be identical, because, under a certain aspect, they can be expressed by one and the same concept. Secondly, the real identity of an individual thing in virtue of which it persists through changes and is the bond of many attributes, is confused with the conceptual identity of the universal. A theory of Logic based on errors such as these, whatever be the subtlety with which it be defended, must needs be radically fallacious.

CHAPTER IX.

THE CATEGORIES.

§ 1. **The Categories in their Metaphysical Aspect.** The **Categories** (or **Predicaments**) may be considered both as belonging to the real order, and as belonging to the conceptual order,—in other words, both metaphysically and logically. It will be convenient to treat of them under their metaphysical aspect first, and to deal with their logical aspect afterwards.

Viewed as belonging to the real order, they were termed by the Scholastics the ten *summa genera* of things—the ten classes, to one of which all things whatever could be referred. But in this account of the Categories, the word 'things' must be rightly understood. In ordinary discourse, by things we mean substances: and a division of things into classes would signify a classification of substances. This is not what is meant here. The word 'things' includes accidents as well as substances. For accidents are real entities, though they cannot exist apart from substances. It is one thing to be a horse, another to be black, another to be carrying a rider. Yet Bucephalus may be all these things at one and the same time. It is in this sense we must understand the word, when we term the following list a classification of 'things.'

1. **Substance**:—natures which exist not as mere determinations, but in their own right, e.g. Socrates, man, animal.
2. **Quantity**:—the spatial extension, height, breadth of a substance.
3. **Quality**:—determinations which characterize the nature.

Aristotle in the Categories distinguishes four kinds of qualities, viz. :—

(1) *Habits* and *Dispositions*, i.e. determinations of the nature itself, e.g. knowledge in the intellect, health in the body.

(2) *Capacity* and *Incapacity*, i.e. determinations of the active powers, e.g. the capacity to walk. The term *Incapacity* signifies not the total absence of the determination, but its possession in an undeveloped and immature degree.

(3) *Passive Qualities*, i.e. determinations consequent on or productive of physical change, e.g. the sensible qualities of cold, heat and colour.

(4) *Figure*, i.e. determinations of quantitative extension.

It should however be observed that these four classes are not put forward by Aristotle as mutually exclusive, nor even as necessarily exhaustive (*Categ.*, c. 8, §§ 3, 6, 23). They merely embody the distinctions recognized in the current vocabulary of the Greek language.

4. **Relation** :—the order which holds between one substance and another. Thus two substances may be *alike* in quality, *equal* in quantity, the *same* in specific nature.

5. **Place** :—position in relation to surrounding space, e.g. at London, in Westminster Abbey.

6. **Time** :—position in relation to the course of events, e.g. last year.

7. **Posture** :—the relative position of parts in the object itself, e.g. sitting, lying down.

8. **Habit** :—the determinations accruing from the physical adjuncts, which belong to the full integrity of the substance as a necessary equipment for its work in Nature, e.g. armed, cloaked.

9. **Action** :—the production of a change in some other object, e.g. digging.

10. **Passion** :—the reception of change from some agent, e.g. being struck.

These ten Categories may be illustrated in the case of an individual. We find that in order that the individual may fill his place as a part of Nature, he must be determined in all these ten ways. Here, for instance, is some one whom I know. As man, he is *substance*—something that is not a determination, but can receive determina-

tions. As regards *quantity*, he is six feet high. Among his *qualities* it may be noted that he is (1) a mathematician, (2) a skilful carver, (3) swarthy, (4) square-shouldered. He closely resembles his father, to whom he is thus related by the *relation* of likeness. He is in the city of York (*place*); and it is October, 1907 (*time*). He is stooping (*posture*); dressed in cloth, and provided with a hammer and chisel (*habit*); he is carving wood (*action*), and his tool has just cut him (*passion*).

In what sense are the nine categories of Accidents called 'things'? How can we regard the quality of swarthiness or the relation of likeness as 'a thing'? If indeed we are speaking of these accidents considered as separate entities, then they are not things at all: there is no such thing as 'walking' or 'sitting' or 'health' existing independently. These accidents are said to *be*, not because they possess existence themselves, but because the concrete subject is what it is, through them.[1] The concrete subject *is* walking, and *is* healthy. Hence, as Aristotle says, "They are called 'things,' because in their 'respective ways, they determine that which is a thing in 'the sense of being a substance." [2]

In speaking of the Categories as the '*summa genera* of things,' we have employed a traditional expression. But their nature would perhaps be more clearly indicated, by calling them the ten *modes of being* in which an individual thing is realized. For the various kinds of determination —the classes of 'things'—differ one from another by the mode of being, which they confer on the individual.

It is now clear why there is not one *summum genus* only, viz.: Thing or Being, of which these classes are subordinate species. Genus and species are only found, when the various classes can be expressed in a common univocal concept, and are distinguished by specific differen-

[1] St.Thomas, *Summa Theol.*, I., Q. 45, Art. 4. "Illi proprie convenit Esse, quod 'habet Esse, et est subsistens in suo Esse. Formae autem et accidentia et alia 'hujusmodi, non dicuntur entia quasi ipsa sunt, sed quia *eis aliquid est*: ut 'albedo eâ ratione dicitur ens, quia eâ subjectum est album."

[2] *Met.*, VI., c. 1, § 2. τὰ δ' ἄλλα λέγεται ὄντα τῷ τοῦ οὕτως ὄντος τὰ μὲν ποσότητας εἶναι, τὰ δὲ ποιότητας, τὰ δὲ πάθη, τὰ δὲ ἄλλο τι τοιοῦτον.

tiae. There is no univocal concept of 'thing,' nor can we find differentiae, by which the notion is determined. 'Thing' is an analogous term. It has a different meaning as applied to each of the ten categories : and though we can form a universal notion expressive of Thing in general, it possesses the universality of an analogous, not of a univocal concept. For the same reason there is no common genus 'Accident.' The nine kinds of accident are irreducible.

It has been asserted by some authorities that the order of the Categories is based on no intelligible principle, that Aristotle apparently drew up his list in a haphazard fashion.[1]

Most assuredly, this would be a very extraordinary feature in a doctrine to which Aristotle recurs so frequently. As a matter of fact those who say this, have missed a most important feature in the doctrine of the Categories. The order in which they are enumerated, reveals to us **the law governing the synthesis of forms in the individual.** This will easily be seen on consideration. The substantial nature gives us as it were the starting point. In virtue of this, the thing possesses independent existence, and is the kind of thing it is. But the substantial nature cannot exist without those accidental forms which constitute the complement of its being.[2] Of these the first is magnitude or extension—the second Category. Physical qualities presuppose extension : they are supported by the substance as extended. From these three primary Categories result three kinds of relation, which give an order and harmony in the manifold of the universe, viz.: likeness of quality, equality of quantity, sameness of specific form. The remaining categories are extrinsic determinations. They add nothing to the entity itself. But in the physical universe, each entity is determined not merely by its

[1] Thus Dr. Wallace says (*Outlines of the Philosophy of Aristotle*, p. 25), "These ten Categories would seem to be arranged on little or no principle."

[2] The order of the Categories is of course an order of relative subordination. It is in no sense an order of time. St. Thomas, *Opusc.* 63 *in Boeth. de Trin.* Q. 5. art. 3.

inherent characteristics, but by its status as a part of the whole. It must be at a definite point in space, and at a definite period in time. Its own parts are disposed either after one order, or after another. Further, in the case of man, a special Category arises—that of habit. For man, unlike the beasts, is not fully equipped by Nature. The incompleteness of his equipment is a necessary consequence of his intelligence. Irrational creatures are provided with the instruments and the extraneous covering requisite to enable them to attain the ends to which instinct guides them : for these ends are confined within definite limits. Man in virtue of his intelligence pursues ends which are unlimited in their range. Hence his equipment cannot be a natural endowment, but must be the work of the inventive reason (*Summ. Theol.* I. q. 76, art. 5, ad. 4).

The entity is now determined as a part of Nature. But between the different physical substances there is mutual interaction. Hence arise the two last Categories, action and passion.

* It is often asserted that the last six Categories are all relations, that they have no right to be reckoned as independent classes. It may be of interest to indicate in a very brief manner on what grounds St. Thomas would have founded the claims of these Categories to independence.

In regard to the Category of place, it is true that when we state where an object is, we do so by indicating its relation to other bodies. But the thing is not constituted in place by these relations. It is possessed of these relations because it is in this place. Local position itself is not a relation. For us the place of a thing is defined by its distance from certain assigned points. Thus the place of an object in a room is defined by its distance from the enclosing walls : the place of an object in the air by its height above such and such a spot on the earth's surface. The case of time is analogous to that of place. This, too, is an extrinsic determination irreducible to one of any other kind. As concerns posture, a change of position among the various parts of one and the same thing, cannot be called a change of relations : for here also we have but one substance, not two. The Category of habit would indeed be reducible to relation, if man could be

looked on as a complete substance related to the extraneous substance which is predicated of him. But, as we have seen, as a part of Nature he is incomplete until he has received the additional determination conferred by the adjacent 'habit.' Action and passion include relations, but are differentiated from that Category, inasmuch as they further involve the production of change, or its reception.

It may be noted that there has always been considerable divergence of opinion among Scholastic authors regarding the value of this tenfold division, and as to whether certain of the number can justly be reckoned as independent classes. The reader may be referred to the treatment of the subject by Domet de Vorges, *Abrégé de Métaphysique*, II., cc. 25–36.

§ 2. **The Categories in their Logical Aspect.** In their logical aspect the Categories are no longer modes of real being: they belong to the conceptual order. It is as thus regarded that they belong properly to our subject. They claim our attention as *a classification of things as mentally represented.*

It is under this aspect that Aristotle treats the Categories in his famous work known by that name. As thus understood, they are defined as **the orderly classification of genera, species, and individuals from the summa genera to the individual entities.**[1] The treatment of this subject belongs of course to the Logic of the Concept, not to the Logic of the Judgment.

Things in the real order are all singular. The substances, qualities and quantities which are found in the external world cannot be universals. The singular alone exists. But when we pass to the order of knowledge we find not merely things singular but things universal. Our intellect shews us universal natures such as *man, animal, substance.* These terms signify real substances, for they can one and all be affirmed of the concrete individual: and what is identical with the individual is real. Yet at the same time they are universal; though *as universal* they exist in our mind alone. In precisely the same

[1] Joan. a S. Thoma. (1590–1644), *Log.*, Q. 14, Art. 1. "Praedicamentum nihil 'aliud est quam series seu co-ordinatio praedicatorum superiorum et inferiorum ab 'uno supremo genere quod praedicatur de omni inferiori, usque ad individuum quod subjicitur omni superiori." Cf. Suarez, *Disp. Met.*, XXXIX. Proem.

way, the mental order reveals to us not merely the singular quality *this whiteness*, but the universal quality *colour*. The store of these conceptual representations of the real order which at any time is found in our minds, constitutes the elements into which our knowledge is resolvable. They are that out of which our mental furniture is built up. Of them our judgments and our reasonings are all formed.

A careful scrutiny of the character of these concepts reveals the fact that they stand in a hierarchical subordination one to another. Some are such that they cannot be predicated universally of any other ; some again, while they can be predicated of others, can themselves stand as the subjects of predication : while a last group can only occupy the position of predicates, and are incapable of receiving predication (*An. Prior I.*, c. 27, § 2). The first of these three classes consists of individual things—singulars. In the second are found all those universal terms, which themselves stand as species to wider generic conceptions. In the third class are the *summa genera* alone. Thus we can say, 'Socrates is a man,' 'Man is vertebrate,' 'Vertebrates are animals' : but we cannot reverse the order and say, 'Animals are vertebrates,' 'Vertebrates are men.'[1] Of what fundamental importance in Logic is this law, in virtue of which the wider concept is predicated of the more restricted, we shall see in subsequent chapters.

It further appears that this hierarchical arrangement falls into distinct groups,—the groups we have designated as the Categories. We cannot predicate a term be-

[1] This point is well put by Mr. Joseph (*Introd. to Logic*, p. 236). "We say 'that diamonds glitter, rather than that some glittering things are diamonds : 'that blue is a colour, rather than that a colour is blue. To say that a colour 'may be blue is natural enough : just as it is to say that a stone may be a dia-'mond : but still we predicate the genus of the species, and not the species of 'the genus : it is not the genus colour, but colour in some particular case ; not 'the genus stone, but some particular mineral that is blue or that is diamond. 'Commonly, except where they are mere coincident attributes, the predicate 'is a wider term or more generic than the subject in judgment." The fact to which Mr. Joseph here calls attention, viz. : that when we say 'Some stones are diamonds,' we are predicating the species of *individuals*, not of the genus, at once appears, if we reflect that we cannot say, 'Stone (*as such*) is diamond,' 'The triangle (*as such*) is equilateral.'

longing to one Category of another, except where the two determinations happen to belong to one subject. The natures expressed by the different Categories, are fundamentally distinct.[1]

The investigation of the conceptual order has thus revealed the fact that it is not a chaos of concepts, but that, no less than the real order, it is governed by definite laws. This discovery must ever rank as one of the greatest results of Aristotle's genius.[2]

Each Category then consists, as we have said, of an ordered series of terms significative of a special mode of being, commencing with the term that designates the *summum genus*, and ending with singular terms, e.g. *this man*. Each series develops into a tree of Porphyry. But, as we have noted on more than one occasion, the universals of the Categories must not be confused with the universals of the Predicables. In the Predicables we view the universals as universals : we consider in what relation the abstracted and universal nature stands to the subjects of which it is predicated. Thus the Predicables are all terms of Second Intention. The Categories on the other hand are terms of First Intention. We are concerned only with the nature expressed, and not with the character of the nature so far as universal. As belonging to the Category of substance the nature ' man ' is not ' a species,' but is ' rational animal.'

§ 3. **The Categories in their Relation to the Sciences.** The important bearing of the predicamental lines on

[1] Hence as arranged in ascending series according to their several categories, the terms belonging to the nine genera of accidents are expressed in the abstract, not in the concrete form, e.g. as ' prudence,' ' virtue,' not ' prudent,' ' virtuous.' The latter form expresses the accident as qualifying something else. Thus ' white ' signifies some substance which is qualified by whiteness. It is by the abstract term alone that we can signify the accident as a determination distinct from the subject in which it inheres (vide Pesch. *Inst. Log.*, § 1449). We do not, however, as has been objected, transform the accident into a substance by employing the abstract term. The concrete term alone expresses the subsistent reality which belongs to substance (S. Thomas, *Summa Theol.*, I., Q. 3, ad. 3).

[2] The Categories provide a complete classification of concepts. Negative and Privative terms belong to the category to which belongs the positive term, by means of which they are conceived. Terms of Second Intention belong to the Category of Relation.

those organized bodies of knowledge which we term sciences, should not be overlooked. In any such line, the concepts in their ascending scale of wider and wider generality furnish us with so many distinct objects of scientific consideration. Provided that our abstractions are not arbitrary, but are grounded on the nature of things, the objects expressed by the several concepts have each of them a group of attributes peculiar to itself. Thus in the series *horse–equidae–ungulate mammal–vertebrate*, etc., each of the types expressed has many distinctive properties. These properties are affirmed of the type in a system of true universal, i.e. generic judgments: e.g. 'Equidae *as such* tread only on the hoof upon the last phalanx of the third digit.' The group of generic judgments known in regard of any object, constitutes the science of that object.

It is not merely in regard of substances that the predicament has this function. In other Categories also, science organizes itself in our mind on this basis. Thus *quantity—continuous quantity—extension—plane surface—triangle*, provide us with so many heads of scientific knowledge. Did science in any case reach its unattainable ideal, it would consist of the accurate definition of some object thus universally conceived, together with the complete series of attributes predicable of it, whether as constituting its own properties or as belonging to it in virtue of some type higher in the scale. Thus the Categories provide us with the principle on which the analysis of the sciences must be based. They shew how the various sciences are correlated in our mind: and they shew how their respective objects are distinguished from each other, either as falling within different Categories, or as differentiated by various degrees of abstraction. The *Posterior Analytics* of Aristotle, which we have described above (Ch. 1, § 5) as a treatise on the logical analysis of science, throughout considers the sciences under this aspect and in the light of these principles.

§ 4. **The Categories as a Classification of Predicates.** The Categories may further be viewed in their bearing

on the logical proposition. As such they form a classification of possible predicates. It is under this aspect they are introduced by Aristotle in his *Topics* (I., c. 9). And it is as thus conceived that they have received the name of Category (*κατηγορία, κατηγορεῖν to predicate*), and its Latin equivalent *Predicament*. The very purpose of the proposition, is to tell us what the subject *is*—to assert some form of *being* in its regard. Since then the Categories are our mental representations of ' being ' in its various modes, it follows of necessity that the predicates of all propositions may be grouped according to the particular kind of ' being ' asserted, in other words according to the ten Categories.[1]

From this it appears that the doctrine of the Categories throws a new and most important light on the meaning of the ' is ' of the copula. The discussion as to the import of the proposition, is in fact incomplete, apart from the doctrine of the Categories. The copula, as we saw, signifies that the subject is (or is not) determined in some way. The Categories shew us that it does not express one mode of determination only, but a variety, according to the different modes of ' being.' According to the predicate employed, it signifies that the subject is determined either substantially, or quantitatively, or qualitatively, or by some relation, etc., etc.[2] Nothing can be more erroneous than to hold that the determination is always of one kind.[3] This is the mistake which we have noticed on the part of those who hold the equational theory as to the import of propositions. They regard the copula as always signifying that the subject is determined by a relation of equality.

[1] Cf. St. Thomas *in Met.*, V., lect. 9. " Propter hoc ea in quae dividitur ens 'primo, dicuntur esse *praedicamenta* quia distinguuntur secundum diversum 'modum praedicandi."

[2] St. Thomas, *Opusc.* 39, *De Natura Generis*. " Eorum autem quae praedican-'tur, quaedam significant quid, quaedam quantum, quaedam quale, et sic de ' ceteris : ideo oportet quod unicuique modo praedicandi ' esse ' idem significet : ' ut cum dicitur ' homo est animal,' esse significet substantiam, cum vero dicitur ' homo est albus,' esse significet qualitatem, et sic de aliis praedicamentis." Cf. *in Met.*, V. lect. 9.

[3] Such a theory of the copula may be found in Sidgwick's *Fallacies*, p. 53.

It may at first sight appear inaccurate to speak of the predicate as *determining* the subject, when it is a more generic term within the same category. We do not *determine* the subject when we say, e.g. 'Man is an animal,' 'A triangle is a plane figure': for the predicate in these cases shews us the same nature as the subject but more abstractly conceived. But it must be remembered that in any proposition the predicate alone is understood in comprehension, the subject being understood in extension without formal advertence to the attributes it connotes. Hence even where the predicate is a generic term, it is conceived as one of the attributes or determinations belonging to the subject.[1]

* § 5. **Mill's Scheme of Categories.** Mill, after enumerating the Categories of Aristotle, remarks that "the imperfections 'of this classification are too obvious, and its merits not 'sufficient to reward a minute examination." He therefore proposes to "recommence under better auspices, the attempt 'made with such imperfect success by the great founder of 'logic."

After a lengthy discussion, the following is the scheme he gives us. It should be premised that the point of view is altogether metaphysical. It is a list of things, and is not proposed in any way as a classification of concepts :—

"(1) Feelings, or States of Consciousness.'

'(2) The Minds, which experience those feelings.'

'(3) The Bodies or external objects, which excite certain of 'those feelings, together with the powers or properties whereby 'they excite them: these last being included rather in com-'pliance with common opinion, and because their existence is 'taken for granted in the common language from which I can-'not prudently deviate, than because the recognition of such 'powers or properties as real existences appears to be warranted 'by sound philosophy.

'(4) The Successions, and Coexistences, the Likenesses and 'Unlikenesses between feelings or states of consciousness."

It would be beyond the scope of this work to enter upon a criticism of Mill's metaphysical views. It must be sufficient to note that this scheme is not even consistent with his own philosophical position. He holds strongly that the mind has no reality save that of the feelings which it experiences: that it has no substantial existence of its own. He has, therefore, no justification for placing minds in a separate category from feelings. The same is true in regard to bodies. Bodies, as existing realities apart from the sensations by which we are

[1] Cf. Joan. a S. Thoma., *Logica*, II., Q. 5, Art. 2, ad. 4.

conscious of bodies, are, according to him, a figment of the imagination. They too then should have been relegated to the category of Feelings. Relations fare no better. He tells us that no relation is anything but our feeling of such a relation. The philosophy of Mill, is in fact incompatible with any scheme of Categories at all : for it reduces a reality of whatever kind to mere subjective feeling.

* § 6. **The Categories of Kant.** Kant, as we have noted (Ch. 1, *note* (5)), assumed as certain the hypothesis that the data of knowledge consist solely in the subjective states of our own mind : and further that these states are mere instantaneous and unconnected feelings, neither in time nor in space. The problem, therefore, before him was to explain how, if these are our data, our experience can present us with the world such as we know it. The result, he held, is due to subjective principles of our cognitive faculties, which operating on the unconnected mental impressions, fashion them within us into the appearance of a world. The internal knowledge thus afforded, is all we can hope to possess. It is due, he taught, to the sensitive faculty that our impressions appear in time and space. The intellectual faculty, also, provides twelve 'forms' of its own, corresponding to the twelve different species of judgments.[1] These twelve 'forms' he termed Categories. They are shewn in the following list :—

Forms of Judgment.	Categories.
I. Quantity.	
1. *Singular* (*This S is P*)	1. Unity.
2. *Particular* (*Some S is P*)	2. Plurality.
3. *Universal* (*All S is P*)	3. Totality.
II. Quality.	
4. *Affirmative* (*S is P*)	4. Reality.
5. *Negative* (*S is not P*)	5. Negation.
6. *Infinite* (*S is not-P*)	6. Limitation.
III. Relation.	
7. *Categorical* (*S is P*)	7. Substance and Attribute.
8. *Hypothetical* (*If S is P, Q is R*)	8. Cause and Effect.
9. *Disjunctive* (*S is P or Q*)	9. Reciprocal Action.

[1] Kant believed that all thought is judgment, and that for that reason the various kinds of judgment must necessarily shew the various modes in which it is possible for the intellect to shape our knowledge.

Forms of Judgment.	Categories.
IV. Modality.	
10. *Problematic* (*S may be P*)	10. Possibility and Impossibility.
11. *Assertoric* (*S is P*)	11. Existence and Non-existence.
12. *Apodictic* (*S must be P*)	12. Necessity and Contingency.

Thus, by way of illustration, if I judge that 'All men are mortal,' this is not, as I fondly imagine, because there is an actual world outside me, in which there is a race of men, all of whom have this attribute of mortality : it is because within my mind the four Categories of Totality, Reality, Substance and Accident, and Existence and Non-existence, have shaped my knowledge, and have thus produced a judgment which is universal, affirmative, categorical, and assertoric.

We are not called on here to enter on a discussion of the Kantian philosophy. But apart from the more fundamental errors with which the system may be charged, the scheme of Categories, considered as an analysis of logical judgments, contains, as we have already pointed out (Ch. 3, §§ 3, 9), serious imperfections.

* § 7. **The Concept of Being.** We have already noted (§ 1) that the term *Being* cannot be the *summum genus* in regard to the Categories, inasmuch as it is predicated of the ten classes in distinct though analogous senses. It seems desirable to indicate the true relation of this concept to these classes. The Categories express as we have seen the nature or *essence* of things : they tell us *what the thing is*. The *infima species* gives us the complete nature : the higher classes express it as less completely determined. The concept of Being on the other hand has nothing to do with the essence : it is concerned with the *existence*. When I affirm *Being* of a subject, I mean that that subject can exist *in rerum natura*. The concept of existence can have no place in a scheme, which, while it expresses essences in their different degress of determination, prescinds from the question as to whether they exist or not.

That this view of the concept *Being* is the true one may be seen from the following consideration. In regard to the nature of an object our knowledge progresses from the indeterminate to the determinate. We recognize that it is corporeal substance : then that it is a living animal : finally we arrive at the complete type. There is no advance whatever in regard to our recognition of it as Being. From the first we realize that it is a Thing—it exists. No increase of knowledge can reveal a higher degree of existence.

CHAPTER X.

DEFINITION AND DIVISION.

§ 1. **Definition.** We are concerned in this chapter with two processes, both of which belong, not to the Logic of the Judgment, but to that of the Concept. Neither Definition nor Division, however, can be satisfactorily treated, unless the Predicables have previously been explained. This, therefore, seems to be the most convenient place at which to deal with them.

The definition of an object is **the declaration of its essential characteristics.** Hence, a definition is given in the form of a proposition, in which the object defined stands as the subject, and the essential characteristics form the predicate. It is this predicate which is the definition properly so called. The discussion of the question belongs, as we have said, to the Logic of the Concept : for considered as a mental act, the definition is the concept which expresses the true nature of the thing defined.

It is carefully to be observed that the definition is concerned with *the nature of a thing*. For by some logicians it is explained as being simply the connotation of the subject term, as it is understood by competent thinkers. Now it is, of course, the case that whenever the essential characteristics of a thing—its true nature,—are known, these will constitute the intension of its name. Thus the intension of the term ' triangle,' is ' a plane figure contained by three straight lines.' But it will often happen that a name is applied to a group of objects, which we are perfectly able to identify by certain common properties they possess, while at the same time, we are ignorant of their real nature. Thus,

for instance, when a new disease, e.g. the sleeping sickness, makes its appearance, doctors recognize it and give it a name, long before they are able to define it. The term in this case, has a connotation, viz.: the symptoms by which the disease is known : but we have not yet found the definition *of the thing*. The true definition must do more than enable us to recognize it. It must unfold its nature.

Aristotle expressed this, by saying that the definition gives us the ' why ' of the thing.[1] The definition ' Man is a rational animal,' is a case in point. If we are asked what makes Socrates a man, we reply that he possesses these characteristics. It is not because he is ' a tool-using animal,' that he is a man, nor yet because he is ' an animal that cooks his food,' though these statements are true. He is a man *because* he is a rational animal. The ideal definition will then contain the essential characteristics. To what extent we are able actually to realize this ideal in our definitions, we shall see when we study the various kinds of definition.

Definition is always of *the universal*. Nature gives us general classes, and phenomena which occur subject to general laws. The individual members of these classes, the individual instances of the phenomena, are all different : each has accidental characteristics, by which it differs from every other. The aim of definition is to seize on the type, which is constant amid all this variety. One attack, e.g. of sleeping sickness, or of malarial fever, differs from another in a hundred particulars,—in duration, in intensity, in collateral effects, etc., etc. These are of no importance to the definition ; for it is concerned alone with what is essential—with the permanent type.[2]

Hence definition is rightly said to be the aim of science.[3]

[1] *An. Post. II.*, c. 2, § 5. *Ὥσπερ οὖν λέγομεν τὸ τί ἐστιν εἰδέναι ταὐτό ἐστι καὶ διὰ τί ἐστιν.* Cf. *ibid.* c. 8, § 1.

[2] *An. Post. II.*, c. 13, § 19.

[3] Cf. Rabier, *Logique*, p. 180. La définition, au sens de concept résumant la science, est la fin de la science. Mercier, *op. cit.*, § 153. La définition est avant tout un moyen d'asseoir les bases de la science.

Science has achieved its object, when it has accurately determined the nature of some substance, or the law of some phenomenon.

It is often said that all definition should be by genus and differentia. There are indeed certain cases in which we can assign the genus and differentia, understanding those words as they were employed in connexion with the Predicables. In some cases, however, this is impossible. Thus an eclipse can be defined, but it has not properly speaking a genus or differentia. Hence these terms are here employed with a certain amount of latitude. Genus should be understood as meaning no more than such attributes as are common alike to the class of objects in question, and to other classes : differentia signifies the notes which are proper to the class, and distinguish it from others.[1]

§ 2. **Various Kinds of Definition.** (i) **Real and Nominal.** In the last section we shewed in what a Real definition consists. It is an expression which declares the nature of a thing. A Nominal definition on the other hand, is an expression declaring the meaning of a word. Some logicians, as we have already noticed, have maintained that no definitions are intended to do more than this ; that one and all they merely unfold the connotation of terms. Aristotle considers this question at length, and distinguishes two kinds of Nominal definitions. In the first place, there are (1) definitions of names which signify imaginary objects, to which nothing either actual or possible corresponds. We may find an example in the definition of a dragon as 'a serpent breathing flame.' An impossible self-contradictory concept cannot provide us with a Real definition : for that must state the essence. An essence which contains repugnant characteristics is no essence at all.[2] Indeed the expression, 'A dragon is a

[1] Cf. Joan. a S. Thoma., *Logica*, Bk. II., c. 3. "Definitio fit per genus et 'differentiam; et cum haec conditio non solum complectatur definitionem 'essentialem, in qua proprie invenitur genus et differentia, sed etiam descrip-'tivam et accidentalem in qua non est proprié genus et differentia, sed aliquid 'loco illius; ideo intelligitur nomine generis aliquid commune, nomine differentiae 'aliquid distinctivum particulare."

[2] *An. Post. II.*, c. 7, § 2. τὸ γὰρ μὴ ὄν, οὐδεὶς οἶδεν ὅ τι ἐστίν, ἀλλὰ τί μὲν σημαίνει ὁ λόγος ἢ τὸ ὄνομα, ὅταν εἴπω τραγέλαφος, τί δ' ἔστι τραγέλαφος, ἀδύνατον εἰδέναι.

serpent breathing flame,' is elliptical. Fully stated, the proposition should be 'A dragon (as imagined by the writers of fables) is a serpent breathing flame' (Ch. 7, § 4).

Aristotle further reckons as Nominal definitions, (2) those in which we are unable to assign the essential properties of a thing, and are merely able to indicate it by a description. Thus, the definition of thunder, as 'a noise in the clouds,' is nominal: nominal, because it describes what is meant, and yet does not unfold the nature of the object (*An. Post. II.*, c. 10, § 1, c. 8, § 7).

All Nominal definitions, therefore, do not deny existence to their objects. In this latter class existence is presupposed. But in Aristotle's view, none save those which express the essential characteristics, can rightly be termed definitions of the thing. This distinction of Real and Nominal has largely fallen into disuse for reasons to be mentioned presently. But it should be carefully noticed, for the principle it embodies is one of importance.

* Mill holds strongly to the position that definitions are merely explicative of names. Hence he is led to consider at some length the alleged importance of definitions. Is it the case, he asks, that some sciences, e.g. Geometry, are deduced from statements as to the meaning of names? His answer is that the sciences are not derived from the definitions at all, but from the implied postulate of the existence of the thing in question. Thus the definition of triangle "obviously comprises not one but two 'propositions, perfectly distinguishable. The one is, 'There 'may exist a figure bounded by three straight lines'; the other, ''And this figure may be termed a triangle.' The former of 'these propositions is not a definition at all. The latter is a 'mere Nominal definition. It is the former which is the basis 'of all reasoning about the triangle."

Mill's analysis is here at fault. Often it is true we do presuppose the existence of the object. This occurs when, previous to definition, the existence of the object has been manifested to us. For instance men knew that the relative motion of earth and sun existed as a fact, long before they knew how to define it. When at length, it was expressed in a definition accurately setting forth the law of the process, they then possessed a sure basis for reasoning. But the ground of reasoning was not the

existence of the phenomenon, nor even the nature of the phenomenon in itself, but its nature *as known*—in other words, the definition. Mill has combined the postulate of existence and the definition of the nature, in his first proposition : and then he triumphantly points to a second proposition, which is neither postulate nor definition, and styling it a definition, informs us that it cannot be made the foundation of a train of reasoning. As a matter of fact in the case of a triangle, the postulate of existence is unnecessary, since the statement of its essential characteristics is quite sufficient for the mind to recognize that we are dealing with a *possible* essence. It is not the existence of objects, but their essences that are the foundation of science : many mathematical figures have never existed. When the essence is once known, we can deduce the properties which flow from it. From the definition of a right-angled triangle Pythagoras deduced *Euclid*, I., 47.

(ii) **Essential Definitions.** These are the definitions which are formed by genus and differentia in the stricter sense. Such for instance is our definition of man as 'a rational animal.' Such too are our definitions of mathematical figures. The limits by which a plane figure is bounded, constitute its specific differentia. 'A plane figure contained by three straight lines,' is the essential definition of a rectilinear triangle.

In the case, however, of every other natural type except man, it is impossible to obtain an Essential definition. The specific differentia, from which its peculiar properties flow, is unknown to us. While we recognize that the substantial principle, which determines the distinctive characteristics of, e.g. a lion, must needs be totally different from that of a horse, we can never hope to penetrate to any knowledge of the two principles, except in so far as they are manifested by their properties. We must then be content with definition by properties, or as it is often called :—

(iii) **Distinctive Definition.** It is at definitions of this kind that the student of natural history or of physical science, aims. He seeks to state the most characteristic properties of the type with which he is dealing.[1] More-

[1] Cf. Arist. *De Anima*, I., c. 1, § 8. τὰ συμβεβηκότα συμβάλλεται μέγα μέρος πρὸς τὸ εἰδέναι τὸ τί ἐστιν, 'The knowledge of the properties contributes in a

over he recognizes that he can scarcely hope to attain finality in his quest. For the number of distinctive properties in every natural type is vast, and it is always possible that he may discover some property of primary moment hitherto overlooked.

These definitions are of the highest importance in science. For it is on them that is based the scientific classification of natural types. This application of definition will form the subject of a later chapter.

It is because these Distinctive definitions are the only definitions attainable in the case of natural classes, that Aristotle's division of Real and Nominal definitions has been discarded. It would be manifestly unsatisfactory if a class of definitions, which, within their own sphere, are the highest result of science, were ranked as merely Nominal, because they fail to reveal the specific essence of the nature. Yet between the Distinctive definition which states properties only, and the Essential definition which unfolds the essence, there is a great difference. For wherever we possess the Essential definition we can reason from the essence to the properties. But we cannot from the properties given in the Distinctive definition, conclude to the essence. Moreover, the same substance under different circumstances, manifests itself by different properties. Thus the properties of phosphorus are totally different from those of what is termed amorphous phosphorus : while it is needless to point out how much those of carbon, graphite and diamond differ from each other. Yet in each of these cases, we are supposed to be dealing with the same substance, manifesting itself by different properties when affected by different conditions.

(iv) **Genetic Definitions** give us neither the essential nature of the thing nor its properties, but the elements which, taken in conjunction, result in its production.

great measure to a knowledge of the essence' : and St. Thomas, *Opusc.* 26, *De Ente et Essentia*, c. 5. In rebus enim sensibilibus ipsae differentiae essentiales nobis ignotae sunt : unde significantur per differentias accidentales quae ex essentialibus oriuntur, sicut causa significatur per suum effectum.

The term is most frequently used in reference to certain mathematical definitions, which express the nature of a figure by a statement of the manner in which it may be constructed. Thus a circle may be defined, as a figure formed by the revolution of a line in a plane round one of its extremities. Here the several segments gradually traced by the line are not themselves a circle. Yet when the process is complete, and all the parts are seen in conjunction, they constitute a circle. But the employment of Genetic definitions is by no means limited to this special case. The definitions employed in chemistry are of this type, as, e.g. the definition of water as 'two atomic weights of hydrogen chemically combined with one of oxygen.' For these definitions do not inform us regarding the qualities of the substance. They tell us what constituents are requisite to its production.

(v) **Causal Definitions** are of two kinds. (*a*) One class defines by indicating the *Final cause* or purpose of the object (Arist., *Met.*, VII., c. 2, §§ 7, 8). This form of definition is the one ordinarily used in the case of the works of human ingenuity. We define a clock by saying that it is 'a mechanism destined to indicate the hours of the day,' a stirrup by terming it 'a metal hoop for the purpose of supporting the foot when riding.'

(*b*) The other class of Causal definitions defines by stating the *Efficient cause*. It may often happen that the most satisfactory explanation of the nature of the thing, is given in this manner. Thus since the days of Koch, who first discovered that certain diseases were due to the presence of specific microbes, it is a true definition of anthrax (wool-sorter's disease) to say that it is 'an illness caused by the introduction into the body of the *bacillus anthracis*.' Some things, e.g. natural products, can hardly be defined in any other way. We naturally define the double cocoa-nut as 'the fruit of the tree *Lodoicea Seychellarum*.'

(vi) **Accidental Definitions.** These are employed to define those sub-classes, which do not constitute distinct

species. Thus a negro may be defined by his colour as 'a black man,' even though colour be an accident and not a property of the nature. These are, however, rather to be regarded as *descriptions*, than as definitions.

(vii) **Analytically-formed and Synthetically-formed Definitions.** This distinction has been employed by recent logicians : it is only applicable when definitions are regarded not as declaring the nature of the thing, but simply as stating the meaning of the word. If the definition gives the recognized intension of the term, it is said to be analytically-formed. It, however, sometimes happens that a special meaning is attached to a word, which hitherto it has not borne. Such for instance is the employment of the term 'wave' by physicists, as signifying 'a change periodically recurring in time and space.' When a new definition of this character is introduced, it is said to be synthetically-formed.[1]

The Aristotelian doctrine of the four causes throws considerable light on this long list of definitions. Every material thing has four causes—the *efficient*, the *final*, the *formal*, and the *material* cause. The meaning of these terms may be illustrated in the case of a statue. The efficient cause of the statue is the sculptor : the final cause is the motive, be it honour or profit, that leads him to execute the work. These are known as the *extrinsic* causes. The formal and material causes are termed *intrinsic*, since they are internal constitutive principles of the thing itself. The formal cause is the determining principle which gives the thing its specific character. The shape of the statue, e.g. Apollo, Julius Caesar, Charles I., may be viewed as such. In a natural entity such as e.g. a man, or any one of the animals, the formal cause is not, of course, the mere external shape. The unifying principle of the material object, that which makes it to be the kind of thing it is, is the vital principle, the soul. The material cause is the substratum to which

[1] *Vide* Ueberweg, § 61. The distinction was first given by Kant, *Logic*, § 100. He however interprets it somewhat differently.

the formal principle gives its character. In the statue it is the marble.

We stated above that the true definition always gives us the cause of the thing defined. Definitions are formed from each of the four causes which we have mentioned.[1] Sometimes a more satisfactory explanation is given by assigning one cause rather than another : sometimes only one cause is fully known to us. The definitions drawn from the efficient and final causes have been explicitly noticed in our list, and need not be further discussed. Essential definition is definition by the formal cause. We are not speaking here of the formal cause in the real order, which is, as we have seen, the soul. But we can form an abstract notion of ' humanity,' expressing those attributes alone, in virtue of which an individual is a man. In the conceptual order this ' humanity ' is viewed as though it were a formal cause by which the individual is constituted man. He may have many other attributes : he may be a Greek, a philosopher, etc., etc. But it is only those attributes which belong to his ' humanity,' which make him a man. The definition ' rational animal ' expresses the notes which are found in this formal cause.[2]

Finally Genetic definition is definition by the material cause. Even when the conjunction of constitutive parts does not, as in the case of chemical combination, result in internal changes, the parts always stand to the whole in the relation of material cause. For taken separately they lack the distinctive attributes of the thing in question. It is only to the whole produced by their conjunction, that the attributes belong.[3]

[1] Cf. St. Thomas, *An. Post. II.*, lect. 4.

[2] The abstract specific essence conceived as though it were a form, was termed the **forma totius.** On this subject see St. Thomas, *Opusc.* 26, *De Ente et Essentia*, c. 3, and *An. Post. II.*, lect. 20. In the latter passage St. Thomas guards his readers against supposing that ' humanitas ' is something real, common to the various members of the species ' homo,' and points out that it belongs to the conceptual order, ' Socrates est similis Platoni in humanitate, *non quasi una humanitate numero in utroque existente.*'

[3] For this aspect of the material cause, see *An. Post. II.*, c. 11, §§ 1, 2. *Physics*, II., c. 3, § 7.

§ 3. **Limits of Definition.** It is manifest that the highest genera will be incapable of definition properly so called. We cannot assign any higher genus under which they may fall. Much more is this the case with concepts such as Being, Unity and the like, which transcend the limits of the highest genera, and are found in all of them. At the other end of the scale, there can be no definition of individuals. They can only be described, not defined: for definition is always of the universal. Further it is impossible to define the simple qualities which are the immediate data of sense-perception, e.g. sweetness, cold, whiteness, pain, etc. The purpose of definition is to unfold the nature of the object by an analysis. Here there is no room for analysis. These are the elements from which knowledge begins.[1] We can, of course, enumerate the properties of such qualities, as when, e.g. we say that red is the colour which manifests itself through vibrations varying from 360 billions to 500 billions per second. But this is not properly speaking a definition: for apart from this information we have a perfectly clear conception of the colour itself.

§ 4. **Rules of Definition.** We must now consider the traditional rules of definition. These are four in number:—

(1) **The definition must be adequate to its object; erring neither by excess nor defect.** It must, that is, be applicable to every member of the class defined, and to no other objects. It must thus be *convertible with the class name.* We constantly come across definitions which are too wide or too narrow. Thus the definition, 'Logic is a machine for combating fallacy,' is far too narrow. It reduces the whole science to what is, in fact, one of the least important of its properties. Mill's definition of

[1] Hence it follows, as both Aristotle and St. Thomas point out, that we cannot define the terms of a definition *in infinitum.* "Definitiones habent principium et finem, quia non est ascendere in infinitum in generibus, sed accipitur 'quasi primum genus generalissimum: nec etiam est descendere in infinitum 'in speciebus, sed est stare in specie specialissima." *De Anima,* I., lect. 8.

eloquence as 'the power of influencing the feelings by speech or writing,' is too wide. It is possible to influence men's feelings by speech without eloquence. So too, to define religion as 'the totality of man's relations with God,' is to err by excess.

(2) **The definition must not be obscure.** This rule must not be misunderstood. It does not signify that the definition must in no case appear obscure to the uninstructed. Sometimes the elements of the definition may to the uninstructed be less comprehensible than the thing defined. In a scientific definition, e.g. of lightning, or a philosophic definition, say of free-will, obscurities are unavoidable.[1] A definition is the result of long study, and many definitions will, to those whose minds are unprepared for them, seem more obscure than the thing they profess to explain. When the mind of the learner has been prepared by the requisite instruction, he will realize that in the definition, he has received the summarized results of science. Such definitions do not really offend against this rule; for they are true analyses of the phenomenon into its simpler elements. In regard to this class of definitions, the purpose of the rule is to forbid ambiguous and metaphorical expressions.

Where, however, the definition is intended to serve as a brief explanation for those not versed in the special subject under consideration, it is requisite that it should be couched in simple terms. Those who violate this rule, are said to explain *obscurum per obscurius*.

(3) **The definition must not be tautologous.** This fault is committed when the subject of the definition reappears either explicitly or implicitly in the defining predicate. Thus to define a horse as 'a member of the species equus,' would convey no information whatever. This rule is violated when we have what is termed *circulus in definiendo*, e.g. 'A day is a period of time consisting of twenty-four hours,' and 'An hour is a twenty-fourth part of a

[1] Rabier, *Logique*, p. 198. 'Les trois quarts des définitions étant données d'emblée, seront pour qui les entend pour la première fois, *Verba et voces praetereaque nihil* . . . à moins cependant qu'elles ne soient une suggestion d'idées bizarres ou fausses.' Cf. *An. Post. II.*, c. 13, §§ 19, 20.

day.' It is not, however, regarded as a circular definition when, in the case of two relative terms, each appears in the definition of the other; since their concepts are mutually dependent. We cannot define 'antecedent' without mention of 'consequent,' nor 'consequent' without mention of 'antecedent.' [1]

(4) **The definition must not be negative if it can be positive.** We are not to define wisdom by saying that it consists in the 'avoidance of folly'; nor health as 'the absence of sickness.'

There are two cases where positive definitions cannot be given. The first of these is when we have to define objects which are incorporeal or unextended. Our cognitive faculties have direct knowledge only of what is corporeal and extended. We are therefore compelled to define the unextended by negative expressions.[2] We define a line as 'length without breadth'; a point as 'that which has no parts;' [3] a spirit as 'an immaterial substance.' The second case occurs in regard to negations and privations. These consist essentially in the mere absence of a positive quality, and hence must be defined in that manner. Blindness is 'the absence of sight in an animal usually found with that sense.' Darkness is 'the absence of light.'

§ 5. Logical Division.

A Logical Division is the analysis of a logical whole into its parts; in other words, the analysis of a genus into its species. It is expressed in a proposition in which the generic term occupies the place of subject, while the subordinate species are enumerated disjunctively in the predicate, e.g. 'Substance is either corporeal or incorporeal,' 'Quantity is either continuous or discrete,'

[1] *Topics*, VI., c. 4, § 13.

[2] Cf. St. Thomas, *Summa Theol.*, I., Q. 11, Art. 2, ad. 4.

[3] This definition (*οὗ μέρος οὐθὲν*) is that of Euclid. Fault has been found with it on the ground that it contains no positive element at all. A point has position, and belongs to continuous quantity. Aristotle, *An. Post. I.*, c. 27, defines it *στιγμὴ δὲ οὐσία θετός*: and St. Thomas, *Summa Theol.*, I., Q. 52, Art. 2, 'Punctum est indivisibile habens situm.' Pythagoras had also defined it by its positive aspect as 'a unit having position (*μονὰς θέσιν ἔχουσα*). Cf. Arist., *De Anima*, I., c. 4, § 7.

'The Vertebrate is either amphibian or reptile or bird or fish or mammal.' [1]

Logical divisions are ordinarily distinguished into (1) Natural, and (2) Artificial. **A Natural division** (*divisio per se*) **is based on the essential modifications of the constitutive attributes;** while on the other hand **an Artificial division is one based on the modifications of a few attributes, or even of a single one.** The older logicians distinguished three kinds of such artificial divisions, which they termed *divisiones per accidens*.[2]

(1) Divisions of substances according to the presence of some accidental quality, e.g. 'Men are either Europeans, or Asiatics, or Africans, or Americans, or Australians, or Polynesians.'

(2) Division of an accidental quality according to the substances in which it is found, e.g. 'Beings with vocal organs are either men or brutes.'

(3) Division of things having some accidental quality according to the modifications of some other accidental quality, e.g. 'Inhabitants of Scotland are either English-speaking, or Gaelic-speaking, or bilingual.'

The generic notion which forms the subject of a logical division is necessarily *indeterminate*. It expresses no one of its subordinate species in particular: but it must be realized in one or other of these species. For this reason the division is correctly expressed by a disjunctive proposition. The subordinate species are termed **members of the division.** Each of them removes the indetermination of the genus in some manner: and it is the variety of mode in which the indetermination is actualized that constitutes the difference of species. The characteristic whose varieties thus modify the indeterminate genus is called **the basis of the division** (*fundamentum divisionis*).

[1] Definition and Division may be distinguished by saying that the purpose of the former is to reply to the question *Quid sit?* (What is the thing?), and the purpose of the latter to reply to the question *Quotuplex sit?* (In how many forms is the thing realized?).

[2] Boethius, *De Divisione*, ed. Migne, col. 877.

The essential difference between *divisio per se* and the *divisiones per accidens*, lies in the fact that in the former the genus is divided into the species which are naturally subordinate to it, and belong to its own category. In the latter the differentiating note is something accidental to the genus and belonging to some other category.

It is most important to observe that in logical division we are not concerned with extension. The species are not parts of the genus because each of them contains a certain number of individuals, which taken all together constitute the larger class. The relation of **logical whole** and **logical part** arises entirely from the indeterminateness of generic nature as realized in the conceptual order, and the determination of the subordinate species. The whole contains its parts *potentially* : in other words, it is capable of being determined by various differentiae to each of them. Thus the notion ' triangle ' contains potentially ' equilateral,' ' isosceles ' and ' scalene.' [1] To a logical whole of this kind belong two noticeable characteristics, viz., (1) it is found in each of its parts.[2] As the whole contains the part *potentially*, so the part contains the whole *actually*. The generic nature ' triangle ' is actually contained in the specific notion ' scalene triangle.' And (2) it is, as we saw when treating of the Categories (Ch. 9, § 2), predicable of all its parts.[3] The generic notion is predicated of the specific notion, not the specific notion of the generic. We say, ' Mammals are vertebrates,' not ' Vertebrates are mammals.'

A further point to be noticed is that logical division properly so called stops short at the *ultima species*. It is by sense, not by intellect that we distinguish the individual members of a species : the function of the intellect is to reveal unity in multiplicity. The division of the universal by its own differentiae stops short at the com-

[1] On the logical whole, cf. Hamilton, *Logic*, I., pp. 204–207.

[2] Cf. St. Thomas, *Summa Theol.*, I., Q. 77, Art. 1, ad. 1. Totum enim universale adest cuilibet parti secundum totam suam essentiam et virtutem, ut animal homini et equo.

[3] Arist., *Met.* IV., c. 26, § 2.

plete specific nature.[1] Yet though this is the case a looser terminology is sometimes found : and the expression 'logical part' is applied to all subordinates of the class-notion, whether they be subordinated as species to a genus, or as individuals to a species. Owing to the convenience of this terminology, we shall occasionally employ it.

Several recent logicians, failing to distinguish between the science of the conceptual order and the sciences which deal with the real order, have identified logical division with the general theory of Classification. The subject of Classification will occupy us when we come to consider the Method of Science. It deals mainly with the principles on which the student of the real order should co-ordinate the types offered by Nature to his contemplation. The co-ordination of these types is effected through long and careful comparison of numerous individuals, the results of the comparison being registered in definitions. As the definitions are framed, the mutual relationships of the natures as conceptually expressed reveal themselves as a logical division. Together with the attainment of a scientific classification of real things, we attain a logical division in regard to our concepts. The whole and the parts in the two cases, are widely different. The one is a whole of many concrete individuals, the other whole is a single generic nature. The treatise on Classification belongs to the natural sciences ; that on logical division belongs to the science of the conceptual order.

Subdivision. It is evident that each of the subordinate species enumerated in an act of division, may itself be subjected to a similar process. This is called a subdivision. These subdivisions are of importance to science. For it is by a series of subdivisions that systematic classifications are mentally represented. Hence, our knowledge of the elaborate classifications, which play so large a part in many sciences, is summed up in logical divisions.

[1] Joseph, *Introduction to Logic*, p. 116. Thus Aristotle terms the *ultimae species* 'indivisible.' *An. Post. II.*, c. 13, § 6, χρὴ δὲ . . . διελεῖν τὸ γένος εἰς τὰ ἄτομα τῷ εἴδει τὰ πρῶτα.

Co-division occurs when a single class is variously divided by divisions founded on different bases (*fundamenta divisionis*). Thus, e.g. 'man' may be divided according to political divisions, or according to racial divisions. The classes founded on the two systems will manifestly not be in accord with each other.

In the proposition, the mind views the subject and predicate as standing in the relation of logical part to logical whole. Indeed the term **subjective part** is used as synonymous with that of logical part.[1] This relation between subject and predicate is manifest in regard to those propositions in which the predicate belongs to the same category as the subject, e.g. 'Mammals are vertebrates.' But even the propositions in which the terms are of different categories, e.g. 'Tigers are ferocious,' may be thus regarded. The subject is regarded as a species subordinated to the predicate in virtue of a *divisio per accidens*. This aspect of the proposition will be found to be of great logical importance, and should by no means be overlooked.[2]

§ 6. **Rules of Division.** The rules of Division are variously enumerated. The following four rules are ordinarily accepted :—

(1) **Each act of division must have but one basis.** The violation of this rule involves either that some members appear in more than one species, or that some are altogether omitted. Usually, both faults occur. Thus, if we divide American citizens into Republicans, Democrats, and Immigrants, many citizens will be found to have a place in two classes, while others, who are, it may be, little interested in politics, will not be included under any of the species enumerated.

(2) **The constituent species must together be equal to the genus.** The rule prescribes that the division should

[1] The term *Subjective part* is a little wider in its reference than *Logical part*. An individual is a subjective part in regard to the species.

[2] Cf. the important passage, S. Thomas, *Summa Theol.*, I., Q. 85, Art. 5, ad. 3.

be complete, that none of the subclasses should be omitted. If, e.g., we divide men into Aryan, Semitic, and Hamitic races, we leave out many varieties, and the genus is not exhausted.

(3) **The constituent species must be mutually exclusive.** This condition is fulfilled, if the first rule be observed. Since the division is constituted by different determinations of the indeterminate generic notion, it follows that as long as one basis is adhered to, one determination is necessarily exclusive of the other. Thus, if we divide, e.g., the genus 'nation' into English, French, German, etc., etc., it is plain that the members of this division *considered as nations* are mutually exclusive. This is true, even if it be the case that certain individual men have a right to citizenship in more than one country.

(4) **Each step in a continued division must be a proximate one** (*Divisio ne fiat per saltum*). When this rule is neglected, some species are omitted. The rule would, for instance, be broken, if after dividing quantity into continuous and discrete, we should at once proceed to divide continuous quantity into triangles, quadrilaterals, etc., etc. By neglecting the step, in which continuous quantity is first divided into plane and solid, we should have omitted all the species of solid figures.

§ 7. **Division by Dichotomy.** This name has been given to a method of division consisting of successive steps, in each of which a division is made into two classes according to the presence or absence of some given attribute. By thus repeatedly subdividing, it was held that we might arrive at last at a fully determined notion of the last species. Plato advocated this method of division as a means of discovering definitions.[1] Thus the definition of man, might be supposed to be reached by successive subdivisions of the notion of substance. Aristotle severely criticized this view of Plato's, and hence some mention of dichotomous division is found in most logical treatises.[2]

[1] Cf. Plato, *Protagoras*, 332. *Phaedrus*, 266.
[2] Cf. *An. Prior I.*, c. 31. *De Part. Anim.*, I., cc. 2, 3.

Regarded as a method of finding the definition of an object, it is clearly useless. In order to know to which member of the division we should assign the entity, we must already know its definition to that extent. Thus, unless we knew that man is corporeal, it would avail nothing to divide substance into corporeal substance and incorporeal substances; for we should not know under which member we should class man.

Again the method is invalid, because the mere absence of an attribute, is no basis on which to constitute a class. Let us consider what is involved in dividing a class *A* into *AB* and *A non-B*. One of two things must be the case. Either (1) we know that in this class there are real objects marked by the absence of B. If this be so, these objects have their own specific differentiae. In all probability they belong to many different species, and to rank them all together as *A non-B*, is calculated to mislead people into supposing they have a generic unity. Moreover, if we distinguish the class *A non-B* by subsequent division, the species will appear in their wrong position in the classification. For they will not be placed where they should be placed, viz., as co-ordinate species with the class *AB*. Or (2) we may be supposed to be unaware of the existence of other objects save those we are considering. In that case, we shall, at each step of the division, be introducing a class, whose very existence is doubtful. At this rate, our dichotomous division might be carried to any length. We might divide men into men with wings (if any) and men without wings.

Division by dichotomy is ranked by Kant (*Logic*, § 113) as a method which is applicable in purely formal Logic, and which is entirely independent of material considerations. Mansel has rightly pointed out that this is not so.[1] We cannot, e.g., divide animal into irrational and rational, unless we know that there are rational and irrational animals. We cannot divide any generic notion into two classes, unless we are acquainted with the determinations which that notion is capable of receiving.

[1] *Proleg. Logica*, pp. 192, 193.

§ 8. **Various Kinds of Division.** We have seen that logical division is the analysis of a logical whole into its parts. With a view of rendering quite distinct the difference between this kind of division and the other processes which are so termed, Boethius in his *Liber de Divisione* calls the attention of his readers to the other cases in which a whole is resolved into its parts. The following are those now usually mentioned in this connexion. They differ but very slightly from those enumerated in his list.[1]

(1) *Physical partition*, as e.g. the partition of a house into its parts—roof, bricks, timber, glass, etc.

(2) *Metaphysical distinction*, viz., of the genus from the differentia, as when we distinguish between the 'animality' and the 'rationality' in man. The term is sometimes employed to signify further the distinction of the qualities of an object, e.g. between the colour, fragrance, shape, etc., of an orange.

(3) *Distinction of faculties*, e.g. of the soul into memory, intellect, and will.

(4) *Distinction between the various meanings of a term*, e.g. between 'church' as a building, and 'church' as signifying the whole body of the faithful.

[1] Boethius, *op. cit.*, ed. Migne, col. 877, 878; and cf. S. Thomas, *Summa Theol.*, I., Q. 76, Art. 8.

CHAPTER XI.

THE CATEGORICAL SYLLOGISM (I.).

§ 1. **The Categorical Syllogism.** We have now discussed the Logic both of the Concept, and of the Judgment. It remains for us to treat of the third of the mental processes mentioned at the beginning of this work (Ch. 1, § 2), viz., Reasoning or Mediate Inference. This has already been defined as *the process by which from certain truths already known, the mind passes to another truth distinct from these.* The two principal forms of inference are termed **deduction** and **induction** respectively. In deduction, the mind passes from the more general to the particular or the individual. In induction, we argue from the individual or particular to the general. It is with the former of the two, that we are at present concerned.

The deductive inference is known as the **syllogism.** A syllogism is defined as **an inference by which, from two given propositions, we proceed to a third proposition the truth of which follows from these two as a necessary consequence.**[1] Thus I may argue:

'All mammals are vertebrates.
All horses are mammals.'
Therefore, All horses are vertebrates.

This syllogism, consisting, as it does, of three *A* propositions, may be represented by the formula :—

$$\frac{\begin{array}{l} M\ a\ P. \\ S\ a\ M. \end{array}}{S\ a\ P.}$$

[1] Arist. *An. Prior I.*, c. 1, § 5. Συλλογισμὸς δέ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι.

The two given propositions are known as the **premisses** (*προτάσεις*, *praemissae*); that which follows from them as the **conclusion** (*συμπέρασμα*, *conclusio*) or *consequent*.[1]

In all deductive inference, we argue, as has been just stated, from the more general to the less general. Hence, one at least of the premisses must be a universal proposition. And the essence of syllogistic reasoning lies in the application of a general principle to a case that falls under it. In the example just given, it will be seen that the general principle, 'All mammals are vertebrates,' is applied to the particular case of the horse.

It is evident that no inference is possible unless there is a term which is common to the two premisses. This term is known as the **middle term** (*τὸ μέσον*). Aristotle gives us two reasons for his selection of this name. The middle term, he says, is the term which is itself contained in one of the others (*P*), while the remaining term (*S*) is contained in it. The relation of which he here speaks, is that of logical part to logical whole. The middle term is a logical part in regard to one of the other terms, and a logical whole in regard of the third. A logical part, as we saw in Ch. 10, § 6, stands to its whole in the relation of a subordinate concept to one that is more general. It cannot be too clearly recognized that this is what is meant by the terms 'whole' and 'part' in this connexion. The relation belongs, not to extension, but to the hierarchical subordination of species and genus in *the conceptual order*. When we predicate one term of another, it is the wider notion which is the predicate: the genus is affirmed of the species. Hence, since the middle term is subject of one premiss, and predicate of the other, it is a logical part in relation to the term predicated of it, a logical whole

[1] Some of the earlier writers employ a different terminology, calling the two premisses respectively the *sumptio* (or *sumptum*) and the *assumptio*. "Quoniam omnis syllogismus ex propositionibus texitur, prima vel propositio vel sumptum vocatur: secunda vero dicitur assumptio: ex his quae infertur conclusio nuncupatur." Boethius, *De Syll. Hypoth.* (Migne, t. 64, col. 844).

[2] *An. Prior I.*, c. 4, § 3. καλῶ δὲ μέσον μὲν ὃ καὶ αὐτὸ ἐν ἄλλῳ καὶ ἄλλο ἐν αὐτῷ ἐστίν, ὃ καὶ τῇ θέσει γίνεται μέσον. Cf. also c. 41, § 5.

in relation to the term of which it itself is predicated.[1] Aristotle in his treatment of the syllogism ever keeps in view the typical case in which the terms stand in this order. It is entirely erroneous to say, as do many logicians, that he looked on the terms of the syllogism in their extension, and called the middle-term by that name because in extension it lay between the two others.

The second reason, which Aristotle gives, is that the middle term is such by position. This reason is of less moment, and need not detain us. It rested on the Aristotelian method of stating the syllogism. Where we say *M is P, and S is M, therefore S is P*, he employed the form, *P is predicated of M, M is predicated of S, therefore P is predicated of S.*

Regarding a syllogism of the same character as the standard type, the term which is subject in the conclusion is aptly known as the **minor term,** and the term which is predicate in the conclusion as the **major term.** These names are adhered to even in negative conclusions, and in other cases where the relation of whole and part, is not found. The major and minor terms are sometimes spoken of as the **extremes** (*τὰ ἄκρα*). The premiss in which the major term occurs, is called the **major premiss :** that which contains the minor term is called the **minor premiss.**

A distinction is sometimes drawn between the **matter** and **form** of a syllogism. The matter of the syllogism consists of the terms and propositions employed in it. The form lies in the special arrangement of the terms in the three propositions, in virtue of which the conclusion follows from the premisses by necessary consequence. It is evident that a syllogism may be formally valid, while in regard to the matter, every proposition may be untrue. The greater part of the Logic of the syllogism, is concerned with the form alone. We treat of the various ways in which a process of reasoning may be con-

[1] *An. Prior I.*, c. 1, § 7. *τὸ δὲ ἐν ὅλῳ εἶναι ἕτερον ἑτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι θατέρου θάτερον ταὐτόν ἐστιν.* 'To say that one thing is contained in another as a whole, is the same as to say that the second can be predicated universally of the first.'

structed so as to conclude validly. It is chiefly because this important branch of Logic deals wholly with the self-consistency of our thought—with securing conclusions consistent with their premisses,—that the Formal logicians were led to hold that Logic deals only with self-consistency, and abstracts altogether from the question of truth. The answer to this contention is that the theory of syllogistic form, is but one portion of Logic ; and that even in it, we treat of formal truth only as a means by which the mind passes from true premisses to a true conclusion.

§ 2. **Relation of Premisses to Conclusion in regard to Truth.** It is quite possible that false premisses may furnish a true conclusion. Thus if I argue, 'All mammals are vertebrates, All owls are mammals, *therefore* All owls are vertebrates,' the minor premiss is false, yet the conclusion is true. The way in which this occurs,

is easily seen from the diagram. I assert that S is P, *because it is* M. It is true that S is P, but it is not so because of the reason alleged. A conclusion of this kind is said to be true *per accidens* ; i.e. its truth is not connected with the reason assigned in the argument.

Hence it appears that we cannot argue that the premisses must be true because the conclusion is true. On the other hand, if the premisses are true, and the argument is rightly stated, we may be sure that the conclusion is true. For a conclusion legitimately drawn, contains nothing that is not implicitly contained in the premisses (*An. Prior II.*, c. 2, § 1, c. 4, § 13).

§ 3. **General Rules of the Syllogism.** The following

eight rules hold good of all syllogisms, in whatever form they may be stated [1] :—

I. *Relating to the structure of the syllogism.*

Rule 1. A syllogism must contain three, and only three, terms.

Rule 2. A syllogism must consist of three, and only three, propositions.

II. *Relating to quantity.*

Rule 3. The middle term must be distributed in one, at least, of the premisses.

Rule 4. No term may be distributed in the conclusion, which is not distributed in a premiss.

III. *Relating to quality.*

Rule 5. No conclusion can be drawn, where both premisses are negative.

Rule 6. If one premiss be negative, the conclusion must be negative : and to prove a negative conclusion, one of the premisses must be negative.

IV. *Corollaries.*

Rule 7. No conclusion can be drawn, where both premisses are particular.

Rule 8. If one premiss be particular, the conclusion must be particular.

We shall now deal with each of these rules separately.

Rule 1. This rule prohibits what is known as an **ambiguous middle.** We have stated above (Ch. 2, § 12) that an equivocal term is really equivalent to two terms. Hence, when the middle term is equivocal, and is employed in different senses in the two premisses, we have not three terms but four. The following example will serve to

[1] The following mnemonic verses, summing up the rules of the syllogism are traditional in English works on Logic :—

Distribuas medium, nec quartus terminus adsit,
Utraque nec praemissa negans, nec particularis,
Sectetur partem conclusio deteriorem,
Et non distribuat, nisi cum praemissa, negetve.

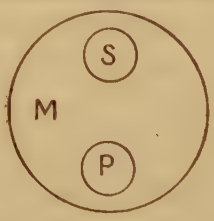
The third line, to the effect that the conclusion must always follow the inferior alternative, signifies that where one premiss is negative, the conclusion is negative, and where one is particular the conclusion is particular. Negation is regarded as inferior to affirmation, and the particular as inferior to the universal.

illustrate what is meant. 'Beings, who are not free, are incapable of sin, Slaves are beings who are not free, *therefore* Slaves are incapable of sin.' Here the word 'free' in the major premiss denotes the freedom of the will, in the minor premiss civil freedom.

The example, given in Ch. 8, § 4, 'Man is a species, Socrates is a man, *therefore* Socrates is a species,' provides us with another case of ambiguous middle. The term 'man' is not equivocal. But when a term is employed in the one case in reference to the real, in the other in reference to the conceptual order, ambiguity necessarily arises.

Rule 2. The second rule follows immediately from the definition of a syllogism, which states that it is an inference, in which from two given propositions, we pass to a third proposition. There are, as we shall see, forms of argument, which contain a number of propositions. These are not syllogisms, though they consist of syllogisms. A syllogism must have but three propositions.

Rule 3. The object of the third rule is to guard against the fallacy known as **undistributed middle.** It prescribes that in one premiss at least the middle term should refer to the whole of its extension. The reason for this is obvious. In the syllogism, we discover the relation existing between the major and minor terms, by comparing each with the middle term. It is therefore absolutely essential that the term of comparison should be the same in both cases, otherwise we have no means of comparing major and minor. When the middle term is undistributed, we have no guarantee that both propositions refer



to the same part of the extension. This may be seen in the accompanying diagram. I may assert with truth

that *S is M*, and that *P is M*. But I am not justified in concluding that *S is P*. Hence premisses, such as 'All men are mortal, All negroes are mortal,' give no ground for the conclusion that 'All negroes are men,' though that proposition happens in fact to be true. This appears at once, if in place of the term 'negroes' we substitute the term 'birds' or 'fishes.'

Rule 4. The reason for this rule is obvious. It forbids us to go beyond our data, and assert more in the conclusion than is warranted by the premisses. It can be violated both as regards the minor and the major term. When the major term receives illegitimate distribution in the conclusion, that is, when the conclusion takes it in its whole extent, though the premisses gave information about a part only, the fallacy is termed **illicit process of the major.** Should it be in the case of the minor term that the error occurs, it is known as **illicit process of the minor.** As an instance of the former, we may take the following argument, 'All men are beings with an immortal destiny, Brutes are not men, *therefore* Brutes are not beings with an immortal destiny.' The invalidity of such an argument is manifest, if we take an example which is precisely similar, save that the conclusion is false: e.g. 'All men are animals, Brutes are not men, *therefore* 'Brutes are not animals.' Illicit process of the minor may be illustrated by, 'No birds are viviparous, All birds 'are bipeds, *therefore* No bipeds are viviparous.' In this syllogism the premisses only justify us in concluding, 'Some bipeds are not viviparous.'

Rule 5 forbids that both premisses should be negative. Should this occur, we have no means of drawing a conclusion. In a negative premiss, we deny the connexion of the middle term with the extreme contained in that premiss. If both extremes are declared to be unconnected with the middle term, we have no means of comparing them. We cannot say whether they are found conjoined or not.

Certain recent logicians have called in question the universa validity of this law. Prof. Jevons (*Principles of Science*, p. 63) gives us the following syllogism as an exception to it :—

What is not metallic is not capable of powerful magnetic influence.
Carbon is not metallic.
∴ Carbon is not capable of powerful magnetic influence.

Mr. Bradley concurs with Prof. Jevons. "The fact remains," he says, "that from two denials you somehow *have* proved a further denial." The solution to the difficulty will be easily seen, when it is observed that if the proposition, 'Carbon is not metallic' is taken as it stands, we have no middle term. In the major premiss we have 'what-is-not-metallic,' and in the minor 'metallic.' The mind replaces this proposition with the equivalent affirmative, 'Carbon is a-thing-which-is-not-metallic,' and the syllogism concludes without breach of rule.[1]

Rule 6. The reason for the sixth rule is evident. When the one premiss is affirmative, and the other negative, the connexion of one extreme with the middle term is asserted, the connexion of the other extreme with the same term is denied. It follows that the conclusion must necessarily deny that the two extremes are connected *inter se*. The truth of the second part of the rule, viz.: that a negative conclusion involves a negative premiss, is shewn in a similar way. If the conclusion denies the connexion of the two extremes, this cannot be because both of them are connected with one and the same middle term: it must be because the one is connected with this term, and the other is not.

Rule 7. This rule may be proved by examining the possible cases of two particular premisses, and testing them by the preceding rules. Since there are but two particular propositions, *I* and *O*, the possible combinations are limited to three *II*, *IO*, *OO*.

Of these the first *II* contains no distributed term, and hence violates Rule 3. The third *OO* gives us two negative premisses, and thus contains a breach of Rule 5. The combination *IO* has one term alone distributed, viz.: the predicate of *O*. But it is necessary that two terms, the middle and the major, should be distributed in the premisses. A distributed middle is required by

[1] The difficulty is of respectable antiquity. Ueberweg, § 106, calls attention to its solution by Boethius, *in Lib. de Interp., Ed. 2da* (ed. Migne, t. 64, c. 551); and Boethius refers us back to Alexander of Aphrodisias.

Rule 3 ; and a distributed major is also needed, for, since the conclusion must be negative by Rule 6, the predicate must be a distributed term.

Rule 8. The rule that, if one premiss is particular, the conclusion is particular, is also proved by an examination of cases. The possible combinations are four, *AI*, *AO*, *EI*, *EO*. Since *EO* contains two negatives, it may be disregarded. *AI* contains but one distributed term. Since the middle term must be distributed, it follows that no term is distributed in the conclusion. The conclusion therefore cannot be universal. *AO* and *EI* both contain two distributed terms. One of these must be the middle term. Hence one term alone is distributed in the conclusion. But the conclusion must necessarily be negative, since in both cases there is a negative premiss. But the universal negative *E* has *both* terms distributed. The conclusion therefore, since it has but one distributed term, can only be particular.

§ 4. **Figures and Moods of the Syllogism.**

(i) **Figure is the form of the syllogism as determined by the position of the middle term in the two premisses.** The arrangement of figures now usual supposes that in stating the premisses, we are already aware which of the extremes is to be the subject of the conclusion, and which is to be the predicate, in other words, which of our two propositions is the minor premiss, and which the major. This gives us a fourfold division. (1) In the first figure, the middle term is subject in the major premiss, and predicate in the minor premiss. (2) In the second figure, the middle term is predicate in both premisses. (3) In the third figure, the middle term is subject in both premisses. (4) In the fourth figure, the middle term is predicate in the major, and subject in the minor premiss.

The figures are represented by the following forms :—

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
M P	*P M*	*M P*	*P M*
S M	*S M*	*M S*	*M S*
S P	*S P*	*S P*	*S P*

* For many centuries logicians employed a division of figures in which no account was taken as to which premiss contained the major term and which the minor. Nothing was considered save the position of the middle term, and three figures only were recognized, which were thus distinguished (1) *Fig.* 1, *M*, subject in the one premiss, predicate in the other, (2) *Fig.* 2, *M*, predicate in both, (3) *Fig.* 3, *M*, subject in both. The figures may be represented as follows :—

(1)	(2)	(3)
M B	*B M*	*M B*
A M	*A M*	*M A*

The first of these figures, however, necessarily provides two varieties, according as we make *A* or *B* the subject of the conclusion. The forms in which *B*, the term which is predicate in its premiss, becomes the subject in the conclusion, are those which, according to the system now in vogue, constitute the fourth figure. This arrangement was that of Theophrastus, a disciple of Aristotle. It differs a little from Aristotle's own treatment of the figures, in a point which will be mentioned in Ch. 12, § 2 below. The Theophrastean arrangement is still preferred by many logicians.

(ii) **Mood is the form of a syllogism as determined by the quantity and quality of its premisses.** Since there are but two premisses in a syllogism, and each of these must be one of the four propositions, *A*,*E*,*I*,*O*, it follows that there are but sixteen possible arrangements of premisses, from which the mood of the syllogism can be selected, viz. :

AA	**IA**	**EA**	**OA**
AI	*II*	**EI**	*OI*
AE	*IE*	*EE*	*OE*
AO	*IO*	*EO*	*OO*

Not all of these sixteen combinations, however, can be employed in the premisses of a syllogism. The rule of the syllogism prohibiting two negative premisses, excludes four of the sixteen, viz. : *EE*, *EO*, *OE*, *OO*. Four also are excluded by the rule, which tells us that two particular premisses give no conclusion, viz. : *II*, *IO*, *OI*, *OO*. The last of these had already been rejected on the previous count. Of the nine remaining cases, one, *IE*, can be shewn to involve an illicit process of the major

term. For since it contains a negative premiss, the conclusion must be negative. A negative conclusion distributes its predicate, and hence necessitates that the major term should be distributed in its premiss. But in *IE*, no term is distributed in the major premiss, *I*. Therefore a conclusion drawn from these premisses must needs be vitiated by an illicit process of the major.

The possible moods are therefore eight in number, and consist of the combination *EI*, and the seven possible combinations, which contain the premiss *A*. We must now examine, which of these moods may be employed in the several figures.

§ 5. **Special Rules of the Four Figures.** Though, as we have seen, there are eight possible moods, not all of them can be employed in each figure. Every figure has its own special rules, involved in the arrangement of terms peculiar to it, and only admits those moods which conform to these rules. In this section, we shall deal with the rules of the four figures in succession.

Figure 1.

$$\begin{array}{l} M\ P \\ \underline{S\ M} \\ S\ P \end{array}$$

Rules. (1) The minor premiss must be affirmative.

(2) The major premiss must be universal.[1]

(1) If the minor premiss were negative, the conclusion would be negative. A negative conclusion requires that the major term should be distributed in the conclusion, and therefore in the major premiss also. But the major term is predicate in its premiss : hence were it distributed, the premiss must be negative. Thus we should have two negative premisses.

(2) Since the minor premiss is affirmative, the middle term in that premiss is undistributed. It must therefore be distributed in the major premiss. In that premiss, it

[1] The older logics give us these rules in the following mnemonic hexameter *Sit minor affirmans, major vero generalis.*

stands as the subject : the subject is distributed in universal propositions alone.

Rule (1) excludes the moods *AE* and *AO*, and Rule (2) excludes *IA* and *OA*. There remain therefore four available moods in this figure, viz. : *AA*, *AI*, *EA*, *EI*. Or, if we express them with the letter denoting the conclusion to be drawn from them, **AAA, EAE, AII, EIO**.

Figure 2.

$$\begin{array}{l} P\ M \\ S\ M \\ \hline S\ P \end{array}$$

Rules. (1) One premiss must be negative.

(2) The major premiss must be universal.[1]

(1) Since the middle term is predicate in both premisses, were both affirmative, it would not be distributed in either. Therefore one must be negative.

(2) Since the conclusion is negative, the major term is distributed. It must therefore be distributed in its premiss. In the major premiss, it stands as subject. The distribution of the subject involves a universal proposition.

Here Rule (1) excludes *AA*, *AI*, *IA* ; and Rule (2) *IA* and *OA*. The valid moods of this figure will therefore be **EAE, AEE, EIO, AOO.**

Figure 3.

$$\begin{array}{l} M\ P \\ M\ S \\ \hline S\ P \end{array}$$

Rules. (1) The minor premiss must be affirmative.

(2) The conclusion must be particular.[2]

(1) The former of these rules is established by the same considerations as shewed the necessity of an affirmative minor premiss in Fig. 1. A negative minor would involve a negative major, and therefore there would be two negative premisses, contrary to the fifth rule of the syllogism.

(2) Since the minor is affirmative, its predicate is undistributed. This predicate is the minor term. The con-

[1] *Una negans esto, major vero generalis.*

[2] *Sit minor affirmans, conclusio particularis.*

clusion therefore has an undistributed subject, and thus is particular.

The valid moods of this figure are six in number, **AAI, IAI, AII, EAO, OAO, EIO.**

* *Figure* 4.

$$\begin{array}{ll} P & M \\ M & S \\ \hline S & P \end{array}$$

The fourth figure, as will be seen in the sections which follow, is both theoretically and practically of very minor importance. Its rules however are somewhat more complex than those of the other figures.

Rules. (1) If the major is affirmative, the minor must be universal.

(2) If the minor is affirmative, the conclusion must be particular.

(3) If the conclusion is negative, the major must be universal.

A breach of Rule (1) would involve undistributed middle: a breach of Rule (2) an illicit process of the minor; and of Rule (3) an illicit process of the major.

There are five valid moods, **AAI, AEE, IAI, EAO, EIO.**

§ 6. **The Mnemonic Lines.** The nineteen moods, whose validity we have established, are enumerated in the following mnemonic lines. It will be noticed that the moods are expressed in succession by the vowels of each word. Thus the mood *AAA* is signified by Barbara, *EAE* by Celarent. The significance of the consonants will be explained in § 7 :—

Bárbara, Celárent, Dárii, Ferió*que prióris*,
Césare, Camés*tres*, Festíno, Baróco, *secundae.*
Tertia Darápti, Dísamis, Datísi, Felápton
Bocárdo, Ferí*son*, *habet. Quarta insuper addit*
Bramántip, Cámenes, Dímaris, Fesápo, Fresíson.[1]

Five of these moods, viz. Barbara, Celarent, Cesare, Camestres, and Camenes, give universal conclusions. We

[1] These particular mnemonics in an earlier form (Ch. 12, § 2, *note*) occur first in the work of William Shyreswood (d. 1249). They are contained also in the *Summulae Logicales* of Petrus Hispanus, afterwards Pope John XXI. (d. 1277). This was one of the most widely known of mediæval logical treatises, and through it they acquired universal notoriety. Many others of the old mnemonics are found in this work.

may however, if we will, draw from the premisses a conclusion that is particular; since there is nothing to prevent us from asserting less than our premisses would warrant. These moods, Barbari, Celaront, etc., are known as **subaltern moods.** They are, of course, of no importance.

The name of **strengthened syllogisms** is occasionally given to the moods, Darapti, Felapton, Bramantip, and Fesapo, for the reason that in each of them, the same conclusion could be obtained by a particular premiss in lieu of one of the universal premisses employed. In Darapti, Felapton, and Fesapo, the middle term is twice distributed. In Bramantip, the major term is distributed in the premiss, and undistributed in the conclusion. The remaining fifteen moods are termed **fundamental syllogisms.**

§ 7. **Reduction.** Aristotle held that only in the first figure is the validity of our conclusion absolutely evident. Figs. 2 and 3 he regards as valid, but as destitute of the peculiar evidential quality, which marks Fig. 1. That alone is the perfect (τέλειος) syllogism. The others are imperfect (ἀτελεῖς). Their conclusiveness is only manifest, when after the conversion of one or other of the premisses, the syllogism is rearranged in some mood of the first figure. This process was called **Reduction** by the Scholastics, and may be defined as **the process by which a syllogism in one of the imperfect figures is expressed as a syllogism of the first figure.**

The names of the various moods as they are given in the Mnemonic lines, "Barbara, etc.," are constructed on an ingenious plan, so as to indicate the moods of the first figure, to which they are to be reduced, and what operations are necessary to achieve the result.

It will be observed that every name begins with one of the four letters, **B, C, D,** or **F.** These *initial* letters signify respectively that the mood in question is to be reduced to **B**arbara, **C**elarent, **D**arii or **F**erio. Of the consonants composing the body of the word, the letters

s, p, m, c are employed to tell us what changes are required to obtain a syllogism in one of these four moods :—

s (= *simpliciter*) signifies that the premiss indicated by the preceding vowel must be converted simply.

p (= *per accidens*) signifies that the preceding premiss must be converted *per accidens.*

m (= *muta*), that the premisses are to be transposed.

c (= *per contradictoriam propositionem*), that the reduction is to be indirect or *per impossibile.*

These four letters were, therefore, not selected arbitrarily, but in each case denote the Latin name of the process employed.

One or two examples will illustrate the application of these methods. We give first a syllogism in Cesare of the second figure :—

No Englishmen are negroes.
All Hottentots are negroes.
∴ No Hottentots are Englishmen.

The letter **s** in Cesare tells us that the major premiss must be converted simply, the initial C indicating that the syllogism will be in the mood Celarent of Fig. 1. The process may be symbolized—

$$\begin{array}{ll} P\ e\ M & M\ e\ P \\ S\ a\ M \quad \text{——} & S\ a\ M \\ \hline S\ e\ P & S\ e\ P \end{array}$$

The conversion of the major premiss gives us,

No negroes are Englishmen.
All Hottentots are negroes.
∴ No Hottentots are Englishmen.

The following example is in Disamis. Here we have to convert the major premiss, and to transpose the order of the two premisses. Moreover, the resulting conclusion in Fig. 1 is not the actual conclusion of the syllogism in Disamis, but, as the final **s** indicates, is shewn to be equivalent to it by simple conversion.

Some murderers are unsafe companions.
All murderers are men.
∴ Some men are unsafe companions.

Here the symbolic representation becomes,

$$\begin{array}{ll} M\ i\ P & M\ a\ S \\ M\ a\ S & P\ i\ M \\ \hline S\ i\ P & P\ i\ S \end{array}$$

The syllogism as expressed in Darii will therefore be,

All murderers are men.

Some unsafe companions are murderers.

∴ Some unsafe companions are men.

By conversion this conclusion is shewn to be equivalent to the conclusion of the Disamis syllogism.

This process, which gives us a syllogism in the first figure precisely equivalent to the original syllogism, is termed **Direct** or **Ostensive Reduction.** There are, however, two moods, Baroco (Fig. 2) and Bocardo (Fig. 3), to which it cannot be applied. In these moods, another method, that of **Indirect Reduction,** is employed. This consists in admitting by way of hypothesis that the conclusion of the mood may be false, and in showing by a syllogism in Barbara that this supposition involves the falsity of one of the original premisses. The original premisses, however, are *ex hypothesi* known to be true. Hence we are forced to admit that the conclusions in Bocardo and Baroco are valid. Thus, we may take the following syllogism in Baroco,

All whales are aquatic animals.

Some mammals are not aquatic animals.

∴ Some mammals are not whales.

If the conclusion is false, its contradictory must be true, i.e. we must admit that 'All mammals are whales.' We now form a syllogism in Barbara, using as our premisses this proposition, and that one of the original premisses which is a universal affirmative. This gives us,

All whales are aquatic animals.

All mammals are whales.

∴ All mammals are aquatic animals.

This conclusion is, however, the contradictory of the original premiss, 'Some mammals are not aquatic animals,' and is therefore false. But the error does not lie in the reasoning, for that is in the first figure. One of the premisses must therefore be false. The premiss 'All

whales are aquatic animals' is, however, given as true. The premiss 'All mammals are whales' is consequently the erroneous one; and since it is false, its contradictory, the original conclusion of the Baroco syllogism, is true.

This method of indirect Reduction may be applied to any mood, in lieu of the ostensive process. Let us take, e.g., a syllogism in Felapton, $\frac{M\ e\ P}{M\ a\ S.}{\overline{S\ o\ P.}}$ If the conclusion $S\ o\ P$ be false, $S\ a\ P$ is true. We may now form a syllogism in Barbara $\frac{S\ a\ P}{M\ a\ S}{\overline{M\ a\ P.}}$ But $M\ a\ P$ is inconsistent with the original premiss $M\ e\ P$, and is therefore false. It follows that the supposition $S\ a\ P$ is false, and that $S\ o\ P$ is true.

Indirect Reduction is the only method employed by Aristotle for dealing with Baroco and Bocardo. If, however, we make use of Obversion, they may be treated ostensively. The mnemonic words *Faksoko* and *Doksamosk*, indicate the necessary operations for the direct reduction of syllogisms in Baroco and Bocardo to Ferio and Darii respectively. The letter *k* signifies the obversion of the preceding premiss. The combination *ks* signifies that the premiss must be first obverted, and then converted. The following syllogism will serve as an illustration :—

Some ministers are not straightforward.
All ministers are privy-councillors.
∴ Some privy-councillors are not straightforward.

This will become :—

All ministers are privy-councillors.
Some men, who are not straightforward are ministers.
Some men, who are not straightforward are privy-councillors.

* *Further employment of Reduction.* Aristotle also shews us (*An. Prior I.*, c. 7, § 4) how Darii and Ferio may be reduced respectively to Barbara and Celarent. It may appear remarkable that he should think that these moods gain anything by reduc-

tion, since he explicitly recognizes them as being among the 'perfect' moods of the syllogism. But his reason will be seen, if we remember that the two universal moods alone give us what he holds to be the absolutely typical inferential process. In them the subject and middle-term are related as logical part and logical whole, while the major-term either contains the middle or absolutely excludes it. In the particular moods we no longer have three universal concepts. The minor is particular: and particular propositions always depend in the last resort on sensible experience.[1] Only a universal judgment can become the object of an entirely intellectual intuition. For this reason a syllogism in which one premiss is particular does not attain to what in Aristotle's view is the absolute standard of the inferential process of the intellect.

The method of reduction Aristotle here employs, is somewhat cumbrous. A simpler method is possible. We may illustrate it in the case of Darii. If $S\ i\ P$ be false, $S\ e\ P$ is true. This may be converted to $P\ e\ S$, and obverted to $P\ a\ \overline{S}$. We may now, by employing one of the original premisses form the syllogism in

Barbara $\dfrac{\begin{matrix}P\ a\ \overline{S}\\ M\ a\ P\end{matrix}}{M\ a\ \overline{S}}$ which gives the conclusion $M\ a\ \overline{S}$ or $M\ e\ S$,

equivalent to $S\ e\ M$, the contradictory of the original premiss $S\ i\ M$. Ferio may be dealt with in a precisely similar way.

§ 8. **Superiority of Fig. 1.** Not only does the first figure excel the others as a mode of inference. It has two other characteristics which render it superior to them. (1) By its means we may arrive at conclusions in all the qualitative and quantitative varieties that are possible. We are not limited to negatives as in Fig. 2, nor to particulars as in Fig. 3.[2] More important still, is the fact, that by it we can obtain a universal affirmative conclusion, viz. in Barbara. When we consider how great an importance attaches to conclusions of this character, we see at once that its utility exceeds that of all the other figures. Every argument in which we bring a special case or type of case under a general rule, is an argument in Barbara—whether our subject be Mathematics, Physics, Ethics, Law or what not. If the student will, for instance, be at the pains to analyse a short proposition of Euclid, he will find that each step of advance is a syllogism in this mood.

[1] *An. Post. I.*, c. 24, § 15, ἡ μὲν καθόλου νοητή, ἡ δὲ κατὰ μέρος εἰς αἴσθησιν τελευτᾷ. "The universal proposition is the object of intellectual intuition: the particular ends in sense-perception."

[2] *An. Prior I.*, c. 5, § 15.

CHAPTER XII.

THE CATEGORICAL SYLLOGISM (II.).

§ 1. **Canon of Syllogistic Reasoning.** We stated in the last section that Aristotle held the first figure to be the only one in which the conclusiveness of the inference is absolutely evident. His Scholastic followers expressed the principle, which governs the reasoning of the figure, in a canon to which they gave the name of the **Dictum de Omni et Nullo.** This they held to be the fundamental principle of the reasoning process. It may be thus stated: **Whatever is affirmed (or denied) universally of any subject is thereby affirmed (or denied) of every logical part of that subject.** The Dictum is based on a passage of Aristotle, and accurately represents his theory of inference, though his expression of it does not give it as the ultimate canon of reasoning.[1]

It must be carefully noticed that in this principle, the subject of which the affirmation or denial is made, is not regarded as a collection of individuals. This would

[1] The principle is thus given by John of St. Thomas, who may be taken as fairly representative of the Thomistic school. *Quidquid universaliter dicitur de aliquo subjecto, dicitur de omni quod sub tali subjecto continetur: quidquid negatur de aliquo subjecto, negatur et de omni contento sub tali subjecto* (*Logica. Pars I., Lib.* 3, *c.* x). The various forms in which it is stated, differ but slightly from each other. Cf. St. Thomas *An. Post. I.*, lect. 9. Scotus, *Sup. Lib. I. Priorum, Q.* 7. The actual words of Aristotle, which it represents, are found in *An. Prior I., c.* 1, § 8. Λέγομεν δὲ τὸ κατὰ πάντος κατηγορεῖσθαι ὅταν μηδὲν ᾖ λαβεῖν τῶν τοῦ ὑποκειμένου, καθ' οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως. 'We say that an attribute is predicated of *All* a subject, 'when there is no one of the parts of the subject, of which the attribute is 'not predicated; and similarly in the case, in which it is predicated of *None*." The passage explains the significance of the universal proposition, which constitutes the major premiss. As Scotus says (l.c.), it gives us the Nominal definition (*quid nominis*) of the universal.

Some recent writers have erroneously identified Arist., *Cat.*, c. 3, § 1 with the *Dictum de Omni et Nullo.* On the true meaning of that passage, see a valuable note in Mr. Joseph's *Introduction to Logic*, p. 275.

reduce inference to a barren tautology : for if the subject is regarded as a collection of individuals, no inference is required to affirm the attribute of them taken singly. The general proposition has already affirmed it of *each*. Such judgments as these, which are really a matter of counting heads, are called *Enumerative* judgments. In the Dictum the universal subject is the species or genus as such, the logical whole. This appears clearly if we state the proposition in the generic form, ' Man is vertebrate,' ' The salmon has scales,' ' The nightshade is poisonous.' Our reason for employing the prefix ' All,' is to emphasize the distributive force of the subject, not to show that our knowledge is to be viewed as the result of an enumeration. It is possible to attain a knowledge of the species without knowing each member of the class. We cannot have experimental knowledge of all men, or of all salmon, or all plants of nightshade. There is therefore no tautology, when we infer from the logical whole to one of its parts.[1]

It is manifest that all reasoning in Fig 1, is of this type : the major premiss affirms (or denies) some attribute of a universal subject in its distributive acceptation, e.g. ' All mammals have lungs.' The minor premiss states that some class or some individual is a subordinate of this universal subject. The conclusion affirms the attribute of the class or the individual in question.

The Dictum is an immediate deduction from the principle of Contradiction. Were we to assert any attribute of a generic whole, and then deny it of some part of this whole we should violate that primary law

[1] That the Scholastics understood the universal terms constituting the subject and the predicate, not as classes reckoned in extension, but as logical wholes, may be seen from the following passage from Versorius Parisiensis, a Scholastic commentator on Petrus Hispanus. When dealing with the *Dictum de Omni et Nullo*, he distinguishes as follows, between the two expressions employed by Aristotle (*An. Prior I.*, c. 1, § 8) of the universal proposition. " Primo scien- ' dum quod ' dici de omni ' est conditio praedicati in ordine ad subjectum . . . ' sed ' esse in toto ' est conditio subjecti in ordine ad praedicatum, quia *sub- ' jectum est in praedicato sicut pars subjectiva in toto suo universali*." Petrus Hispanus (*Venice*, 1597), p. 217. If the subject is a *pars subjectiva* of the predicate it must be viewed as a logical unity, and not as a collection, even though it be qualified by the term ' All.'

of thought. We should simultaneously assert an *A* and an *O* proposition.

Why, it may be asked, did Aristotle regard the first figure as the only one possessed of perfect evidence? The answer must be that in this figure alone is the natural synthesis of the terms given in the premisses themselves. The premisses of the first figure give us '*S* is a logical part of *M*, *M* a logical part of *P*.' The subordination of the concepts, gives us the synthesis we require; and we pass at once to '*S* is a logical part of *P*.' The transition from the mental acts which constitute the premisses, to the conclusion, is spontaneous, immediate, and necessary.

In Figs. 2 and 3, the premisses do not give us the terms in the order of subordination which we need. The synthesis we are seeking, is not offered to us in the mere conjunction of the premisses. The scheme of Fig. 2 makes both *S* and *P* subordinate to *M*: that of Fig. 3 gives us *M* as a logical part of *S* and *P*. Hence the logical relation of the terms *S* and *P* is not given to us in the premisses themselves. Doubtless our data are sufficient for us to draw the conclusion which *de facto* we do draw. But the mental operation here is not the perfect process of inference: for in that the premisses are *two causative principles whose mere conjunction in the mind gives us the conclusion as an immediate result*.[1] It was for this reason that Aristotle was led to regard these forms as imperfect, and to maintain that only in Fig. 1 does the full necessity of the inferential act appear. It need not however be supposed that he believed Reduction to represent the actual working of

[1] Aristotle tells us that the premisses are the *material* cause of the conclusion. *αἱ ὑποθέσεις τοῦ συμπεράσματος ὡς τὸ ἐξ οὗ αἰτιά ἐστιν.* II. *Phys.* 3, § 7, cf. *Met.* IV., c. 2, § 7. They are the material cause in a sense analogous to that in which the parts are the material cause of the whole. Thus e.g. the parts of a circle separately are merely the material of the circle: but set together in due order they result in the circle. In this sense in *An. Post. II.*, c. 11, § 1 he calls the material cause to '*τὸ τίνων ὄντων ἀνάγκη τοῦτ' εἶναι.*' The similarity between this definition and that of the syllogism (p. 169, note) is patent. The premisses taken separately are mere 'material': but taken together they give a complete inference.

the mind in reasoning. We employ, it can hardly be doubted, Figs. 2 and 3 as independent instruments of thought. Where e.g. we have such premisses as,

All heliotrope is sweet-smelling.
This flower is not sweet-smelling,

we need not convert the negative premiss: we recognize that any other conclusion than 'This flower is not heliotrope' would involve a contradiction. But the process by which the premisses result in the conclusion is not the connatural inferential process of the mind. That is only found when the three concepts which together constitute the object of the intellectual act, stand in such a relation to each other, as themselves to give us the logical relation of the terms of the conclusion.[1]

It is asserted by many recent logicians that Aristotle was in error in holding the first figure to be the only one in which the inferential process is absolutely evident. Independent canons are given for the other figures: and the whole process of Reduction is declared to be useless. The following canons are among those which have been suggested.

Fig. 2. *Dictum de diverso.* "If a certain attribute can be predicated affirmatively or negatively of every member of a class, any subject of which it cannot be so predicated, is not a member of that class" (Mansel, *Aldrich*, p. 84).

Fig. 3. (I) *Dictum de exemplo.* "If a certain attribute can be affirmed of any portion of the members of a class, it is not incompatible with the attributes of that class" (*ibid.*)

(II) *Dictum de excepto.* "If a certain attribute can be denied of any portion of the members of a class, it

[1] No argument can be drawn against the doctrine of Reduction from those syllogisms in the third figure, in which the middle term is a singular term—the *syllogismus expositorius*, e.g. Socrates is poor, Socrates is wise ∴ Some wise men are poor. This is not an inferential process at all, but an appeal to experience in a particular case. The mind does not pass to a new truth it did not possess. "Syllogismus expositorius non est vere syllogismus, sed magis 'sensibilis demonstratio, seu resolutio facta ad sensum, ad hoc quod conse-'quentia quae vera est secundum intellectualem cognitionem, declaretur in 'sensibili." St. Thomas, *Opusc.* 43, *De Natura Syllogismi.* It is in that manner that it is employed by Aristotle in the proof of Fig. 3.

is not inseparable from the distinctive attributes of that class" (*ibid.*)

Fig. 4. Considerable difficulty has been found in discovering any principle, which shall serve as a canon for this figure. Lambert's *Dictum de reciproco* runs as follows: "If no M is B, no B is this or that M: if C is or is not this or that B, there are Bs which are or are not C."

Those who reject Aristotle's doctrine, regard the syllogism from a totally different point of view, and in most cases appear to be ignorant as to the nature of his theory of inference.[1] In the canons just cited, the general proposition is a mere statement about the "members of a class" taken in extension. The idea of the terms of the premisses as objects of thought related one to another as logical whole and logical part, was quite foreign to them. It is easy enough to draw up canons on the basis of extension. But if the canons are to be strictly logical in their character, it may be questioned whether any save that of the first figure can claim to be absolutely self-evident.

Mill rejects the *Dictum de omni* on the ground that it "merely amounts to the identical proposition that 'whatever is true of certain objects, is true of each of 'those objects" (II., c. 2, § 2). He prefers the canon "Whatever is a mark of any mark, is a mark of that 'which this last is a mark of." In regard to this it is sufficient to say that in the conceptual order we are not concerned with *things* and *marks*, but with *subjects* and *attributes*. The principle is not a logical principle at all.[2]

* § 2. **The Fourth Figure.** The Fourth Figure calls for

[1] The Aristotelian theory of the syllogism fell into almost complete oblivion. It is interesting however to note that it was explained and defended by the eighteenth century Scottish writer Lord Monboddo. See Dugald Stewart's *Philosophy of the Human Mind*, vol. II., ch. 3, § 1.

[2] Many recent Scholastic writers give the principles: '*Quae sunt eadem uni tertio sunt eadem inter se: Si ex duobus unum cum tertio identicum est, alterum non est, neque inter se illa duo identica esse possunt.*' But in the logical order the subject and predicate are not the same. The identity is in the real order, not the conceptual. To justify the syllogism we need a principle referring to thought not things.

separate treatment. It corresponds to no natural process of inference. The premisses tell us that '*A* is a logical part of *B*, *B* a logical part of *C*'; and we are asked to regard a conclusion in which *C* is subordinate to *A*, as a normal working of the mind on a parity with a conclusion in Fig. 1. It may safely be asserted that the mind never acts in this way: though, of course, the conclusion that *C* is in part subordinate to *A*, is not invalid. Three of the moods (*Bramantip, Camenes, Dimaris*), are in fact simply the first three moods of Fig. 1 with the conclusion converted. The two remaining moods (*Fesapo, Fresison*) are not of this character. But they need not therefore be relegated to a special figure. Aristotle (*An. Prior I.*, c. 7) views them as cases of Fig. 1, in which the minor premiss is a universal negative, $\begin{matrix} M & a & (i) & P. \\ S & e & & M. \end{matrix}$ Here, he says, a conclusion can be obtained, if the premisses be converted: but in the conclusion the minor term will be predicated of the major,

$M\ e\ S.$

viz. $\underline{P\ i\ M}$ The arrangement made by his disciple Theophrastus,

$\therefore P\ o\ S.$

was, it would appear, more satisfactory. He recognizes the possibility, not merely of the normal conclusion in Fig. 1, '*S* is subordinate to *P*,' but also of the abnormal but valid conclusion, '*P* is subordinate to *S*.' This adds the five moods in question to the four Aristotelian moods of Fig. 1, but reckons them in a different class. This system was followed by the mediæval logicians, who termed them *Indirect Moods* of Fig. 1.[1] It is stated that Galen was the first to rank them apart as an independent mode of reasoning. But this method did not become prevalent till the decadence of Scholasticism, when excessive attention was paid to the mechanical arrangement of terms and premisses, and philosophic considerations were neglected.[2]

Aristotle's conception of the figures of the syllogism is perhaps best understood, if they be considered in their application to the mental organization of our knowledge along the predicamental lines (Ch. 9, § 3). In Fig. 1, employing a genus as middle term we establish a connexion between a species on the one hand, and on the other either a property of the genus or some higher

[1] The mnemonic lines as given by Petrus Hispanus follow this arrangement:—

Barbara, Celarent, Darii, Ferio. Baralipet
Celantes, Dabitis, Fapesmo, Frisesmo. *Deinde*
Cesare, Camestres, Festino, Baroco. Darapti.
Felapton, Disamis, Datisi, Bocardo, Ferison (*Petrus Hisp.*, Venice, 1597, p. 249).

[2] On Fig. 4, *vide* Joseph, *Introduction to Logic*, pp. 301–305. Ueberweg, § 103.

genus under which it falls : in Fig. 2 we shew the negative relation between species which do not fall under the same genus ; in Fig. 3 we shew the relation which holds between two genera, if the same species is subordinate to both. This last relation can, of course, only give a particular conclusion.

§ 3. **Expression in Syllogistic Form.** The student will find it a valuable exercise to throw into syllogistic form some of the reasonings he may meet with in the books which he reads, and in ordinary conversation. Whenever we conclude from a general principle to a particular case which falls under that principle, the argument is syllogistic, though the syllogism may not be expressed in full. An argument stated with its full paraphernalia of major, minor, and conclusion may be compared to a specimen beetle set out for exhibition. When at large and alive, it is not accustomed to display its members in a manner so convenient for the entomologist. Nor do we, either in the spoken or written word, express every step of our argument. We may omit one of the premisses, leaving our hearers to supply it. Or we may state the premisses, and leave the hearer to draw the conclusion for himself. Or we put one member, not as a judgment, but as a rhetorical question. One or two examples will serve to illustrate the point:—

'This undertaking is doomed to failure. No enterprise succeeds whose promoters lack foresight and prudence.'

This may be expressed by the syllogism,

No enterprise, whose promoters lack foresight and prudence, is successful.

This undertaking is an enterprise, whose promoters lack foresight and prudence.

∴ This undertaking will not be successful.

The following example is a little more complex :—

'There is a real reason for distrusting many common-sense judgments, since these are largely the general opinion of men based on mere sense-perception, without the correction which mature reflection affords.'

The syllogistic expression will be :—

All general opinions, based on mere sense-perception and without, etc., etc., are to be distrusted.

Many common-sense judgments are general opinions, based on mere sense-perception, etc., etc.

∴ Many common sense judgments are to be distrusted.

The next example provides a case in which one of the members is expressed as a rhetorical question :—

'Can any general, whose army is disheartened, attain victory? That was his case ; and there lay the cause of his failure.'

This will become :—

No general whose army is disheartened is victorious.

He was a general whose army was disheartened.

∴ He was not victorious.

§ 4. **Progressive and Regressive Syllogisms.**

Progressive Syllogisms are those in which we reason from a cause to its effect. Regressive Syllogisms are those in which we reason from the effect to the cause. Thus I may argue from the ascent of the mercury in the tube of a barometer, to the conclusion that it must be subject to atmospheric pressure. Or I may argue from a previous knowledge of the existence of atmospheric pressure, to the ascent of the mercury as a necessary result. The Regressive syllogism will take the form :—

A liquid, which (under the given circumstances) ascends in an exhausted tube, must be subject to atmospheric pressure.

The mercury of a barometer is a liquid which ascends under the circumstances stated.

∴ The mercury of a barometer is subject to atmospheric pressure.

A Progressive syllogism will proceed in the converse way :—

What is subjected to atmospheric pressure will (under given circumstances) ascend in the tube.

The mercury of the barometer is subjected to atmospheric pressure.

∴ The mercury of the barometer will ascend in the tube.

In Progressive syllogisms it is the middle term which expresses the cause.

The cause from which we conclude to the effect may be either the immediate or the remoter cause. Thus, if

I argue that Socrates is capable of sensation *because he is a man*, I argue from a remoter cause. He is endowed with the capacity to feel, not because he belongs to the narrower class 'man,' but because he belongs to the wider class 'animal.' It is in his being an animal that we find the immediate cause of his sensation.

When we know a thing through its immediate cause, our knowledge of it is of a higher character than when we reach it in any other way. It is when we know the precise immediate cause of a thing, the reason why it takes place, that we regard ourselves as fully understanding it.

In the syllogism in which we argued that the mercury must rise in the barometer because of the presence of the atmosphere, our reasoning was based on the efficient cause. But each of the four causes (Ch. 10, § 2) may be employed in a Progressive argument (*An. Post. II.*, c. 11).[1]

The syllogism is thus seen to afford us an inference based on our knowledge of law. It is our knowledge of a law which enables us to infer either from effect to cause, or cause to effect. A syllogism in which the major premiss is a mere enumerative judgment, is an inference of another and far inferior kind. Arguments such as, e.g. 'All the apostles were Jews; St. Paul was an apostle; ∴ St. Paul was a Jew,' have for those who know the grounds on which the major is based, no right to the name of reasoning. For those who themselves have made the enumeration, there is no passage from one truth to another. They knew the conclusion before they asserted the major.[2]

§ 5. **Validity of the Syllogism.** It is necessary for us to consider a view taken by certain logicians, belonging

[1] Syllogisms in which the middle term is the immediate cause of the attribute which constitutes the major, are styled by Aristotle *Syllogisms of the Cause* (συλλογισμοὶ τοῦ διότι). All others are classed as *Syllogisms of the Fact* (συλλογισμοὶ τοῦ ὅτι). Cf. *An. Post. I.*, c. 13. It is Syllogisms of the Cause which he regards as the true syllogisms of science: in them alone our mental expression of things is in harmonious correspondence with the order of Nature.

[2] On the two kinds of universal, see *Summa Totius Logicae*, Tract 9, c. 2.

to the Empiricist school, to the effect that the syllogism as a method of inference, is formally invalid. The most conspicuous recent logician who has adopted this position is Mill. In this section we shall notice the objections which he urges against the syllogism, and the account of the mind's process in reasoning which he regards as presenting a more satisfactory analysis.

He argues that the major premiss, if literally interpreted, is a manifest begging of the question. In it, we assert the very fact, which in the conclusion, we profess to prove. The conclusion, as we state it, contains nothing new: it is merely a reassertion of what we have already affirmed in the major. "In every 'syllogism, considered as an argument to prove the con-'clusion, there is a *petitio principii*. When we say, ''All men are mortal, Socrates is a man, therefore Socrates 'is mortal,' it is unanswerably urged by the adversaries 'of the syllogistic theory, that the proposition Socrates 'is mortal, is presupposed by the more general assump-'tion, All men are mortal: that we cannot be assured 'of the mortality of all men, unless we are already assured 'of the mortality of every individual man" (*Logic*, Bk. II., c. 3, § 2). It is clear, that if this be admitted, we do not reach the conclusion that Socrates is mortal from the major premiss, but on some other grounds. Mill holds that the true grounds for our conclusion, are in every case to be found in individual facts. We argue from the deaths of individual men, of John and Thomas and every other person in whose case the experiment has been fairly tried, that the person of whom we are speaking is mortal like the rest. We may indeed pass through the stage of asserting that All men are mortal, but we need not do so: not one iota is added to the proof by interpolating a general proposition. Indeed, experience assures us, that as a matter of fact, we habitually do reason from particulars, without the interpolation of the general proposition. "All our 'earliest inferences are of this nature. . . . The child, 'who, having burnt his fingers, avoids to thrust them

'again into the fire, has reasoned or inferred, though 'he has never thought of the general maxim, Fire burns. '. . . He is not generalizing : he is inferring a particular 'from particulars" (*ibid.* § 3).

From these considerations, Mill concludes that the reasoning process is but ill represented by the syllogistic form. The true and universal type of ratiocination may be thus expressed, "Certain individuals have a 'given attribute : an individual or individuals resemble 'the former in certain other attributes : therefore they 'resemble them also in this attribute."

If then, there is no inferential process in the descent from the major premiss to the conclusion, where, it may be asked, does the inference take place, when we use the syllogism in argument? Mill tells us that the whole of the inferential process is found in the assertion of the major premiss. The assertion of the universal proposition shows that the grounds on which we draw an inference in regard of some one individual, are sufficient for us to infer the same conclusion in regard to any individual, who, in certain attributes, resembles the cases we have observed. A general proposition serves not merely to preserve in the memory the particular facts it records, but also to contain our inference from these facts. But "the inference is finished when we have 'asserted that all men are mortal. What remains to 'be performed afterwards is merely deciphering our 'own notes" (*ibid.* § 3).

Nevertheless, while denying the claim of the syllogism to be a correct analysis of reasoning, Mill holds those who look on it as useless, to be mistaken. The universal statement renders us cautious in our inferences. Were we not to make use of it, we might easily, in cases in which we feel a special interest, give way to the natural inclination to draw a conclusion in accordance with our wishes. The universal proposition shews us that "the experience, which justifies a single prediction, 'must be such as will bear out a general theorem" (*ibid.* § 5). It may well be that this general theorem

is manifestly untrue. Hence we discover our error by a *reductio ad impossibile.*

Our criticism on this theory shall be confined to three issues :—

(1) It is not the case that the universal proposition is a mere record of individual facts, together with an inference to other particulars. The true universal proposition, (for we are not here concerned with the enumerative judgments mentioned in § 1) relates, not to a collection of observed instances, but, as we have often said, to the nature regarded in the abstract. When e.g. we say, ' The cow is a ruminant,' we mean that the attribute ' ruminant ' belongs to the nature of the cow, and is to be found in any number of individuals, past, present or future, in whom that nature is realized. The Empiricist school deny that we possess this power of intellectual abstraction. By so doing they cut away the foundations of Logic.

(2) It is erroneous to say that the whole inference is finished, when we have asserted the major premiss. The major premiss may be well known to us, while, through ignorance of the minor, we may be unaware of the conclusion. I may have learnt that all mammals breathe through lungs, and yet because I imagine that a whale is a fish, I may not be aware that whales breathe through lungs. It is true that the major universal proposition, asserting that all mammals breathe through lungs, covered the *fact* that whales do so. But inference has to do, not with the facts as they are in the real order, but with our knowledge of the facts. And if I do not know that whales breathe through lungs, I certainly did not infer it when I made the assertion that all mammals breathe through lungs. It may even happen that I know both the major and the minor premiss, and yet may be ignorant of the conclusion. For unless I think of the two premisses *together*—unless I make the *synthesis*, I may never reach the conclusion involved in them.[1]

[1] Cf. *An. Prior II.*, c. 21, § 9.

(3) If we examine the nature of the inference, by which we pass from particulars to a conclusion about another particular which resembles them, we shall find that a universal proposition is involved. We argue from one case to another, because they possess certain common attributes. That is to say, we base our conclusion on these attributes. But this involves a universal proposition applicable to each and every case in which such attributes are present.

* § 6. **Mathematical Reasoning.** Several recent logicians have maintained that much of the reasoning employed in mathematics is not syllogistic. As cases in point they allege inferences of the following character, common enough in geometry and in arithmetic :—

The triangle *ABC* is equal to the triangle *DEF*.
The triangle *GHI* is equal to the triangle *DEF*.
∴ The triangle *GHI* is equal to the triangle *ABC*.

and

$12=7+5$, $12=20-8$, ∴ $7+5=20-8$.

Here, it is argued, the process is clearly non-syllogistic. Our data do not give us a subject-attribute relation, but a relation between two quantities. The argument if carefully inspected, has four terms, not three. And the principle on which the reasoning is based, is not a canon of the syllogism, but the axiom 'Things which are equal to the same thing are equal to each other.'

We believe this view to be erroneous. In the first place the data most certainly give us a subject-attribute relation : for this is inseparable from judgment. We predicate the attribute 'equal to *DEF*' of the two subjects *ABC* and *GHI*. Secondly, it is impossible that the axiom 'Things which are equal to the same thing, etc., etc.' should be a principle of inference. It is a truth relating to the real order, not to the conceptual. It is necessary to the inference, but it is not a canon governing the inferential process itself. A canon of inference must have explicit reference to the conceptual order. Here, as elsewhere, we find our ultimate justification in the *Dictum*, which tells us that what is predicated of the logical whole, may be predicated of its parts. Doubtless, in practice we do not need so to arrange our data as to show how the argument is justified by its conformity with this canon. The mind has no more need of this than it has of reduction. But if this be required of us, the argument must be evolved in the form :—

Any two quantities, each of which is equal to the same third quantity, are equal to each other.
ABC and *GHI* (as being equal to *DEF*) are two quantities each of which is equal to the same third quantity.
∴ *ABC* and *GHI* are equal to each other.

§ 7. Inferences other than Syllogistic. Some defenders of the syllogism have gone so far as to maintain that there is no such thing as non-syllogistic inference. They represent the syllogism as the only legitimate form of reasoning, and hold that whenever the mind infers, it must of necessity proceed syllogistically. If we restrict the term inference to signify the derivation of a truth from propositions in which it is implicitly contained, this is true. In the syllogism, and in the syllogism alone, is the conclusion implicitly contained in given premisses. But inference is frequently used in a wider sense, to include also the derivation of a conclusion from concrete facts by an act of the critical judgment. When a jury after weighing a mass of evidence acquit or condemn a man accused of burglary, they may be rightly said to infer. But their concern has not been to find out what is implicitly contained in certain propositions, but to find out what is involved in certain concrete facts. They decide that these facts are compatible or incompatible, as the case may be, with the man's innocence. Their conclusion is simply a critical estimate of what the facts involve. Cardinal Newman has discussed this form of inference at length in the *Grammar of Assent*. But the difference between the two forms of reasoning was familiar to St. Thomas, and is carefully noted by him more than once.[1]

* Mr. Bradley, dealing with this question, gives the following as specimens of non-syllogistic inference: (1) *A* is to the right of *B*, *B* is to the right of *C*, therefore *A* is to the right of *C*; (2) *A* is due north of *B*, *B* is due west of *C*, therefore *A* is north-

[1] St. Thomas holds that as these inferences are solely concerned with particular facts, they are effected by the *vis cogitativa*, in which sense and intellect meet See *Summa Theol.*, II. II^ae., Q. 2, Art. 1.

west of *C*, etc., etc. In regard to these, we believe him to be mistaken. It seems to be indubitable, that in these cases, as in the mathematical inferences considered in the last section, the mind relies on an unexpressed, but self-evident first principle. It is as self-evident that what is locally to the right of an object, which is situated to the right of a third entity, is itself to the right of that entity, as that things which are equal to the same thing are equal to each other.

* § 8. **Mr. Bradley's theory of Inference.** The view taken by Mr. Bradley as to Universals (Ch. 8, § 5) will have to some extent prepared the reader for his theory regarding the closely connected question of inference. "Inference," he tells us, "rests on the principle that what seems the same, is the 'same" (*Principles*, p. 264). In virtue of this 'axiom,' wherever we have two premisses with a common term, we are able to effect 'an ideal construction,' which unites the two judgments into an individual whole, and so permits the mind to pass to the assertion of a new relation. "The premisses . . . are two 'or more judgments, and the operation on these data will con-'sist in joining them into a whole. We must fasten them to-'gether, so that they cease to be several, and are one construc-'tion, one individual whole. Thus instead of *A*—*B*, *B*—*C*, we 'must have *A*—*B*—*C*" (*ibid.*, p. 236). "If we are to construct, 'we must have identity of the terminal points. Thus in *A*—*B*, '*B*—*C*, *B* is the same, and we connect *A*—*B*—*C*. The operation 'consists in the extension and enlargement of one datum by 'others, by means of the identity of common links. . . . Hav-'ing thus turned our premisses into one whole we proceed to 'our conclusion by mere inspection. . . . By inspection we 'discover and select a new relation, and this intuition is the 'conclusion. . . . It is useless to lay down rules for either 'part of this process. . . . No models for construction can 'possibly be invented" (*ibid.*, pp. 237–8).[1]

The view taken by Mr. Bosanquet differs in various respects from Mr. Bradley's theory. He rejects the account of the process as a 'construction.' He is, however, at one with Mr. Bradley in holding that inference depends on a real identity shared by the various differences of a common universal. "It is possible," he says, "to proceed from content to content, because the world 'as known consists of universals exhibited in differences, and

[1] Mr. Bradley's respect for the theory of the syllogism is small. He denies that a major premiss is necessary in inference. "Begotten by an old meta-'physical blunder, nourished by a senseless choice of examples, fostered by 'the stupid conservatism of logicians, this chimaera has had a good deal more 'than its day." "The syllogism itself like the major premiss is a superstition." "Perhaps we may say that a man who cannot use more than three terms in 'reasoning, is unlikely to do much in any subject" (*Principles*, pp. 228-9).

'the contents from which and to which we proceed, are not 'shut up within their respective selves, but depend on a per- 'vading identical character or universal, of which they are the 'differences" (*Logic*, II., p. 2). The simplest example of an inference through a universal he finds in the syllogism in which the middle term is a proper name, as when we argue that since Socrates is both good and a Greek, therefore a Greek may be good (*ibid.* p. 200).[1]

As we have already pointed out, this view that things which can be brought under the same universal are united by a pervading identity, is based on the confusion of the real and conceptual order. In the real order the things are similar, not identical. But in virtue of the abstractive power of the intellect, we can prescind from individual varieties and represent them by a common concept. This universal concept enables us to make statements valid in regard to all objects to which it may be applied, and thus provides us with a basis for inference.

[1] The reader will not need to be reminded that the Scholastic authors denied that there was any inferential process in these *syllogismi expositorii*: see p. 190, note.

CHAPTER XIII.

HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS.

§ 1. **Mixed Hypothetical Syllogisms.** Syllogisms, in which one or both of the premisses are hypothetical propositions, are termed hypothetical syllogisms. If both premisses are of this character, the syllogism is known as a *Pure Hypothetical*. This is of far less importance than the *Mixed Hypothetical Syllogism*. The latter is defined as a **syllogism, in which the major premiss is a hypothetical proposition, and the minor premiss a categorical proposition either affirming the antecedent or denying the consequent of the major.** The following formulas show the character of the reasoning : in Ex. 1, the antecedent is affirmed ; in Ex. 2, the consequent is denied.

Ex. 1.	If *A* is *B*, it is *D*.	or If *A* is *B*, *C* is *D*.
	A is *B*.	*A* is *B*.
	∴ *A* is *D*.	∴ *C* is *D*.
Ex. 2.	If *A* is *B*, it is *D*.	or If *A* is *B*, *C* is *D*.
	A is not *D*.	*C* is not *D*.
	∴ *A* is not *B*.	∴ *A* is not *B*.

The affirmation of the antecedent, and the denial of the consequent constitute the two moods of the mixed hypothetical syllogism. These are known as the **modus ponens** (or the *constructive* hypothetical syllogism), and the **modus tollens** (or the *destructive* hypothetical syllogism) respectively, these names being derived from the canon of the syllogism, which is thus stated :—

To posit the antecedent is to posit the consequent ; and to sublate the consequent is to sublate the antecedent.[1]

It is more convenient to employ the terms ' posit ' and

[1] Posito antecedente *ponitur* consequens : sublato consequente *tollitur* antecedens.

'sublate,' in this connexion, than 'affirm' and 'deny. For the antecedent may contain a negative; and in this case the proposition positing it, will also be of negative quality. To speak of negative propositions as affirmations, could only cause confusion. This feature is illustrated by the following syllogism,

If the doctor is not skilful, he will cause this patient much pain.

He is not skilful.

∴ He will cause this patient much pain.

Here we posit the antecedent by a negative sentence.

It is further to be observed that when the consequent is denied, the conclusion will be the *contradictory*, not the contrary, of the antecedent it denies. Thus in the syllogism,

If all men were actuated by the highest motives, the sanction of punishment would be unnecessary.

The sanction of punishment is not unnecessary;

we can only conclude that '*Some* men are not actuated by the highest motives.' It would be fallacious to argue that ***none*** are so actuated.

There are two fallacies to which these syllogisms are liable. These are (1) the fallacy of denying the antecedent, and (2) the fallacy of affirming the consequent. From the fact that the antecedent can be denied, we cannot conclude to the falsity of the consequent: for it may well be that the same result may take place, but owing to some other cause. I cannot argue thus:

If his rifle is damaged, he will fail to hit the bull's-eye.

His rifle is not damaged.

∴ He will not fail to hit the bull's-eye.

Similarly, to assert the truth of the consequent affords us no ground for concluding to the truth of the antecedent. We cannot, for instance, argue

If the novel possesses real literary merit, it will be widely read.

It will be widely read.

∴ It possesses real literary merit.

This fallacy has, at all times, been one of the most frequent pitfalls in men's ordinary reasonings. The old proverb, *Cucullus non facit monachum*, warns us that, though it is safe to argue that because a man is a monk, he will wear a habit, there is risk of error, when we conclude if he wears a habit, therefore he must needs be a monk.

The fallacy of denying the antecedent is often said to be equivalent to illicit process of the major, and that of affirming the consequent to undistributed middle. It is in fact the case, that if we express the major of a Barbara syllogism as a hypothetical judgment, the fallacies of illicit major and undistributed middle will be thus represented. For example, 'Every man is a biped; this bird is a biped: ∴ This bird is a man,' will become, 'If anything is a man, it is a biped: this bird is a biped: ∴ This bird is a man.' But it cannot be said there is any real equivalence. The error in thought is not the same. Moreover the transformed categorical syllogism is only in appearance a hypothetical syllogism. It has no true right to the name: for the minor affirming the consequent, contains a term 'bird,' which is not found in the major. In the true hypothetical, the minor introduces no new terms.

Several logicians of eminence, amongst others Kant, Hamilton and Bain, have held that the mixed hypothetical syllogism is an Immediate and not a Mediate Inference. The reason for this view, is that in the hypothetical we have only two distinct clauses, both of which appear in the major. Hence it has been argued that the true mode of stating the hypothetical syllogism is,

If *A* is *B*, *C* is *D*.
∴ *A* being *B*, *C* is *D*.

The view cannot be maintained. The judgment '*A* is *B*, is not part of the conclusion, but is one of the data. From the conditional statement, 'If the afternoon prove fine I shall go for a walk,' we cannot draw as a conclusion the judgment that 'The afternoon is fine.' The data of the syllogism are two in number, and neither is sufficient alone.

Pure Hypothetical Syllogism. In the Pure Hypothetical

Syllogism, as has already been observed, all the propositions are hypothetical. The argument is of the form :—

If *C* is *D*, *E* is *F*.
If *A* is *B*, *C* is *D*.
∴ If *A* is *B*, *E* is *F*.

Since the hypothetical proposition admits no variety as regards quantity or quality, but is necessarily both affirmative and singular, this syllogism has but one form and no more.

§ 2. **Reduction of Hypothetical Syllogisms.** In these syllogisms, the *modus ponens* may without difficulty be reduced to the *modus tollens*, and *vice versa*. All that is required is to substitute the negation of the consequent for the antecedent, and the negation of the antecedent for the consequent. Thus,

Modus ponens	*Modus tollens*
If *A* is *B*, *C* is *D*, may be expressed	If *C* is not *D*, *A* is not *B*.
A is *B*.	*A* is *B*.
∴ *C* is *D*.	∴ *C* is *D*.

It will be seen that the second form is a true *Modus tollens*, since the conclusion '*C* is *D*,' contradicts the antecedent of the major premiss.

Reduction to categorical form. It is sometimes said that all these syllogisms can be reduced to categorical form. In regard to this point, the essential difference between the two classes of proposition must be borne in mind. The categorical proposition gives us two concepts related as subject and attribute : the hypothetical proposition gives us two judgments related not as subject and attribute—as a thing and its determination, but as reason and consequent. It may safely be said that to express the one in terms of the other, is to do violence to our thought. This appears plainly in the following example.

If the opium habit is innocuous, the whole medical faculty is deluded.
The whole medical faculty is not deluded.
∴ The opium habit is not innocuous.

A syllogism of this character cannot naturally be

expressed as a categorical. The result, it is true, is sometimes achieved, as follows :—

The case of the opium habit being innocuous is a case of the whole medical faculty being deluded.
The case of the whole medical faculty being deluded is a supposition not to be admitted.
∴ The case of the opium habit being innocuous is a supposition not to be admitted.

But it is rightly urged that the altered major premiss has no meaning, unless it is translated back into its hypothetical significance. The latter clause does not really express an attribute of the former. The mind must understand the two clauses as reason and consequent, whatever be the clumsy phraseology we elect to employ. Hence, there is here no reduction properly so called. The mental process remains the same. It is the words only that have been changed. And the result of the change is thoroughly misleading, since it induces us to regard as identical, processes which are fundamentally distinct.

§ 3. **The Disjunctive Syllogism.** (*a*) *Import of the Disjunctive Judgment.* Before explaining the disjunctive syllogism, a word must be said as to the import of the disjunctive judgment. This point could not be conveniently treated in Ch. 7, where the import of categoricals and hypotheticals was discussed: for, as we have seen (Ch. 10, § 5), one form of the disjunctive expresses the relation of the genus to its constituent species, and in consequence, cannot be understood till the nature of that relation has been explained. The disjunctive proposition is of two kinds. One may be termed the *Disjunctive of Logical Division.* In it the subject is a generic whole, while the predicate is formed by the disjunctive enumeration of the subordinate parts. This form of the judgment has been discussed sufficiently already. The other kind may be termed the *Disjunctive of Ignorance.* In this we affirm that one out of two (or more) predicates which we enumerate, belongs to the

subject. Our knowledge is sufficient for us to assert that the truth lies with one of these alternatives, but we do not know with which : e.g. ' This curve is either a circle or an ellipse,' ' He is either very thoughtful or very stupid.' In disjunctives of this class the subject is not the logical whole but a subordinate part. It is the function of the predicate to express the whole, as in the categorical sentence: but we are in doubt to which of two logical wholes the subject should be referred.[1]

Logicians have disputed as to whether in a disjunctive judgment the members of the disjunction are mutually exclusive. In other words, when we assert ' *S* is *P* or *Q*,' do we intend to exclude the supposition that some particular *S* may be both at the same time? In regard to the first class of disjunctives, the mutual exclusiveness is quite evident. The species must be distinguished from each other. But in regard to the disjunctives of ignorance, it would seem that the mere form of the proposition gives us no assurance on the point. If I say ' He must be very clever or very painstaking,' I can hardly be understood to exclude the supposition that the person in question may be both. Often however the members of the disjunction are formed by repugnant terms, e.g. ' The horse was either bay or chestnut.' Here we have mutual exclusion : but it is not due to the disjunctive form as such.

(*b*) *The Disjunctive Syllogism.* This may be defined as **a syllogism in which the major premiss is a disjunctive proposition, and the minor a categorical proposition, either affirming or denying one member of the opposition.** It is to be noted that this syllogism has no application in regard to the first of the two forms of disjunctive proposition. It is concerned with the Disjunctive of Ignorance alone. The argument has two moods, called respectively the *modus ponendo tollens* and the *modus tollendo ponens*. The affirmation in the minor premiss

[1] The form ' Either *A* is *B*, or *C* is *D*,' is also a case of the Disjunction of Ignorance.

of one of the two alternatives, gives us the *modus ponendo tollens* : the denial of an alternative, the *modus tollendo ponens*. The following formulas illustrate these cases :—

Modus ponendo tollens.	*Modus tollendo ponens.*
S is either P or Q.	S is either P or Q.
S is P.	S is not P.
$\therefore S$ is not Q.	$\therefore S$ is Q.

The *modus ponendo tollens* is only valid in those cases, in which the members of the disjunction are known to be mutually exclusive. Otherwise it would be impossible to conclude from the presence of one alternative to the absence of the other. The *modus tollendo ponens* is always valid.

§ 4. **The Dilemma.** The Dilemma is an argument in which both the hypothetical and the disjunctive judgment are employed. It is defined as an argument in which the major premiss is a compound hypothetical proposition, and the minor a disjunctive. In actual use the disjunctive is often placed first. Its chief importance arises from its effectiveness as a rhetorical weapon. The disjunctive apparently exhausts the alternatives open to the adversary ; and the hypothetical, which follows, shows that each one of them leads him to an unwelcome conclusion. Hence it is not surprising that this argument was first analysed and explained by rhetoricians.[1] Where three alternatives are offered, we have the **Trilemma** : where four, the **Tetralemma.** Like other hypothetical syllogisms it has a *constructive* and a *destructive* form, according as the disjunctive proposition affirms that one of the antecedents must be true, or that one of the consequents must be false. Moreover both the Constructive and the Destructive Dilemma may be *Simple* or *Complex*. In the *Simple Constructive*, the consequents of the two hypothetical clauses are identical. In the *Complex Constructive* they are different. It is manifest that in a Constructive Dilemma, there can be no question of the antecedents being identical : for

[1] See Ueberweg, § 123.

were they so, they could not be disjunctively asserted in the minor. In the destructive form, the opposite is the case. The *Simple Destructive* has but one antecedent ; the *Complex Destructive* has two. The following formulas show their varieties :—

(1) *Simple Constructive.* If A is B, E is F : and if C is D, E is F.
Either A is B, or C is D.
$\therefore$ E is F.

(2) *Complex Constructive.* If A is B, E is F : and if C is D, G is H.
Either A is B, or C is D.
$\therefore$ Either E is F, or G is H.

(3) *Simple Destructive.* If A is B, E is F, and G is H.
Either E is not F, or G is not H.
$\therefore$ A is not B.

(4) *Complex Destructive.* If A is B, E is F : and if C is D, G is H.
Either E is not F, or G is not H.
$\therefore$ Either A is not B, or C is not D.

(1) We may find a suitable illustration of the Simple Constructive Dilemma in the argument by which Empson, the notorious agent of Henry VII., is said to have always been able to prove that his victim was capable of paying a large amount into the treasury :—

If the accused lives at a small rate, his savings must have made him very rich : if, on the other hand, he maintains a large household, his expenditure proves him to be wealthy.

But either he lives at a small rate, or he maintains a large expenditure.

$\therefore$ He is rich,—and consequently can pay heavily to the king.

This piece of reasoning was known, as the reader will remember, as 'Empson's *fork*,'—a name, which is a striking parallel to the expression, which speaks of a man as being *impaled on the horns* of a dilemma. The argument leaves no escape. Whichever alternative is selected, the result is equally disagreeable.

(2) As an example of the Complex Constructive Dilemma, we may take Tertullian's argument against the methods employed by Marcus Aurelius in his persecution of the Christians :—

Either the Christians have committed crimes, or not.

If they are guilty of crimes, your refusal to permit a public enquiry is irrational ; if they have committed no offence, it is unjust to punish them.

∴ Your conduct is either irrational or unjust.

Unfortunately from the time of Nero to the present day, neither justice nor reason have weighed much with the persecutors of the Church.

(3) The following may serve to illustrate the Simple Destructive :—

If he would pass the straits unharmed, he must escape both Scylla and Charybdis.

But either he will not escape Scylla, or he will not escape Charybdis.

∴ He will not pass the straits unharmed.

(4) This example of the Complex Destructive Dilemma is given by some authors :—

If he were intelligent, he would see the worthlessness of his arguments ; if he were honest, he would own himself wrong.

But either he does not see that his arguments are worthless ; or seeing it, he will not own himself in the wrong.

∴ Either he is wanting in intelligence, or he is dishonest.

The student will observe that the premiss of the Simple Destructive form differs in a material point from the Simple Constructive. In the Destructive Dilemma, the consequent is copulative, not disjunctive: 'If *A* is *B*, *C* is *D and E* is *F*.' The reason for this is, that were the two consequents alternatives, the denial of one or other of them in the minor premiss would not necessarily involve the falsity of the antecedent. An argument can, of course, be constructed, in which the consequents are disjunctive, and *both* are denied in the minor :—

If *A* is *B*, either *C* is *D*, or *E* is *F*.

C is not *D*, nor is *E*, *F*.

∴ *A* is not *B*.

According to our definition, which postulates a disjunctive minor premiss as essential to the argument, this is not a true dilemma. It should however be noticed that not a few logicians (e.g. Hamilton, Thomson, Ueber-

weg, etc.), hold it to be such, and adopt a different definition of the argument. Thus Mr. Joseph (*Introd. to Logic*, p. 331) defines the dilemma as "an argument offering 'alternatives, and proving something against an adver-'sary in either case." The point is not of first-class importance. We have preferred the view, which requires the disjunctive minor premiss, since the separate treatment of this argument seems to demand that its form should be one and the same in all its varieties.

§ 5. **Answering the Dilemma.** We have already called attention to the great value of the Dilemma to the rhetorician. It is, therefore, not surprising that attention should have been paid to the manner in which it should be answered. Three ways of dealing with it are enumerated :—

(1) *Taking it by the horns.* Here the person against whom it is brought, accepts the alternatives offered him, but shows that they do not involve the consequents attributed to them in the major. Where this is shown in respect of only one of the alternatives, he is said to take the dilemma by one horn. Thus, in the Complex Destructive given in the last section, the controversialist against whom such a dilemma was urged, would probably reply that he would willingly admit himself to be intelligent, but that it by no means followed he would recognize his arguments as invalid : that, on the contrary, he knew them to be valid.

(2) *Escaping between the horns.* This is the method adopted, when it is shown that the disjunction is not complete, and that another alternative remains. For example, suppose it to be pointed out to some man, that he must vote either for the conservative or the liberal candidate in some election, and that in either case he would materially injure his business prospects by giving offence to a section of his clients. He might reply that there was yet another possibility, for it might happen that he would be enjoying a well earned holiday abroad at the time of the election.

(3) *Rebutting the Dilemma.* The Dilemma is said to be rebutted, when reply is made by another Dilemma, in which the same alternatives are taken, but are shown to involve consequents fatal to the original argument. It is, for instance, not difficult to construct a rebutting argument to meet Empson's 'fork.'

If the accused lives at a small rate, his economy is evidence of his poverty : if he has maintained a large expenditure, that must needs have impoverished him.

But either he lives at a small rate, or has maintained a large expenditure.

∴ He is poor—and is incapable of paying much to the king.

To rebut the argument, the two consequents are transposed, and their quality changed. Where the original Dilemma gives us,

If A is B, E is F ; and if C is D, G is H.
But either A is B, or C is D.
∴ Either E is F, or G is H,

the new argument is of the form,

If A is B, G is not H : and if C is D, E is not F.
But either A is B, or C is D.
∴ Either G is not H, or E is not F.

One or two celebrated dilemmas of the ancients still retain their fame as *par excellence* the classical examples of this argument. Of these, the best known is the 'Litigiosus.' It is related that Protagoras the Sophist agreed to train one Euathlus in the art of rhetoric, the condition being that only half the fee should be paid at the time : the payment of the remainder was to depend on Euathlus's success in his first law-suit. Should he fail, the fee was to be forfeited. Euathlus delayed to undertake any suit, and eventually Protagoras himself summoned him before the court. He urged the following dilemma against him :—

If this case is decided in my favour, you must pay me by order of the court : if it is decided in your favour, you must pay me under the terms of our agreement.

But it must be decided either in my favour, or in your favour.

∴ You are in any case bound to pay me.

This argument was met by Euathlus as follows:—

If the case is decided in your favour, I am free by the terms of the agreement: if it is decided in my favour, I am free by order of the court.

But it must either be decided in your favour, or in my favour.

∴ I am in any case discharged of my debt.

How this knotty question should be solved, is still dubious. Perhaps, it should be said that this case was not contemplated by the terms of the original compact, and should have been dealt with independently of it. But various solutions to the puzzle have been suggested. The judges, we are told, left the case undecided.[1]

[1] For a list of the more famous of such dilemmas with references to the authorities where they are found, vide Hamilton, *Logic*, I. 466.

CHAPTER XIV.

INDUCTION.

§ 1. The Nature of Induction. In the chapters on the Categorical Syllogism, we dealt at some length with the subject of deductive reasoning. We saw how it is the process by which the mind passes from a general principle to a particular case, which falls under that principle. In order, therefore, to employ Deduction at all, the mind must have reached the knowledge of some universal truth. The question now arises, how do we come to the knowledge of these general truths. We must reply to this enquiry, by drawing a distinction between such as are analytic, and such as are synthetic in character. The source of our universal analytic judgments need cause us no difficulty. In these propositions the necessary, and therefore universal, connexion of subject and predicate, appears on the analysis of the notions themselves. Every object of which the subject can be affirmed, is necessarily such that of it the predicate too can be affirmed. Thus, for instance, whenever we can term a figure a regular hexagon, we can also affirm that each of its angles equals 120°. It is with the other class of general truths, with those namely that are synthetic, that Induction, the subject of our present chapter, is concerned. In these judgments the predicate is found to belong to the subject not by the analysis of the notions, but by experience. Induction is **the legitimate derivation of universal laws from individual cases.**

Here, it at once appears, there is a problem for solution. Our experience is limited to a comparatively

small number of individual cases. Yet in many instances, we hold ourselves justified in asserting a universal law of nature, which embraces not merely the cases we have observed, but all other members of the same class. We say that 'All diamonds are combustible,' 'All potassium floats in water,' though in each of these cases the number of instances observed is infinitesimal compared with the full number of which the assertion is made. It may even happen that the carefully tested observation of a single case is held sufficient to justify a universal conclusion of the kind to which we refer.

Every law of nature may be viewed as expressing a relation of cause and effect. Now, when is it that after the examination of a few instances in which a certain effect is found to occur, we hold ourselves justified in asserting a universal law attributing this effect to a certain cause? It is when we are certain that we are giving the name of cause to that which is in fact the *reason* of the effect, and not to some circumstance, which although in these instances it is found in connexion with the effect, is not itself the true reason why the effect occurs. When we have discovered the precise quality in virtue of which the effect is produced, we say with confidence that, given these circumstances, this quality will always have this same result. Thus to take a familiar example, a few experiments with hydrogen and oxygen satisfy us that these elements produce water, not in virtue of anything peculiar to the individual instances, but simply because of their character as hydrogen and oxygen: and we have no hesitation in holding that they will always produce water.

The logical process by which we pass to this universal conclusion is purely **abstractive.** Should a result a be produced, we endeavour to determine the particular characteristic A, by which it is effected: and then neglecting all that belongs to the instances simply as individuals, we affirm that A *as such* produces a. This is very different from saying that a concrete object, which among many other qualities has the quality A,

is the cause of *a*. For we abstract one quality, namely *A* from all others, and we assert that it is from this that *a* proceeds. But by the very fact that *A* is thus abstracted, it is no longer conceived as individual : it is the object of a universal concept. Our statement is not that this particular *A* is the cause of *a*, but that *the nature A* is such, that (given similar circumstances to those under review), it has *a* as its effect. Thus in the instance cited, we say that hydrogen and oxygen *as such* produce water. This process is not ratiocinative. We do not argue from premisses to conclusion. Our work is done when we have separated the essential from the non-essential, when we have discovered the true causal relation.[1]

The difficulty of Induction may therefore be said to be practical rather than logical. The passage of the mind from the data of sense to a universal principle, does not admit a variety of forms, as does the syllogism. But as the history of science may tell us, the discovery of universal laws is no easy matter. The complexity of the physical order is prodigious : the circumstances which affect the operation of natural causes, are so multifarious, and their accurate determination surrounded by so many difficulties, that the progress of human knowledge is slow, and its victories won at the cost of prolonged toil. We may illustrate the difficulties of determining a causal relation by an example. Let us suppose that a certain herb produces favourable results on patients suffering from a particular illness. Can it therefore be assumed that the plant possesses definite curative properties in regard to that illness ? May there not be something peculiar to the constitution of the patients in question, owing to which the result

[1] Cf. Ueberweg, § 129. "The formation of valid *inductions* is very closely 'related to the formation of *notions* according to their essential attributes. . . . 'For a great number of properties and relations stand in a causal nexus with 'the essential attributes of the object of which the notion is formed : on this 'causal nexus depends the validity of the induction. From this comes the 'logical right to refer properties to the whole species which have been observed 'in single individuals of a species, in so far as they are not evidently conditioned 'by mere individual relations."

is produced? Even granted that it can be shewn that in this region the results are invariably favourable, can we be sure that the relation between the medicine and the ailment would be the same in a different climate? Further, is the result due to the plant as such, or can we discover some special constituent in it, to which and to which alone the curative effect should be attributed? Even an example so simple as this, will suffice to show us how difficult is the problem before us, when we undertake to separate what is essential from what is non-essential in a physical process.

It is often stated that Induction depends altogether on the great principle of the **Uniformity of Nature,** to the consideration of which our next chapter will be devoted. This principle may be stated as follows:—

All relations of cause and effect are constant and uniform; or in other words, *The same cause will under the same circumstances always produce the same effect; and reciprocally, effects of the same kind are the results of the same cause.* The logicians who tell us that Induction depends on this principle, usually signify that it is actually employed in the inferential process by which we arrive at our universal conclusion. They assert that the Uniformity of Nature is *the major premiss of all Induction,* and that our conclusion is reached by a syllogism of the following type:—

Every relation of causality is constant.

A and a in the instances examined, are related as cause and effect.

$\therefore$ The relation between A and a as cause and effect is constant.[1]

It is easy to see that this explanation is erroneous. The minor premiss in this syllogism is itself the statement of a universal law. For if, when it is said that in the instances examined A is the cause of a, it be still left

[1] This formula is given by Rabier. He denies that it is strictly speaking syllogistic, since the minor term is particular in the premiss, and universal in the conclusion. This distinction, however, depends on the manner of formulation. *Logique,* p. 149. Cf. Mill, *Logic,* Bk. III. c. 3, § 1. Bain, *Induction,* c. 2, § 1.

uncertain whether a was really due to *A as such* or to some other circumstance peculiar to the individual instance, we have no ground for a universal conclusion. The meaning must therefore be that *A as such* is the cause of a. But when this is asserted, we have already left the individual instance far behind, and have by abstraction reached our universal law.

The assertion that Induction depends on the Uniformity of Nature may however be understood in another sense. It may merely signify that in that principle, we have the guarantee that our universal judgment will be verified in fact. Our judgment that *A as such* is the cause of a, would help us but little, unless we further knew that in the real order the same cause does actually always produce the same effect. The principle corresponds to the logical process. Just as in our act of abstraction we judge that the universal nature A is intrinsically connected with a, so the metaphysical principle assures us all A's must produce the same effect, whatever that may be.

Once again, it must not be thought that the principle of Uniformity is the canon of Induction in the sense in which the *Dictum de Omni* is the canon of deductive reasoning. It does not reply to the question, How does the mind pass to the conclusion that the nature A is necessarily connected with the effect a?; but to the question, Does the same real cause always produce the same effect? It belongs to a different order—to the order of things, and not to the order of thought.

§ 2. **Cause and Condition.** The last section has shewn us that every induction depends upon the accurate determination of cause and effect. It is not possible to lay down a universal law, until we have discovered the precise nature of the relation in the case with which we are dealing. Yet here a difficulty meets us. How can we, considering the complexity of Nature, say what is the cause of any phenomenon, and what is not? As I write at my table, can I really say what is the cause of

the letters I form on the paper? Am I the cause? May not the paper manufacturer, or the maker of pens, or the carrier who brings me my materials, all claim to be the cause? Could I write, if it were not for the laws of gravity? Must we, in fine, own that it is a hopeless task to fix on anything special, as the cause *par excellence* of a certain effect, or is it possible to distinguish between cause and condition?

How are we to define a Cause? Many and various are the definitions offered us. One that is given by Dr. Thomson (*Laws of Thought*, p. 218), seems to come very near what we really signify by that term. "We mean," he says, "by the cause of a thing, the sum of the facts 'to which it owes its being." A definition practically identical with this, was in fact frequently employed by the Scholastics. But since this formula is, as will appear, open to a certain ambiguity, it seems preferable to avail ourselves of another expression having the same significance, and define a cause as **that which makes a thing to be what it is.**[1]

Let us consider our definition as it is verified in the four causes enumerated in Ch. 10, § 2. Once again the example of the statue will serve our purpose. Most assuredly, we may say that (1) the final cause—the purpose for which the statue was carved—has made it *what it is*. Its special characteristics are determined in reference to this. The statuary will make it in a different manner, if he intends it to fill a niche over an altar, and if he intends it to stand on a column fifty feet high. Of (2) the efficient cause—the statuary himself—it is hardly necessary to speak. Every detail of the carving is his work. His brain conceived it; and his hand executed it. He too has made it what it is. Similarly, in regard to (3) and (4), the intrinsic causes. If made of a different material, it would be something

[1] The cause is defined as '*Id quod influit esse in rem*,' and as '*Id quo res est id quod est*.' The two definitions may be rightly regarded as equivalent; for that which makes a thing to *be*, is the same as that which makes it *what it is*. No agent can be the cause of the existence, which is not also the cause of the nature.

different from what we now see : while its dependence on the formal cause is yet more intimate. Of the form, even more than of the matter, may it be said that it makes the statue what it is.

It is, of course, the case that in various ways, the statue may be said to owe its being to other agencies. A ship brought the marble from Italy ; an English workman did the ' rough-nobbling.' Yet a very little consideration will show us that there is a great difference between the way in which the object owes its being to the ship or to the skilled workman, and to the four causes we have specified. Some other mode of conveyance might have been substituted for the ship, without causing the least difference in the characteristics of the statue. Granted that similar marble can be obtained in England, the statue would be qualitatively identical, whether the block came thence or from Italy. In other words, in the case of the ship we have to do with an agency which does not make the object *what it is*, which exercises no determining effect on its special characteristics. But to do this, is the distinctive mark of a cause : there is not a single characteristic of the statue which has not been received from one of its causes, and they are its causes only in so far as they communicate these characteristics to it. The ship is not a cause, but **a condition.** A condition is *that which in one way or another enables the causes to act in the production of the effect, but which does not make the thing what it is.*

There is one special kind of condition, which merits separate mention since the name of cause is often applied to it. If, for instance, I strike a match, it is quite true that the friction by which I light the flame, is a condition rather than a cause. The same result might have been produced in many ways—a sure sign that the effect does not owe its characteristics to those of the action in question. Heat, percussion, the application of certain chemical agents would have had the same result. My action merely put the forces, that were present in the match, in the state required for the exercise of the causa-

tive powers they possessed. Yet undoubtedly this act has a special character, since it communicated the impulse, which set free these forces. Other conditions may be requisite for one reason or another; but they do not, as does this one, set in motion the whole process. They are in no sense the *origin* of the effect. In virtue of this originative character, a condition, such as this, is not without reason known as the **determining cause** (*causa determinans*).[1]

§ 3. **The Aim of Inductive Enquiry.** It may be asked, which of the various causes we have enumerated, is the object of inductive investigation. To this it must be replied, that it is not in all cases the same. Sometimes the knowledge of one cause, sometimes that of another, is important to us; and very frequently it happens, that Nature has only put the knowledge of one cause within our reach. As Professor Case has well put it, "Man has to fasten on Nature as best he can. He 'knows more about present facts than about past causes: 'and when he knows causes, it is sometimes the efficient 'cause, as in the mechanics of the forces by which bodies 'act on one another: sometimes the material cause, 'as when the chemist knows the elements out of which 'a body is composed, without having discovered the 'force of chemical affinity so-called, by which they 'compose it: sometimes again the final cause, as in 'morals, where we know for what end we act, without 'fully understanding the voluntary, nervous, and mus-'cular forces by which we perform our actions." [2]

Often indeed, what we wish to discover is the determining cause. For instance, it had long been known that in unisexual plants, fertilization takes place by the communication of pollen from one plant to another. But the method of communication remained a problem for science to resolve, till the determining cause was discovered in the insects that pass from one plant to

[1] De Régnon, *Métaphysique des Causes*, pp. 129, 207, 596.
[2] *Lectures on the Method of Science* (ed. by T. B. Strong), p. 5.

another. When our search is motived by a practical end, the knowledge of the determining cause may be of far greater moment to us than that of the causes properly so-called : for what we desire to know is not so much the inner nature of the phenomenon, as how to bring it about, or how to prevent it. Thus, for instance, if we desire to prevent the spread of some epidemic, to increase the productivity of soil, etc., etc., in all such cases, it is the determining cause, rather than any other, which is the object of the scientist's search.

Colloquially, the word cause is used in a loose sense. It may be applied to any one of the antecedent conditions, in the absence of which the event would not have taken place, and on which for some reason it may be desirable to lay stress. We say that the construction of a railway in the neighbourhood, is the cause of the increase in the value of land. But the mere construction of the railway would not in itself increase the value, unless the railway were such as to bring the neighbourhood into connexion with some populous centre. Inductive enquiry is not concerned with causes in this popular sense : and we must be on our guard against confusing the word as thus colloquially employed, with its more accurate signification. It is the failure to draw this distinction, which alone has given plausibility to the erroneous assertion contained in not a few works on Logic, that there is no difference, scientifically speaking, between cause and condition.

§ 4. **Recognition of the Causal Relation.** The preceding sections have, it is hoped, given the reader a clear notion as to the object of inductive investigation. We must now examine the way, in which the mind recognizes a relation between a cause and its effect.

There are certain cases, in which this recognition is easy, and after considering a few concrete cases of the causality in question, we are able without hesitation to abstract the relation ; and thus from the particular instances, induce a general proposition. We say ' a

few cases,' for ordinarily a single instance leaves us in doubt, whether there may not be some error in our observation. Were we convinced that we had avoided this danger, one instance would suffice us.[1] It is an induction of this kind, if, for instance, I see the impression made by my foot on the sand, and judge that as often as a human foot is pressed on this yielding material it will make a mark like this : and conversely, that such a mark must be the effect of a human foot. I do not pass to my conclusion by a syllogism. I see my foot make and leave its impression : and my mind recognizes the relation between the shape of the foot as cause and the shape of the impression as effect. In the knowledge of this causal relation it has the ground for a universal conclusion. Robinson Crusoe, like other men, had formed this induction. So when he saw the footprints on the shore, he knew for certain that there was another man on the island.

We are often able to understand the nature of the causal relation—the reason why such an effect should follow from such a cause, even when the action is of a less simple character than in the case just described. The cause of malarial fever long escaped observation because of its minuteness. But as soon as its origin was discovered in a special microbe, and the activity of this microbe in the body had been studied, the nature of the relation which united the malady and its cause was evident. When it was known what havoc this minute creature worked among the blood-corpuscles, it immediately became evident how the illness followed from its cause. Here too, the mind abstracts the relation, and induces a universal law that wherever this cause operates, this effect must follow.

[1] " Great scientists rarely mistake the worth of a significant fact, though ' only occurring once. It is said that Sir C. Bell would not repeat the famous ' experiment that established the difference between the motor and sensory ' nerves, so much did his feelings recoil from causing animals to suffer. . . . ' The Abbé Haüy lets fall a piece of quartz, and merely by observing the ' fracture, he concludes that he has discovered a law of nature. . . . So in ' a thousand cases. The knot then is not in the repetition itself, but in the ' fact of the coincidence : only the repetition evidently adds much to the ' value of the coincidence." Janet, *Final Causes* (English trans.), p. 431.

Yet there are a vast number of cases, in which we can see no reason why a particular effect should follow rather than some other, and in which therefore the causal relation cannot be recognized, as it is recognized in these examples. This is especially noticeable in regard to physical constituents, considered as the cause of the resulting compound. There is no resemblance between hydrogen and oxygen on the one side, and water on the other. The properties of the effect bear no likeness to the properties of the cause. Again, we cannot hope to discover any reason to account for the properties possessed by the different natural substances. Why, for instance, should galena crystals be cubical while quartz crystallizes in hexagonal prisms? Why should gold resist the action of solvents, which are efficacious against other metals? Why should sulphur, when heated, first liquefy, then thicken, and then liquefy again? In all these cases, our experience assures us as to the *fact* of the relation. But why such a cause should produce such a result, remains altogether concealed from us. We must be content to know the bare fact that such and such things are causes of such and such effects. We analyse water, and hydrogen and oxygen are found: we synthesize hydrogen and oxygen, and the result is water.

In saying that our knowledge in these cases relates not to the reason of the causal connexion but to the fact, we must not be thought to mean the singular fact as perceived by sense. The fact here is recognized by the intellect. In the singulars which sense perceives, the intellect recognizes and abstracts the universal relation.

In thus determining experimentally a causal connexion, it is usually the case, even more than where the nature of the causal relation is recognized, that a number of experiments is necessary, before the law is looked on as established. But here too the repetition is not itself part of the logical process. It is preliminary to it. It is merely a guarantee that we have not been deceived in the nature of the experiment. If for instance we

suppose the experiment to be the synthesis of hydrogen and oxygen, with a view to establishing for the first time the universal truth that these are the causes of water, we should repeat the experiment under a variety of circumstances, in order to satisfy ourselves that no other ingredient but hydrogen and oxygen was present in our synthesis, that the result was not due to something peculiar to the individual case. When once we are satisfied that the fact is as we conceive it, then we proceed to our induction. We induce from the securely established fact that in such and such concrete cases water has been produced from these elements, a universal conclusion. We assert that hydrogen and oxygen are causally related to water ; and the assertion is made, not of the individual particles of hydrogen and oxygen with which we experimented, but of hydrogen and oxygen as such,—of these elements viewed in abstraction from individualizing conditions.

The value of the instances to produce conviction will depend almost entirely on the intellect which considers them. Evidence which to the untrained mind has no meaning, will reveal with certainty a universal law to one who is already versed in the subject under consideration. An investigator of this kind sees the evidence in the light of a hundred facts previously known. He may recognize that the suggestion contained in the evidence, is precisely what these facts would lead him to expect, or he may see that either it or they must be false. The manifestation of the universal law by the particular case, cannot be made to depend altogether on the nature of the evidence. It depends also on the mind to which it is presented. What is sufficient for the mind well stored with knowledge is insufficient for the tyro.

It will follow from what has been said that it is impossible for us to give a canon or a series of canons, which shall be to Induction what the *Dictum de omni* is to Deduction. We can do no more than describe in general outline the principal ways in which evidence of a causal

connexion presents itself, and thus to indicate what may be termed **methods of inductive enquiry.** This subject however does not belong to Logic strictly so called, but to the Method of Science : for the logical ground of the induction is not altered by the different manner in which the evidence appears. We shall therefore defer its consideration to the second part of this work.

To sum up, Induction is not a syllogistic, but a purely abstractive process. It becomes possible as soon as we see in the individual instance (or instances) of the cause in question, that the effect is not due to some accidental circumstance, but that it belongs essentially to the type. And this is known either (1) by rational insight into the nature of the connexion between cause and effect, or (2) by rational recognition of the fact of the causal connexion.

* The induction of the universal from the particular in the manner we have described, is succinctly taught by Aristotle, *An. Post. I.*, c. 31.

"It is owing to our inability to apply our senses, that certain 'questions remain unsolved. For could we but see what takes 'place, we should not still be seeking a solution. Not indeed 'that the mere act of sight would give us scientific knowledge ; 'but sight would be the means through which we should attain 'the universal. Thus if we saw the perforations in the glass, 'and the light-particles coming through, the cause of its illuminat-'ive power would be manifest. Sense would perceive the in-'dividual instance, and the intellect would recognize that this 'was a universal law."

In this passage Aristotle should not be understood as approving the theory of light which he mentions. He uses, as is his wont, a contemporary opinion as an illustration.

A similar passage occurs *An. Post. II.*, c. 1, § 4.

In the well-known chapter *An. Post. II.*, c. 19 he treats the case in which we draw our universal conclusion from the observation of a number of particulars. The memory of many individual instances constitutes, he tells us, one Experience (ἐμπειρία): this Experience results in the possession of an abstract universal principle in the mind. Since the passage itself is so compressed as to stand in need of some exposition, we prefer to give in its place the lucid commentary of St. Thomas :—

"Ex memoria multoties facta circa eandem rem in diversis tamen singularibus, fit experimentum : quia experimentum nihil

aliud videtur, quam accipere aliquid ex multis in memoria retentis. Sed tamen experimentum indiget aliqua ratiocinatione circa particularia, per quam confertur unum ad aliud, quod est proprium rationis. Puta cum talis recordatur quod talis herba sanavit multos a febre, dicitur esse experimentum quod talis herba sit sanativa febris. Ratio autem non sistit in experimento particularium : sed ex multis particularibus in quibus expertus est, accipit unum commune quod firmatur in anima, et considerat illud absque consideratione alicujus singularium, et hoc accipit ut principium artis et scientiae. Puta diu medicus consideravit hanc herbam sanasse Socratem febrientem, et Platonem et multos alios singulares homines : cum autem sua consideratio ad hoc ascendit, quod talis species herbae sanat febrientem simpliciter, hoc accipitur ut quaedam regula artis medicinae. . . . Posset autem aliquis credere quod solus sensus vel memoria singularium sufficiat ad causandum intelligibilem cognitionem principiorum, sicut posuerunt quidam antiqui non discernentes inter sensum et intellectum : et ideo ad hoc excludendum Philosophus subdit, quod cum sensu oportet praesupponere talem naturam animae quae posset pati hoc, id est sit susceptiva cognitionis universalis " (*An. Post. II.*, lect. 20).

The reader will observe that in these passages no use is made of the term Induction. The word *experimentum* (ἐμπειρία) takes its place.

§ 5. **The Inductive Syllogism.** The Inductive Syllogism is a special mode of arguing from the particular to the general, different from Induction properly so-called. The principle of this method of inference is that **an attribute which is affirmed (or denied) of all the logical parts, may be affirmed (or denied) of the logical whole.** The validity of the argument depends on our certainty that we have overlooked no one of the parts. Hence this mode of reasoning is only available for the cases in which we are dealing with the several species of a genus. The individuals belonging to a species cannot be counted : their number is indefinite.[1] But the species are always limited in number.

The syllogism may be represented by the following formula :—

[1] Aristotle held the number of individuals in a species to be infinite. Hence when he tells us that the enumeration of the parts must be complete, he evidently supposes that we are dealing with species, not with individuals.

S_1, S_2, S_3 are P.
S_1, S_2, S_3 alone are M
$\therefore$ All M is P.

The conclusion here is represented not as in the formula of the deductive syllogism 'All S is P,' but as 'All M is P.' For the purpose of this syllogism is to establish the general proposition, which in the deductive syllogism stands in the major premiss. Hence Aristotle calls it an argument *by which we prove the major term to be true of the middle term, by means of the minor term* (*An. Prior. II.*, c. 23). This method of proof is not infrequently employed by Euclid, when he establishes his proposition for the separate cases, in which the supposition he makes can be realized. The twentieth proposition of his third book is of this character: its conclusion may be expressed as an Inductive syllogism:—

The angle at the circumference of a circle is half the size of the angle at the centre on the same arc, (1) when the centre of the circle is within the angle at the circumference, (2) when it is on one of its sides, (3) when it is outside of it.

But these three cases are the only cases in which it is possible to have an angle at the centre and an angle at the circumference on the same arc.

$\therefore$ The angle at the circumference is always half the size of the angle at the centre on the same arc.[1]

An Inductive Syllogism, if we do no more than shew that P is found in each of the logical parts severally, does not give us a universal proposition in the full and complete sense of the word. It does not show us that there is a law connecting M with P: for the reason why P belongs to the several parts of M may be different in the different cases. It is an *Enumerative judgment*, though one valid of an indefinite number of individuals.[2]

[1] Cf. Ueberweg, § 128.

[2] Aristotle points out its enumerative character in *An. Post. I.*, c. 5. He supposes the case of a man who has proved of the equilateral, the isosceles, and the scalene separately, that the interior angles of each are equal to two right angles. This man, he says, cannot be really said to know that *all* triangles, as such, have this attribute. "οὐ γὰρ ᾗ τρίγωνον οἶδεν, οὐδὲ πᾶν

In order that it should be more than this, it would be necessary to shew that the reason for the inherence of *P* is the same in regard to all the parts of *M*.[1]

The example of this Syllogism, given by Aristotle has often, owing to the brevity of its expression, caused some perplexity. It is as follows :—

Man, horse, ass . . . are long lived.
Man, horse, ass . . . are animals without bile.
∴ All animals without bile are long lived.[2]

The example at first sight presents two considerable difficulties. In the first place the minor premiss as it stands, is manifestly untrue. In the second place, a complete enumeration of animals without bile, would appear impossible. As a matter of fact, by the expression 'without bile,' Aristotle doubtless signifies that the animals in question have no excess of *choleric humours*—a different thing from being totally without bile.[3] Further the assertion should be understood, not of animals in general, but of the more highly organized animals : for in *An. Post. II.*, c. 17, § 7 he restricts it to quadrupeds. Thus a complete enumeration would not present insuperable difficulties.

In another passage Aristotle gives the following, as exemplifying an inductive argument : " If we know that the skilful steers-' man is best, and the skilful driver, and so on, we may conclude ' that the man who is skilful is best at every occupation." [4] But, it would be misleading to employ this here as an illustration. For it is given as an example of dialectical argument, in which demonstrative certitude being unattainable, we must be content with probabilities. It is by no means intended as a typical instance of the argument in its perfect form.

τρίγωνον, ἀλλ' ἢ κατ' ἀριθμὸν · κατ' εἶδος δ' οὐ πᾶν." l.c. § 5. (" For he does ' not know the attribute to belong to the triangle, as such : nor does he know ' it to belong to *all* triangles, except in so far as ' all ' is understood of mere ' numerical reckoning. He does not know it to be a property of the species.") He describes this proof (§ 1) as that which occurs " ὅταν τυγχάνῃ ὂν ὡς ἐν μέρει ὅλον ἐφ' ᾧ δείκνυται." (" When it happens that our proof concerns a whole, not as a whole, but in its logical parts.")

[1] Euclid is generally able to do this. The proposition may require different constructions : but the same proof *mutatis mutandis* is ordinarily employed.

[2] *An. Prior II.*, c. 23, § 2 ; cf. *An. Post. II.*, c. 17, § 7 ; *De Part. Anim. IV.*, c. 2.

[3] The reader will not need to be reminded of the importance attached in ancient physiology, to the four humours, on which the temperament of the individual—sanguine, choleric, melancholic, or phlegmatic—was held to depend. Of this doctrine Wundt says, " Psychology borrowed the doctrine of ' the four temperaments from the medical system of Galen. And though the ' theory of the humours, which was its physiological basis is now obsolete, ' yet the distinction of the temperaments seems to have been derived from ' acute psychological observation." *Psychologie*, III., 637.

[4] *Topics*, I., c. 12.

The Inductive Syllogism resembles a syllogism in Fig. 3 with a universal instead of a particular conclusion. This is legitimate, since the enumeration is known to be complete. The minor is sometimes expressed in the form 'S_1, S_2, S_3 are all M.' This form is however inadmissible. It suggests that the significance of the copula here is different from its ordinary import: that it means 'S_1, S_2, S_3 constitute M.' [1] If this were the meaning, the conclusion would be illegitimate. M would be used collectively in the minor premiss, and distributively in the conclusion.

§ 6. **Perfect and Imperfect Induction.** The term Perfect Induction is employed to signify the process by which we affirm a given attribute of all the individuals belonging to a class, and then proceed to affirm it of the class as a whole. Thus I may assert of each of the apostles taken separately, that he was a Jew, and so pass to the general proposition, 'All the apostles were Jews.' The process may, of course, be drawn up in the form of an Inductive Syllogism. But it differs widely from it. The enumeration there is of logical parts. It is an intellectual process: for it is not by sense-perception that we sum up the equilateral, the isosceles and the scalene as completing the list of possible triangles. But in Perfect Induction the enumeration is purely sensible. It is a simple counting of heads. Further, the conclusions obtained are of different value the one from the other. A general proposition obtained by Perfect Induction is a mere summary; and though inferences may be drawn from it, yet this general proposition presupposes the knowledge of every so-called conclusion. On the other hand the conclusion of an Inductive Syllogism is true of an indefinite number of individuals, and affords us a solid basis for inference to new facts.

When we draw a universal conclusion after enumerating some members only of a class, we are said to employ

[1] Mansel, *Aldrich*, App. G.

Imperfect Induction. A conclusion drawn in this way is of such a precarious character as to be valueless, unless we have grounds for supposing some causal connexion.[1] But if we have reason to believe the attribute to be causally connected with the subject, we are relying on something else than mere enumeration. It may illustrate how liable to frustration are conclusions drawn in this manner, if we remember how at one time, experience seemed to indicate that all bodies expand under the influence of heat, and contract when exposed to cold. Numbers of cases had been examined in which the rule was verified, though no reason could be assigned why it should be so. As we are well aware, exceptions are found to this rule. Water expands instead of contracting, when it falls below 39·4 F.; and similarly, india-rubber, clay and certain other substances, contract under the influence of heat. Had a universal conclusion been drawn in this case by Imperfect Induction, it would have proved false.

* It is commonly stated in English text-books of Logic, that the Scholastic philosophers knew of no inductive process save Perfect and Imperfect Induction, and that they believed that our certainty as to the laws of nature, was based on mere enumeration. We have already had occasion to point out how cautiously we should receive what our popular logicians say of Scholastic philosophy. It may indeed be owned that the subject of Induction received far less attention from the mediæval writers than it merits. Yet to say that they believed our knowledge as to the laws of nature, to rest on a process of Imperfect Induction by mere enumeration, argues a remarkable want of acquaintance with their writings. There is indeed a considerable variety of opinion among the Scholastics on this subject. But

[1] This point is forcibly put by Leibniz. "Nam si universalia nihil aliud sunt quam singularium collectiones, sequetur scientiam nullam haberi per demonstrationem (quod et infra colligit Nizolius) sed collectionem singularium seu inductionem. . . . Certitudo perfecta ab inductione sperari plane non potest, additis quibuscunque adminiculis, et propositionem hanc *Totum magis esse sua parte* sola inductione nunquam perfecte scimus. Mox enim prodibit, qui negabit ob peculiarem quandam rationem in aliis nondum tentatis veram esse, quemadmodum ex facto scimus Gregorium a S. Vincentio negasse totum esse majus sua parte, in angulis saltem contactus: alios in infinito: et Thomam Hobbes (at quem virum?) coepisse dubitare de propositione illa Geometrica a Pythagora demonstrata, et hecatombae sacrificio digna habita: quod ego non sine stupore legi." Leibniz, *De Stilo Philosophico*

the more eminent amongst them base our certainty in regard to natural laws on the principle, that when the operation of some natural agent produces regularly and habitually some particular result, this result is not due to an accidental circumstance, but is an effect having for its cause the specific nature of the agent. In other words, they held the enumeration of instances to be of value, inasmuch as it enables the investigator to judge that the phenomenon α is really connected with the nature A, and not merely with some circumstance incident to this individual instance of A. The instances thus understood, do not constitute the premisses of a universal conclusion. They are the condition of our act of abstraction, '*A as such* is followed by α.' Anything more remote than this from the doctrine which is usually asserted to have been held by the whole body of Scholastic philosophers, can hardly be imagined.

The error seems to have arisen from the fact that the most famous of the Scholastics (St. Thomas, Albert the Great, Scotus) do not employ the term Induction as the distinctive name of the inference by which we establish universal laws of nature. Following the terminology of Aristotle, noticed at the end of § 4, they call it the proof from experience (*ἐμπειρία, experimentum, experientia*). The significance of the term Induction was somewhat vague. It covered all argument from the particular to the general. Hence (as e.g. in Scotus, *An. Prior II.*, Q. 8) it might include this meaning among others. But it was more usually employed to denote the formal process of Perfect Induction arranged as an Inductive Syllogism. Moreover it was sometimes pointed out, that our argument might be thrown into the form of an Inductive Syllogism: for though the enumeration was incomplete, yet in these few instances we have equivalently seen all. It was by a later generation that the term Induction was restricted to its present signification. Incautious readers, finding in certain passages the Inductive Syllogism described as the formula of inductive argument, jumped too hastily to the conclusion that the mediæval philosophers rested their knowledge of the laws of nature on no basis but enumeration.

We have already cited St. Thomas on this subject, in § 4. The reader will see that his words imply the view we have here taken. The following brief extracts will show that the position of Albertus Magnus and of Scotus was the same as that of St. Thomas. "[Propositiones] experimentales autem sunt quas accipimus intellectu orto ex sensu, sicut scimus . . . quod scamonea purgat choleram et quod vinum inebriat: sensus enim apprehendit inebriationem post potationem vini saepius

Nizolii, Op. phil. (Erdmann), p. 70. Leibniz's own view would seem to have been that criticized in § 1 above, according to which the uniformity of Nature is a major premiss in a syllogistic reasoning.

factam, et percipit intellectus quod hoc vini virtute accidit : et si esset casuale non contingeret saepissimé : et sic in intellectu generatur illius rei scientia firma, de qua non est dubium," Alb. Mag. *An. Post. I.*, Tract 1, c. 2. "Quamvis sentire non est scire, nec universale est sensibile, sicut in ante habitis dictum est, sed sensibile per abstractionem fit universale, quod in nobis est principium scientiae." *An. Post. II.*, Tract 1, c. 3.

Scotus writes as follows, "De cognitis per experientiam dico quod licet experientia non habeatur de omnibus singularibus, sed de pluribus, nec quod semper sed quod pluries, tamen expertus infallibiter novit quod ita est, et quod semper, et in omnibus : et hoc per istam propositionem quiescentem in anima : *Quidquid evenit ut in pluribus ab aliqua causa non libera, est effectus naturalis illius causae.*" *I. Sent. Dist.* 3, q. 4, n. 9.

Cf. also De Régnon, *Métaphysique des Causes*, pp. 40-54.

CHAPTER XV.

THE UNIFORMITY OF NATURE.

§ 1. **The Uniformity of Nature.** In this chapter we consider the principle of the Uniformity of Nature, or as it is sometimes termed, the Uniformity of Causation. That principle, as we saw in the preceding chapter, assures us that the same cause will, under the same circumstances, always produce the same effect: and that effects of the same kind are to be referred to the same cause. It was further pointed out that this principle is not a logical principle. It belongs to the real order: it tells us about the nature of things, not about the way in which we think and reason. Yet Logic can scarcely be adequately treated without discussing it. It is insufficient to show how the general proposition '*A* as such, is the cause of *a*,' is abstracted from the particulars of experience, unless we also show that in the order of reality, Nature prescribes that given *A*, *a* shall follow.

The principle as we have enunciated it, must be understood with a qualification. It refers exclusively to the material universe governed by natural laws. Hence it can claim no higher degree of necessity than belongs to the constitution of this natural order. The Power which established the universe can suspend the operation of its laws, and bring it about that a given cause shall produce an abnormal effect, or no effect at all. This is expressed in Scholastic philosophy by saying that the principle of Uniformity is *physically* necessary, not metaphysically.

We propose in the following paragraphs to explain (1) the reasons which justify us in holding that the uniformity of Nature is physically necessary; and (2) the

limits of physical necessity, and the grounds it affords for scientific certainty.

(1) It will be remembered that when we considered the distinction between cause and condition, we saw that all the properties of the effect come to it from one or other of its causes. The Hermes of Praxiteles could never have possessed its beauty of form, had not Praxiteles conceived that form in his imagination : it would never have possessed the durability which has preserved it to our own days, had its material cause been clay and not marble. The connexion between cause and effect is not a mere time sequence of antecedent and consequent. We can find in the cause the reason of every characteristic that the effect possesses : and it is by the action of the cause that they are all communicated to the effect.

It is further manifest that every agent acts according to its nature, and can act in no other way. When we see the young cuckoo appear in the nest of the starling, we never dream that it is the starling's offspring. We do not gather grapes from thorns : and we should regard a man as eccentric, to say the least, who told us that he saw no reason why such things should not occur. *Bos locutus est in foro*, says Livy : but even he regarded it as a portent. The active powers are proportioned to the nature of the agent : they are in fact the connatural expression of that nature. Indeed we only know the permanent nature through the manifestation of its active powers. The nature or essence—for nature is but another term for essence—is the principle which determines the character of these powers. Moreover unless an agent possesses free-will, it must act in a manner not merely in accordance with its nature, but absolutely prescribed by that nature. It cannot give what it has not : it must give what it has. This does not mean that it will act in the same way in every variety of circumstances in which it may be placed. The action involves a relation to the object acted on, and hence depends not merely on the agent but on the patient. But it does involve that where the relation

of the agent to its surroundings is the same, there (unless it be miraculously impeded) its action will be the same. Thus the very concept of a natural agent devoid of free-will, involves that under the same circumstances, its action will be of the same kind : and precisely similar considerations show that similar effects must be referred to causal agencies of the same kind. In other words the truth of the principle is evident from its terms.[1]

(2) In order to deal with the question of the limits of physical necessity, we must touch on a matter which belongs properly to Natural Theology, viz. : the relation of the First Cause to the created universe. The very notion of a First Cause involves as its consequence that He is not merely the cause of all things as regards their origination, but that He continually sustains them in being. Their persistence in being is, as it were, a continued creation. Were the conserving action of the First Cause withheld they would fall back into nothingness. This, moreover, holds good not merely in regard to the existence of creatures, but to all exercise of active power on their part. In acting, a creature obtains a further realization of itself—an extension of its being. Hence as the First Cause is the ground of all reality, He must needs be the fountain of all activity. No exercise of causality can take place, save in so far as He communicates the power to act. He must concur in every action. Without this concurrence, neither spiritual nor material agents could perform any of those actions which are connatural to them. Apart from this concurrence the mind could not think nor could one body act upon another. Again, the First Cause may confer on one of these agents some new active power enabling it to produce higher results than its natural endowments could effect. From this it follows that though we may say truly that the same cause must in like circumstances always pro-

[1] Cf. St. Thomas, *Summa Theol.*, Q. 41, Art. 2. "Effectus assimilatur formae agentis per quam agit. Manifestum est autem quod unius rei non est nisi una forma naturalis per quam habet esse. Unde quale ipsum est, tale facit."

duce the same effect, the necessity of this principle is hypothetical necessity. It supposes the First Cause to preserve the ordinary operation of natural laws.

It has occasionally been asserted that to admit the possibility of any interference in regard to the laws of Nature, is to render the principle of Uniformity nugatory. The objection might have weight if we represented the Deity as interfering capriciously with these laws. So far however is this from being the case, that the regularity of Nature's order is recognized in Scholastic treatises on Natural Theology as a mark of Supreme Wisdom. This very regularity affords man a guide without which he could not direct his life with a view to the future. It is one thing to own that the Deity has power to suspend His own laws, and that occasionally He does in fact suspend them, when the striking manifestation of His power may be for man's good. It is a very different thing to hold that a capricious suspension of law is at any moment likely to occur. In the long run, it is better that the stern regularity of law should train men to rule Nature by obeying her, than that they should constantly be able to obtain exceptions in their favour.

We have asserted that there is a higher kind of necessity than belongs to physical law. This is termed metaphysical necessity. It is not like physical necessity, hypothetical, but absolute. In it there is no scope for miraculous intervention. Axioms such as the principle of Causality that *Whatever comes into being must have a cause*, and the principle of Contradiction that, *It is impossible for a thing both to be and not to be at the same time*, together with all the truths of Mathematics possess this higher degree of necessity. What, it may be asked, justifies the distinction? The answer must be that these principles are metaphysically necessary, because they are altogether independent of any *physical process*. In some cases we see that certain concepts statically considered, stand in a relation of identity (or difference) under pain of a contradiction in terms. To assert that the same thing can at the same time both be and not be, is to assert what is

self-contradictory, in other words, meaningless. In other cases a causal relation is involved in the very nature of the abstract concept, apart from any dynamic efficiency. This is exemplified in the case of geometrical figures and their resultant properties. A figure which has three sides must of necessity have three angles. Here no physical activity is in question, and consequently the cause produces its effect by an absolute, and not a merely hypothetical necessity. Where no physical process is involved, to suppose a cause without its connatural effect, is to suppose a contradiction: and even divine power cannot produce what is self-contradictory.

Since the principle of uniformity is evident from the mere consideration of the concepts involved, it might seem that we possess in it an analytical proposition, yet one which differs from other analytical judgments in being liable to frustration. Undoubtedly an analytical proposition can be framed regarding the uniformity of Nature. But this proposition must express the condition which we have just noticed, and it will take the form: 'A natural agent devoid of free-will, granted the normal concurrence of the First Cause, must in similar circumstances always produce the same effect.' The principle thus enunciated possesses the absolute necessity belonging to all analytical judgments. The necessity of the laws of nature themselves is not absolute but hypothetical.

It may perhaps be urged that if the truth of the principle were thus readily apparent, it would be more universally recognized as necessarily valid. Yet savages look for capricious action on the part of natural phenomena, and children recognize the truth of the multiplication table more easily than that of this principle. To this it may be replied that even where analytical principles are concerned, not all are immediately self-evident. Every conclusion in pure mathematics is analytical: yet many of these are unintelligible to the untrained mind. To realize the necessary character of the principle in question, we must see what is involved in the concept of *a natural agent devoid of free-will.* The untutored mind, anthropo-

morphic in all its representations, is apt to attribute free-will not merely to animals but even to the powers of Nature. In so far as children and the uneducated do as a matter of fact expect the same cause to act in the same way, this may be attributed partly to the influence of experience, and partly to the growing capacity to distinguish between the free and the determined agent.

We have throughout treated both parts of the principle, alike that which asserts that similar causes produce similar effects, and that which asserts that similar effects are to be referred to similar causes, as equally valid. Philosophically this is so. But it must be borne in mind that not uncommonly the object of scientific search is a determining cause, and not a cause properly so called. Moreover agents altogether diverse may possess some similar characteristic in virtue of which they produce similar effects. Here strictly speaking the same cause is operative in all. But in common parlance, we speak of the effect as due to different causes.

A few lines should perhaps be added as to the exception made in regard to the will of man. On what grounds, it may be asked, is the exception made? If all action depends on the nature of the agent, must not human action do the same? Each man has his own character partly inherited, partly the result of his past life. Will not the character of his act depend on the circumstances in which he is placed? If we make an exception here, are we not arguing altogether arbitrarily?

It would, of course, be out of place to enter on the subject of free-will in a logical text-book. It must suffice to point out that our argument is only valid for such agents as, *ex hypothesi*, are not free. To extend it to the human will without first discovering whether that will be free or not, would be arbitrary. It would be to prejudge the whole case. We cannot argue from the mode in which natural agents act to the mode in which the will acts. For the human soul is unlike them. It is spiritual, and the mode of its action must be discovered by an appeal to the testimony of consciousness, and not by a mere analogy drawn from natural agents.[1]

[1] On Free-Will see Maher's *Psychology*, c. xix., pp. 394–424.

§ 2. **J. S. Mill on the Uniformity of Nature.** Mill, of all logicians, is the one who has devoted most attention to the enquiry into the process, by which the mind comes to know laws of nature. He recognizes that apart from a belief in the uniformity of causation, such knowledge is impossible, and makes this principle the foundation of all inductive inference. Thus, he tells us (*Logic*, Bk. III., c. 3, § 1) "We must first observe that there is 'a principle implied in the very statement of what Induc-'tion is . . . namely that what happens once, will under 'a sufficient degree of similarity of circumstances happen 'again"; and he says (*ibid.*) "Every induction may be 'thrown into the form of a syllogism, by supplying a 'major premiss. If this be actually done, the principle 'which we are now considering, that of the uniformity 'of the course of nature, will appear as the ultimate 'major premiss of all inductions."

In the present section, we shall deal shortly both with his explanation of the principle, and with his account of the manner in which we arrive at a knowledge of it. His theory is, of course, based on the Empiricist philosophy, of which he was a leading exponent.

He terms the principle the **Law of Causation,** and tells us that it is "the truth, that every fact which has a 'beginning, has a cause" (c. 5, § 1). But in the following section he enunciates it more fully, thus: "The law of 'Causation . . . is but the familiar truth, that invari-'ability of succession is found by observation to obtain 'between every fact in nature, and some other fact 'which has preceded it . . . the invariable antecedent 'is termed the cause: the invariable consequent the 'effect" (*ibid.* § 2).

The explanation which Mill gives to the principle rests entirely on the meaning which he attaches to the terms 'cause' and 'effect.'

The causal relation, as conceived by him, is totally different from the account we have given of that relation in the last chapter. According to his view, the whole question is one of succession in time,—of before and

after. The cause does not exert any influence on the effect. He expressly desires that the causes treated of by him, should not be viewed as possessed of efficiency : " the causes with which I concern myself, are not *efficient* ' but *physical* causes " (*ibid.* § 2). He finds fault with the Greek philosophers, because " they wished to see ' the *reason* why the physical antecedent should produce ' this particular consequent . . . they were not content ' merely to know that one phenomenon was always ' followed by another " (*ibid.* § 9). A cause in fact, in no wise differs from a condition : " The cause, philoso- ' phically speaking, is the **sum total of the conditions, ' positive and negative, taken together** . . . the negative ' conditions may be all summed up under one head, ' namely, the absence of counteracting causes " (*ibid.* § 3). Yet it should be carefully noted that mere invariability of succession is not sufficient to constitute a relation of causality, unless the succession is also *unconditional*, that is to say, unless it would take place whatever supposition we may make in regard to all other things : " Invariable sequence is not synonymous with causation, unless the sequence, besides being invariable, is also unconditional."

There are various points in this account which call for criticism :—

(1) The reduction of efficient causality to a mere time-relation, is totally contrary to reason. Reason tells us that a thing which begins to exist, cannot possibly have its existence from itself, it cannot be self-created : hence it must receive existence from something else. That this must be so appears plainly, if it be remembered that before a thing exists, it is nothing ; and what is nothing cannot give itself existence. Hence, everything which comes into being, must have, not a mere antecedent in time, but an efficient cause which is the reason why it exists. In one case alone is there no need of an efficient cause, namely in the case of the First Cause, Who has never come into being, but has existed from all eternity.

(2) The identification of cause and condition is inadmissible. We have dwelt at length on the difference between these two (Ch. 14, § 2), and it is unnecessary to discuss the point again. It is clear that any account of causation, which fails to distinguish them, must be fundamentally misleading.

(3) The theory is inconsistent with itself. On the one hand the relation between cause and effect is reduced to succession in time. On the other, we are told that we must be cognizant of the *unconditional* character of the connexion, i.e. that it will take place, whatever supposition we choose to make about other things. It is, however, evident, that if we are able to affirm that the connexion is 'unconditional,' we must know of some bond linking the consequent to the antecedent, some *reason* why this consequent *must* follow this antecedent, and that we have passed far away from a mere record of sequences.

(4) Another inconsistency is to be found in his description of the cause as the 'invariable antecedent.' This can only mean, that whenever a particular effect occurs, it is always the result of the same cause. But such a view contradicts a doctrine, on which Mill lays great stress,—that of the **plurality of causes.** "It is 'not true," he tells us, "that the same phenomenon is 'always produced by the same cause : the effect *a* may 'sometimes arise from *A* sometimes from *B*. . . . Many 'causes may produce motion : many causes may produce 'death."

The doctrine of the plurality of causes is, of course, due to the fact that Mill's theory did not admit of his distinguishing between determining causes, and causes properly so called.

(5) Finally, it should be noticed that he has confused two entirely distinct principles, (*a*) the principle of causality—that everything which begins to be must have an efficient cause ; and (*b*) the principle of uniformity. The latter includes in its reference all causes, efficient, formal, material, final. Each one of these will, under similar

circumstances, produce the same effect. The former relates expressly to efficient causation. There is no need for a thing, which begins to be, to have an antecedent material cause. Were it so, either creation could never have taken place, or matter must have always existed.

We must now consider Mill's account of the grounds, on which we accept this principle, 'the major premiss of all induction,' in virtue of which our knowledge as to the laws of nature is based on something firmer than simple enumeration. It is evident that an empiricist philosopher cannot, as we have done, base it on the nature of things. To this school, the sole sources of knowledge are individual sense-experiences; and a general truth does but summarize a number of such experiences.

The Uniformity of Nature, Mill tells us, is itself discovered "by the loose and uncertain method of simple 'enumeration" (Ch. 21, § 2). The principle is, he holds, gathered from a number of particular uniformities observed in nature, which justify us in regarding the law as of universal application. But before the establishment of the wider principle, these particular uniformities, on which it was based, had not the same certainty as now belongs to laws of nature. For since they could not receive confirmation from the principle of uniformity, antecedently to its establishment they were merely based on the enumeration of instances.

Mill recognizes that he will be accused of inconsistency in thus first contrasting the uncertainty of simple enumeration, with the certainty derived from the principle of uniformity, and then teaching that the former is the foundation of the latter: and it certainly looks as if he were building an immoveable house on a moveable foundation. But he argues that the inconsistency is only apparent, because the uncertainty of simple enumeration is due to the possibility that the empirical law which our observations establish, may be true only within certain limits of place, time and circumstance.

Where the subject-matter of a generalization is so wide "that there is no time, no place, and no combination 'of circumstances, but must afford an example of its 'truth or falsity, and it be never found otherwise than 'true," then we may regard the generalization in question as possessing all the force of a scientific law.

Few thinkers now attempt to defend this theory. Two points in it call for special notice :—

(1) Unless the uniformity of nature be already presupposed, how can we be sure that any of the particular uniformities, on which it is based, have any right to that name? We may, for instance, regard it as a case of such a uniformity that hemlock is poisonous. The evidence, on which we rely, is the effect produced by the plant on a number of individual men. But in estimating the evidence, unless we rely on the general principle, what guarantee have we that the poisoning was not due to different causes in the different cases, although the symptoms were so similar. To prove the principle, even on Mill's own shewing, we must presuppose it.

(2) Human experience has extended over a minute portion of space, through a few thousands of years, and the records of that experience are fragmentary and sometimes uncertain. If our grounds for holding the principle of uniformity be nothing more than the imperfectly recorded successions observed by men on this one small planet, it is as insecurely based as ever was an induction by simple enumeration. Indeed, Mill himself owns that he sees no reason why the law should hold good 'in 'distant parts of the stellar regions' (Ch. 21, § 4). Precisely the same reasons would suggest that we have no satisfactory grounds to rely on it for any time beyond the present.

It is worth noting that some Empiricists boldly renounce all attempt to prove this principle. Thus Bain, *Deductive Logic, App. D*, "The fact generally expressed as Nature's Uniformity 'is the guarantee, the ultimate major premiss of all Induction. '. . . We can give no evidence for this uniformity." So Huxley, *Life of Darwin*, II., p. 200, "The one act of faith in the con-

'vert to science, is the confession of the universality of order, 'and of the absolute validity, in all times and at all places, of 'the law of causation. This confession is an act of faith, be- 'cause by the nature of the case, the truth of such propositions 'is not susceptible of proof." To adopt this attitude, is to renounce the hope of finding any secure basis for physical science. Well might Bain speak of it as "the leap in the dark."

One or two terms employed by Mill in his discussion of this question should be observed, as they are occasionally employed by others also. The **intermixture of effects** occurs when two or more causes combine to produce a **complex effect.** Sometimes this joint effect is of the same kind as the separate laws. This constitutes the **Composition of Causes**; and the effect is called a **Compound Effect.** Sometimes—as in chemical compounds—the effect is totally unlike the causes. This is called **Combination of Causes,** and the effect a **Heteropathic Effect.**

§ 3. **"Cessante causa, cessat effectus."** If it can be shewn that cause and effect are ever absolutely contemporaneous the one with the other, it is evident that a theory, which reduces the causal relation to mere time-sequence must fall to the ground. Hence, Mill is led to devote some space (Ch. 5, § 6) to the discussion of this possibility. He quotes against himself the Scholastic saying "*Cessante causa, cessat effectus* (When the cause ceases to operate, the effect ceases)," [1] and says, "the 'necessity for the continued existence of the cause to 'the continuance of the effect, seems to have been a 'once generally received doctrine. . . . Yet there 'were at all times many familiar instances of the con- 'tinuance of effects long after their causes had ceased. '. . . A ploughshare once made remains a ploughshare, 'without any continuance of heating and hammering, 'and even after the man who heated and hammered 'it, has been gathered to his fathers." This criticism is indicative of the temper in which Mill approached the consideration of any Aristotelian doctrine. It never seems to have occurred to him, that he must have failed to understand the meaning of the axiom, and that in

[1] *Vide* Arist., *Met. V.*, c. 2, and *Phys. II.*, c. 3, § 13, and St. Thomas's commentaries on these two passages.

the absurd sense which he attributed to it, it could not have been a dogma of the schools.

A review of its true meaning will help us in the further elucidation of the causal relation. Let us take Mill's own example of the ploughshare, and consider what part the various causes hold in regard to it.

The efficacy of two among the causes—the final and the efficient—ceases, as soon as the thing is made. It *becomes* what it is through them. But as soon as we can say of it that it *is*, it depends on them no longer : their work is at an end. On the other hand, the material and formal causes are what make it to *be* a ploughshare. Had it been made not of iron, but of some yielding substance, it would not be a ploughshare at all ; and if it should be melted down and lose its shape, it would cease to be one.

We have then two kinds of cause—the **Causa in fieri** (Cause of becoming) and the **Causa in esse** (Cause of being), each with its own proper effect. If either of these two causes ceases to operate, its effect ceases. This does not mean that when the smith dies, the ploughshare ceases to be : for the smith is not, as we have said, the cause of its *being*. He is, strictly speaking, only the cause of the change, by which the iron is transformed into the new shape. If, while the change is going on, he ceases to exercise his causality, the process of change stops. But when he has done his task, it remains : for it is no longer in a transition state. It is an existing thing, and depends on other causes.

There are causes which may be said to be at one and the same time *causa in fieri* and *causa in esse* to their effects. Thus the sun both calls into being and sustains in being the ray of light which proceeds from it : and the pressure of the atmosphere both causes the mercury to ascend in the exhausted tube, and sustains it in that position. Yet these instances are useful chiefly as illustrations. The effects in question are mere accidental modifications. No created cause is both *causa in fieri* and *causa in esse* to any complete entity ; and this fact

reveals to us that created agents realize but in an imperfect manner the full concept of a cause. On the other hand the great First Cause—God Himself—is alike *causa in fieri* and *causa in esse* to the whole universe. He called it into being, and sustains it in every instant of its existence. To imagine otherwise would be to imagine that it could possess existence in its own right, and not as an effect communicated from Him.[1]

Each effect then is dependent on the cause which produces it, in so far as they are related as cause and effect. If the cause ceases to exercise its causality, the effect no longer takes place. This does not imply that cause and effect are one and the same. We call attention to this, unnecessary as it may appear to do so, for such a view seems to be implied by the words of certain recent writers. It is plain that if cause and effect are the same, then the effect, if it be caused at all, must be caused by itself—a supposition that is self-contradictory. Further, not only are the two distinct, but though the action of the cause is contemporaneous with the effect, yet the efficient cause itself must be prior to the effect. The cause must exist before it can act.

* § 4. **Unity of Nature.** The principle which we have termed the Uniformity of Nature, is by some logicians spoken of as the **Unity of Nature.** This name is preferred by those adherents of the Idealist philosophy, who form what is called the Neo-Hegelian School. The name Unity of Nature, is regarded by these writers, as better indicating the reason why, in our experience, things appear as governed by uniform laws and as conforming to uniform types. The most eminent exponent of the Neo-Hegelian doctrine was the late Prof. T. H. Green : and it is his teaching on the point that we shall have in view in the present section.

[1] Cf. St. Thomas, *Summa Theol.*, I.,Q. 104, Art. 1, "Omnis effectus dependet a sua causa secundum quod est causa ejus. Sed considerandum est quod aliquod agens est causa sui effectus secundum fieri tantum, et non directe secundum esse ejus : quod quidem convenit in artificialibus et in rebus naturalibus. Aedificator enim est causa domus quantum ad ejus fieri, non autem directe quantum ad esse ejus. . . . Sicut igitur fieri rei non potest remanere, cessante actione agentis quod est causa effectus secundum fieri : ita nec esse rei potest remanere cessante actione agentis quod est causa effectus non solum secundum fieri, sed etiam secundum esse. . . . Ideo Augustinus dicit 'Virtus 'Dei ab eis quae creata sunt regendis si cessaret aliquando, simul et illorum 'cessaret species, omnisque Natura concideret.'"

The Idealist school, as we have already noted, does not admit the difference between thought and reality. Its defenders hold it as indubitable, that we know nothing save what is within us, and that all the objects of our experience are states of our own consciousness. Further, not merely are the objects of knowledge within the mind, they are also the work of the mind. Apart from the activity of the spiritual principle which we term mind or intellect, there could be nothing in our consciousness save transitory sensations. It is the intellect, and the intellect alone, which transforms what in themselves are simply subjective feelings into an orderly world of matter and motion.

The objects of knowledge, it is urged by Prof. Green, themselves testify to their intellectual origin. If we ask of ourselves, in what any object really consists, we shall find that it is wholly constituted by relations; "Abstract the many relations from 'the one thing, and there is nothing. They being many, deter-'mine and constitute its definite unity. It is not the case that 'it first exists in unity, and then is brought into various rela-'tions. Without the relations it would not exist at all" (Green, *Prolegomena*, § 28). Now relations are essentially the work of the mind: as relations they can only exist for a mind, and in a mind. For every relation involves the mystery of the many in one,—a mystery that is inexplicable, unless we admit that it is mind which thus unites them. Thus two entities united by a relation of succession, in so far as they are related, exist not successively but together i.e. in a mind. "Of two objects 'which form the terms of a relation, one cannot exist as so related 'without the other, and therefore cannot exist before or after 'the other. For this reason the objects between which a relation 'exists, even a relation of succession, are just so far as related 'not successive. In other words a succession always implies 'something else than the terms of a succession . . . which can 'simultaneously present to itself objects as existing not simultane-'ously, but one before the other" (*ibid.* § 31).

The question now arises, what can be the meaning of the distinction, which men are accustomed to make between the real and the unreal? The unphilosophic mind is apt to hold that the unreal and the real, fancy and fact, are distinguished as being the one the work of the mind, the other possessed of an independent existence of its own. This distinction is now seen to be untenable. For on the one hand, the real itself consists but in relations, which are themselves mental; on the other the 'unreal' if it be really the work of the mind, consists equally in relations, and has no less a claim to be looked on as real (*ibid.* § 12). Is then the distinction drawn by commonsense wholly without meaning? No. "If from the futile 'question, What is the real? which we can only answer by saying

'that the real is everything, we pass to one more hopeful—
'How do we decide whether any particular event or object is
'really what it seems to be? . . . the answer must be, that
'we do so by testing the unalterableness of the qualities which
'we ascribe to it."

Reality then consists in a certain '*unalterable set of relations*,' by which the spiritual principle within us constitutes the objects of our experience. But to say that the mind organizes our experience in unalterable relations, is to say that Nature is governed by uniform laws. The uniformity of Nature is therefore involved in the very difference between reality and unreality, and is thus a postulate of all knowledge. "That there is an unalterable 'order of relations . . . is the presupposition of all our enquiry 'into the real nature of appearances" (*ibid.* § 26). It is thus seen that the uniformity of the world as known by us, is due to the unity of the spiritual principle which constitutes it. Hence the expression Unity of Nature may justly be looked on as more philosophic than the commoner phrase Uniformity of Nature.

In all this there is not a little which calls for criticism. The theory we have summarized contains, we believe, several fundamental errors. It is based on three main positions: (1) that relations can only have a mental existence, (2) that individual objects consist solely in relations, and (3) that the real differs from the unreal as being an unchangeable, as contrasted with a changeable, order of relations. We shall examine these points in succession. Grave objections may be urged against each of them.

(1) It is true, as Prof. Green points out, that relations involve the mystery of the many in one. A relation is a link, a bond of connexion: it is according to the old definition "an order 'that holds between one thing and another (*ordo unius ad aliud*)." There is in every relation a unity of the manifold, a multiplicity of that which is one. We may freely own that this is inexplicable without a combining intelligence: and yet we may deny altogether that the ordered entities must needs exist only *in* an intelligence. When things are united by a relation of succession, the mind which thus related them, must indeed have grasped them in a single thought: but as regards their own existence, they are not simultaneous but successive. All the motions in the working of a watch, are related to each other. Yet these motions do not exist simultaneously: they follow each other. Although therefore the world of our experience is bound by a myriad relations, this lends no support to the theory that it exists only in and for a mind. It merely proves that it owes its origin to an intelligent First Cause.

(2) It is difficult to understand how it can be maintained

that an individual consists solely in relations. The accuracy of the definition of relation as 'an order holding between one thing and another' can, we think, hardly be disputed. The intelligence recognizes that this, and nothing else, is what it means by relation. Now the entities which are thus connected, are not themselves connexions. They are substances or quantities or qualities. It is true that every substance and every attribute is, for one reason or another, bound by relations to others. It is related as agent to patient, as patient to agent, as similar, as antecedent, as consequent and so on. But it is not constituted by the relations. On the contrary its own distinctive character is the ground of these relations. Had it not certain characteristic qualities, there could be no relations. To empty all the other categories of being and to reduce everything to relation, is as unphilosophic as is the Empiricist doctrine which admits no reality save sensation alone.

(3) The question at issue as to the distinction between the real and the unreal, is misstated by Green, owing to his Idealist assumptions. He has, of course no difficulty in showing that a thought even if it be erroneous, is *a real thought*: and from this he concludes that the figments of the imagination and erroneous judgments are "as real as anything else" (*op. cit.* § 22). But this conclusion is only valid on the hypothesis that the conceptual order embraces all existence. And this has never been proved, and cannot be proved. On the contrary it is an hypothesis which is contrary both to reason and to the testimony of our cognitive faculties. These alike testify to the existence of a twofold order, the real and the conceptual. And when men speak of a judgment or an imagination as 'unreal,' what they mean is, that though the objects represented exist in thought, yet in that real order which thought should represent, there is nothing to correspond to these creations of the mind.

We conclude therefore that the Idealist theory of an unalterable order of relations breaks down at every point, and affords us no means whatever by which we may account for the Uniformity of Nature. Idealism no less than Empiricism leaves physical science destitute of any basis.

CHAPTER XVI.

ENTHYMEME: SORITES: ANALOGY.

§ 1. **Enthymeme.** The fundamental methods of inference have now been discussed. The forms of argument with which we are concerned in this chapter, do not exhibit any new process of the mind. All save one, viz.: Analogy, are purely syllogistic: and Analogy is resolvable into an induction followed by a deduction. Yet because the structure of these forms differs in certain particulars from the standard type, it is convenient to treat them in a separate chapter.

An Enthymeme is a syllogism, abridged by the omission of one premiss or of the conclusion. The Enthymeme is the usual manner in which syllogistic reasoning is verbally expressed. It has already been pointed out that, though we think in syllogisms, whenever we reason from a general principle to a special case, yet we do not ordinarily express each of the three constituent judgments. We are usually satisfied when our words are sufficient to bring our meaning home to our hearers with clearness and precision: and this can be done by employing the Enthymeme in lieu of the fully-stated syllogism. We have only to read any page of reasoned thought, to assure ourselves that the shorter form is employed more frequently than the longer and complete form. Indeed it is precisely this fact, which rendered necessary the section on the expression of arguments in syllogistic form (Ch. 12, § 3). It is plain, therefore, that the distinction between the Enthymeme and the Syllogism is purely one of language, and in no sense one of thought.

According as omission is made of the major premiss, the minor premiss, or the conclusion, the Enthymeme

is said to be of the first, second or third order. The following brief examples are given by Hamilton :—

The Syllogism. Every liar is a coward.
Caius is a liar.
∴ Caius is a coward.

Enthymeme of the First Order. Caius is a liar.
∴ Caius is a coward.

Enthymeme of the Second Order. Every liar is a coward.
∴ Caius is a coward.

Enthymeme of the Third Order. Every liar is a coward.
Caius is a liar.

The Enthymeme of the third order, which merely states the premisses, and leaves it to the hearers to draw the conclusion, is indeed often far more effective rhetorically, than the explicit syllogism would be.

Similar omissions may be made in the hypothetical syllogism. Our meaning is abundantly clear when we say, 'He will see the notice in the *Times*; so he will hear of something to his advantage,' even though we do not insert the hypothetical major implied. Hence, we may have Enthymemes of the three orders in these syllogisms also.

* § 2. **The Aristotelian Enthymeme.** By Aristotle the term Enthymeme is used in a different sense. It is, he tells us, '*a rhetorical syllogism*' (*Rhet.*, I., c. 2, § 8). He recognized that though the processes of reasoning are always fundamentally the same, yet the manner in which we apply them, will differ greatly according to the class of questions treated. "An educated 'man," he says at the commencement of the *Ethics*, "will always 'look for just so much precision of argument as the matter in hand 'admits." Some account therefore of this 'rhetorical syllogism' is desirable in Logic, if only that we may see how, even when demonstrative reasoning is impossible, its place is filled by something closely analogous.

The perfect syllogism is the method of reasoning appropriate to science. Scientific conclusions are worthless, unless we can appeal to some indubitable and universal principle. In any other case, they would not carry conviction. But in rhetoric our object is not primarily to convince the mind, but to sway the will. Hence when the orator appeals to a principle, it is

sufficient for him if it is verified in the majority of cases, and so affords a reasonable motive to his audience for coming to the decision he desires. Where such principles are not to be had, the orator points to some fact, which may serve as a sign or evidence that his view is correct. Nor does he usually trouble himself to secure anything resembling conclusive proof.

The Enthymeme, therefore, is defined as **a syllogism from likelihoods or signs.**[1] The *likelihood* is a proposition, which is usually true, and thus affords a basis for a probable conclusion. An instance of such an argument is to be found in a well-known clause in the will of the late Mr. Cecil Rhodes :—

Those who live secluded from the world are as children in business affairs.
Resident fellows of colleges live secluded from the world.
∴ They are as children in business affairs.

A piece of reasoning, which would not preclude some fellows of colleges from being highly capable in such matters.

The *sign* is some fact, which affords evidence either for the truth of some general principle or for the existence of some other fact. The argument here may fall into any figure of the syllogism, and is of very different value in the different cases. (α) If the sign is an effect of the fact in question, and an infallible indication of its presence, the syllogism is in Fig. 1, and is perfectly conclusive.

Wherever there is smoke, there is fire.
That house is giving out smoke (s).
∴ That house contains a fire.

(β) The sign may be an individual instance (or instances), from which we conclude to the existence of a general law, e.g. 'The wise are good, for Pittacus is good.' This is in fact a syllogism in Fig. 3, and if analysed, is seen to be of this form :—

Pittacus is good (s).
Pittacus is wise.
∴ The wise are good.

The argument is invalid, since there is an illicit process of the minor. The legitimate conclusion is only that 'Some wise men are good.' We meet with this argument, when we are told that 'Hume, Reid and Hamilton were Scotsmen, and therefore Scotsmen in general are metaphysicians.'

(γ) When the argument from the sign falls into the second figure, it is of less value still; since the premisses do not justify even a particular conclusion. For instance,

Murderers tremble in the presence of the murdered man,
This man trembles in the presence of the murdered man.
∴ This man is the murderer.

[1] Ἐνθύμημα μὲν οὖν ἐστι συλλογισμὸς ἐξ εἰκότων ἢ σημείων (*An. Prior II.*, c. 27, § 2).

In countries where it is the custom to confront the suspected man with the body of the victim, this particular Enthymeme still appears to carry some weight.

The etymological meaning of the word 'enthymeme' would seem to be 'the result of an act of reflection.' It is used to signify a thought suggested by a person or thing.[1] Hence, as Mansel says, "the term is naturally enough applicable to the 'suggestions or persuasive arguments of rhetoric, as distinguished from the demonstrations of science." How it ceased to bear this meaning, and was employed by logicians to signify the abbreviated syllogisms considered in § 1, is not so clear. Possibly the explanation may be found in a passage, in which Aristotle says that, if one of the premisses is well known to the audience, the orator should omit that premiss.[2]

§ 3. **Chains of Reasoning.** In any sustained argument, the syllogisms form a connected series, the conclusion of one being used to form the premiss of another. When this occurs, we are said to have a chain of reasoning, or as it is sometimes called a **Polysyllogism.** The syllogism, whose conclusion becomes the premiss of the other, is termed the **Prosyllogism**: the name of **Episyllogism** is given to that which borrows its premiss from the other. Should three syllogisms be thus connected, the second is an episyllogism as regards the first, a prosyllogism in relation to the third.

The following polysyllogism may serve as an example.

(1) Those who prefer the greater to the lesser good, are wise.
Those who sacrifice temporal things to gain eternal, prefer the greater to the lesser good.
(2) ∴ Those who sacrifice temporal things to gain eternal, are wise.
The martyrs were men who sacrificed temporal things to gain eternal.
(3) ∴ The martyrs were wise.
The eighteen Carthusians were martyrs.
∴ The eighteen Carthusians were wise.

In this instance, it is the major premiss which is in each case supplied by the Prosyllogism. The following

[1] Sophocles, *Oed. Col.* 292, 1199, *vide* Mansel, *Aldrich.*, p. 216.

[2] *τὸ δὲ ἐνθύμημα συλλογισμὸς, καὶ ἐξ ὀλίγων τε καὶ πολλάκις ἐλαττόνων ἢ ἐξ ὧν ὁ πρῶτος συλλογισμός· ἐὰν γὰρ ᾖ τι τούτων γνώριμον, οὐδὲ δεῖ λέγειν· αὐτὸς γὰρ τοῦτο προστίθησιν ὁ ἀκροατής* (*Rhet.* I., c. 2, § 13).

example illustrates the case in which the minor premiss is the one to be borrowed :—

(1) All acts of aggression are unjust.
Napoleon's campaign against Russia was an act of aggression.
∴ Napoleon's campaign against Russia was unjust.
(2) All unjust acts deserve to fail of their intention.
Napoleon's campaign against Russia was unjust.
∴ Napoleon's campaign against Russia deserved to fail of its intention.

In each of the previous examples, we have proceeded from the prosyllogism to the episyllogism ; and consequently we have a **Progressive reasoning.** But it often happens that another method is adopted. The final conclusion is first stated ; the premisses, which establish it, are then given as reasons ; and the proof of these premisses follows in a similar fashion. Such a mode of stating an argument is termed **Regressive.** Thus :—

(1) Logic is deserving of study : *for*,
All the sciences are deserving of study,
And Logic is a science.
(2) Now, All the sciences are deserving of study : *for*,
Whatever assists to perfect the intellect is deserving of study,
And all the sciences assist to perfect the intellect.

§ 4. **Epichirema. A syllogism is termed an Epichirema, when to one or both of its premisses is annexed a reason in support of it.** Thus :—

Whatever is spiritual is immortal ; for it is incapable of corruption.
The human soul is spiritual.
∴ The human soul is immortal.

It will be seen that the Epichirema is equivalent to a regressive Polysyllogism : the Episyllogism is expressed in full, the Prosyllogism appears as an Enthymeme. In the example given, the Enthymeme, when written in full, will be,

Whatever is incapable of corruption is immortal.
Whatever is spiritual is incapable of corruption.
Whatever is spiritual is immortal.

Where a reason is attached to one premiss alone, we have a *Single Epichirema* : where both of the premisses are thus supported, it is called a *Double Epichirema.*

§ 5. **Sorites.**

The Sorites is a syllogism in the first figure with many middle terms.[1] In this reasoning the principle of the *Dictum de Omni et Nullo*, that 'whatever is affirmed of any subject, is affirmed of each logical part of that subject,' is extended in its application. The terms which appear in the premisses, denote not three alone, but a number of species arranged in serial subordination ; and the conclusion states that the lowest term in the series is subordinate to the highest. Thus :—

Socrates is a man ;
All men are mammals ;
All mammals are animals ;
All animals are living creatures ;
All living creatures are substances ;
∴ Socrates is a substance.

An argument of this character may be analysed into a number of regular syllogisms in the first figure, there being as many separate syllogisms as there are middle terms in the Sorites. Hence a Sorites is frequently defined as *a polysyllogism abridged by omitting the conclusions of the prosyllogisms.* In the example we have given, the analysis will be :—

(1) Socrates is a man ;
All men are mammals ;
(2) ∴ Socrates is a mammal.
All mammals are animals ;
(3) ∴ Socrates is an animal.
All animals are living creatures ;
(4) ∴ Socrates is a living creature.
All living creatures are substances ;
∴ Socrates is a substance.

It will be noticed that the first proposition is the minor premiss of its syllogism, and that in each case, the omitted

[1] This definition is given by Mr. Joseph, *Introd. to Logic*, p. 385. See also *Summa Totius Logicae*, Tract 8, c. 4, "Et licet in tali discursu," etc.

conclusion forms the minor premiss of the subsequent episyllogism. This is termed the **Aristotelian Sorites.**[1]

Aristotle himself does not treat of this mode of stating an argument; though the discussion in which he shows that there cannot be an infinite series of middle terms between the ultimate subject and the *summum genus* may be held to imply it (*An. Post. II.*, c. 20). Hamilton tells us that among the Greek logicians of the Lower Empire, it was known by the name of *Complex Syllogism* (συλλογισμὸς συνθετός), and that the name Sorites (σωρός—a heap) does not appear in any author before Laurentius Valla (1415–1465).

There are two special rules for the Aristotelian Sorites:—

(1) Only one premiss, and that the last, may be negative.

(2) Only one premiss, and that the first, may be particular.

The reasons for these rules are easy to see. (1) If any other premiss except the last is negative, we break the rule, which forbids a syllogism in Fig. 1 to have a negative minor premiss. For the first premiss of the Sorites is itself a minor; and any subsequent negation—save in the case of the last premiss,—would involve a negative conclusion, itself a minor premiss in its episyllogism. The result would inevitably be an illicit process of the major term.

(2) The second rule is involved in the first. Since the minor premiss in each syllogism is affirmative, and the syllogisms are of the first figure, it follows that, unless we are to have an undistributed middle, the major premiss must be universal. But every one of the propositions, save the first, plays the part of a major premiss.

The two rules, then, are identical with the two special rules of Fig. 1, that *the minor must be affirmative, the major universal.*

The second variety of Sorites differs from the first only in the arrangement of the propositions composing

[1] The Sorites is not a form of reasoning which frequently occurs. Dr. Watts, however, in his *Logic* (1725), calls our attention to a well-known passage in St. Paul, where the argument is thus stated, "For whom he foreknew, he also predestinated, etc., etc." *Rom.* viii. 29, 30.

it. It is called the **Goclenian Sorites**, from Rodolphus Goclenius, professor at Marburg, who first drew attention to it in a work he published in 1598, on the *Organon* of Aristotle. In this Sorites, we begin not with the lowest of the logical parts, but with that which is immediately subordinate to the *summum genus*. Thus :—

All living creatures are substances;
All animals are living creatures;
All mammals are animals;
All men are mammals;
Socrates is a man;
∴ Socrates is a substance.

When the argument, as thus stated, is analysed, it is seen that it is the *major premisses* of the episyllogisms, which are omitted, and not, as in the case of the Aristotelian Sorites, the minor premisses. This will necessitate a restatement of the two special rules. They now become :—

(1) Only one premiss, and that the *first*, may be negative.

(2) Only one premiss, and that the *last*, may be particular.

These changes are necessary, since (1) here all the premisses save the first are minor premisses; hence a negation in any of them would involve an illicit process of the major. And (2) were any premiss, save the last, particular, we should have a particular conclusion to one of the prosyllogisms, and in consequence a particular major premiss involving an undistributed middle.

§ 6. **Analogy.** In **Analogy** (or as it is termed by Aristotle, **Example** (*παράδειγμα*)) we have to deal, not with a logical form of comparatively little moment, such as are those with which we have been hitherto occupied in this chapter, but with a mode of inference, which we not only employ constantly in the practical concerns of life, but which has also pointed the way to many of the discoveries of science. It is however a guide, which, used without much caution, misleads instead of assisting.

Analogy may be defined as **an inference based on a similitude.** The formula for this mode of argument may be thus represented :—

S_1 is P.
S_2 resembles S_1 in being M.
$\therefore$ S_2 is P.

The following is the illustration, with which Aristotle provides us :—

The war of the Thebans against the Phocians proved calamitous.
War between Athens and Thebes resembles the war of the Thebans with the Phocians in being a war with a neighbouring state.
War between Athens and Thebes will prove calamitous.

The value of the inference here depends altogether on the supposition that there is a causal connexion between M and P. If this be the case, the inference is legitimate. If they are not causally related, it is fallacious ; for the mere fact that S_2 is M, would then give us no reason for supposing that it was also P. But in regard to this causal relation, we must have no more than a supposition. Our grounds for believing it must not justify certainty. As soon as we are certain that M is the cause of P, our argument ceases to be an Analogy and becomes a perfect syllogism.

The analogical argument may be said to contain both induction and deduction. By an induction we conclude from the instance S_1 that ' All M are P ' ; and we then argue deductively, ' All M are P, S_2 is M, $\therefore$ S_2 is P.'[1]

[1] Aristotle thus describes the argument from *Example* : " The Example is ' an inference, not from the logical parts to the logical whole [induction], nor ' from the logical whole to its parts [deduction], but from part to part, when ' both fall under a common genus, but one of the two is better known to us ' than the other " (*An. Prior II.*, c. 24, § 3). Thus, in the illustration given, the inference passes from S_1 to S_2, both being logical parts of the genus M. The validity of the argument requires, as we have indicated, that P should be a property resulting from the nature M, and should not be one of the differentiating characteristics of S_1 (cf. Trendelenburg, *Elem. Log. Arist.*, § 38). In the same passage, Aristotle terms it an inference, " in which we prove the major ' term of the middle, by a term which resembles the minor." This description applies to the inductive part of it alone. In that part, we conclude that the major term (P) may be universally predicated of the middle (M), on the ground that it is predicable of S_1, a term which resembles the subject of our eventual conclusion S_2.

The inference from Example is termed by Aristotle the Induction of Rhetoric, just as he regards the Enthymeme as representing Deduction, where rhetorical argument is concerned. It is in fact frequently applicable in regard to those practical affairs, which form the subject matter of deliberative consultation ; and in which Induction strictly so-called has no place. We often argue in this way, when we apply the lessons of history. We may well suppose, for instance, a Frenchman to have made a forecast of the future on such grounds, when at a time of complete political disorganization, he saw Napoleon Bonaparte return to France as the general of a victorious army. He would remember that Julius Caesar under somewhat similar circumstances, had seized supreme power at Rome, and that on a smaller scale, Cromwell had done the same in England. Analogy would suggest that events would follow the same course in France.

The analogical argument is however not merely of value in such matters as these. It has, as we have noted, been one of the most fruitful sources of scientific discovery. When two different classes of objects have certain attributes in common, and one of the two is further characterized by a property, which appears to be in some way dependent on these attributes, the investigator is led by Analogy to enquire whether the same property is not to be found in the other class. It will be sufficient here to give a single example of this scientific analogy, since we shall have to return to the subject in a later chapter. By way of illustration we may take the inference which led to the discovery that lightning is an electrical discharge. The discovery was made by Franklin in 1749. The phenomena of electricity were at that time only known to him through the small electrical machines of the day. But he had noticed the similarity between lightning on the one hand, and on the other the sparks given off by the conductor of the machine. In both cases the light is of the same colour, the motion rapid, and conduction can be effected by metal. It suggested itself to him, that lightning was

electrical in its character, and was due to the clouds being charged with electricity, just as the electrical machine is charged. Were this so, then a conductor sent among the clouds should, he judged, give off sparks, precisely as does the conductor of the machine. The experiment was tried. A kite was sent up during a thunderstorm with a wire attached to it. The anticipated result was obtained. The two phenomena proved to resemble each other in this point also : the analogical inference was correct. It is true that this inference itself only concluded to the fact that the clouds would give off electrical sparks. But this was sufficient to establish a truth of vast moment, viz. : that lightning was to be reckoned among electrical phenomena.

Mill devotes a chapter of his *Logic* (Bk. III. c. 20) to the subject of Analogy. He holds that the force of an analogical inference, depends entirely on the number of points of similarity between the two objects. Thus he says (*l.c.* § 3), "Since the 'value of an analogical argument, inferring one resemblance 'from other resemblances without any antecedent evidence of a 'connexion between them, depends on the extent of ascertained 'resemblance, compared first with the amount of ascertained 'difference, and next with the extent of the unexplored region 'of unascertained properties : it follows that . . . if after much 'observation of *B*, we find that it agrees with *A* in nine out of 'ten of its known properties, we may conclude with a probability 'of nine to one that it will possess any given derivative property 'of *A*." What has been said above will have shown that such a basis for analogy presents many difficulties. The mere number of resemblances, is in fact a point of little moment. The value of the inference depends on the reasons, which we possess for supposing that the characteristic common to the two objects is really connected with the property in question. These reasons must not, as we have already said, be such as to amount to certainty. They must be probable, not conclusive. But it is they that are the true foundation of the argument : and where they are wanting analogy becomes mere guess-work. Moreover, Mill's demand that a comparison should be instituted between the known points of agreement and difference on the one hand, and 'the unexplored region of unascertained properties,' is incapable of fulfilment. Since the region is unexplored, any attempt to form an estimate of the number of properties which it contains, must needs be fruitless.

Analogy is occasionally defined as *an argument based on similarity of relations*. Of this kind is the inference which compares various orders of citizens to the different parts of the human body—the head, the hands, the eyes, etc.,—and then reasons to their respective duties. The whole basis of such an argument is the similarity of relations between the group of citizens and the state on the one hand, and on the other between the organ in question and the body. This account of Analogy adheres closely to the etymological meaning of the term (ἀναλογία), which was used among the Greeks to signify numerical proportion. As a definition it presents some inconveniences. The form of reasoning traditionally known as Analogy, is one which plays a very important part. In many cases the resemblances on which it is based, are not similarities of relations. Nothing would be gained by restricting the use of the term to a narrower scope.

CHAPTER XVII.

FALLACIES.

§ 1. **The Treatment of Fallacies in Logic.** It might perhaps seem that fallacies have no place in this work. We have throughout treated Logic as the science which deals with the conceptual representation of the real order. So far, then, as an argument is fallacious, it is no longer logical. It fails to conform to any of those processes, which it is the work of Logic to explain. It contributes but little to a knowledge of Logic, to be warned against a syllogism whose middle term is ambiguous. For here we have not one of those legitimate operations of the mind, with which the science is concerned, but a mere counterfeit argument, whose plausibility is due to the fact that the same vocal sound represents two different ideas.

The chief reason why fallacies almost invariably form a constituent part of treatises on Logic is historical. The last portion of the *Organon* of Aristotle, the *Sophistici Elenchi*, or *Refutations of the Sophists*, is concerned with them. Hence it came about that subsequent writers on the subject also included them within the scope of their work. We shall realize why Aristotle treated this point at some length, if we recall the circumstances of his day. At that period, the pursuit of knowledge depended far less on written books than on the spoken word. Verbal intercourse or 'Discussion' was not merely the method of instruction, but also the method, by which friends co-operated in the search for philosophic truth, and by which controversy was carried on. There were three main kinds of these reasoned discussions:

(1) The demonstrative arguments relating to some special science, proposed by a teacher to learners.

(2) Friendly co-operation in the search for truth.

(3) Intellectual skirmishes between opponents.[1] To this class, belonged the arguments of the Sophists. The boast of these men was that they could defend either side in any argument, and reduce their opponent to discomfiture. To them reasoning was a means employed, not for the discovery of the truth, but to win popular applause and secure pecuniary gain.[2]

It is therefore not to be wondered at, that Aristotle in his logical treatises deals not merely with the internal reason, but with argument as carried on between contending parties. He discusses the methods, by which in these verbal discussions, truth might be pursued, the claims of suggested solutions tested, and the fallacious arguments of the Sophists avoided. These questions form the subject of the *Topics* and the *Sophistici Elenchi*.

It is not, however, to be thought that the study of fallacies is useless to ourselves. We have not indeed a class of men prepared for a consideration to defend either side of any thesis. But the art of making 'the worse cause appear the better,' still has its adepts. Politics, philosophy, certain aspects of religious controversy, afford these men a field for the exercise of their talents. Some acquaintance with the main types of fallacy may be of real service in guarding us against pitfalls. It is true, this alone will not ensure that we shall detect the sophisms proposed for our acceptance; but it will undoubtedly render us more capable of doing so.

§ 2. **What Errors are reckoned as Fallacies?** It is manifest that in treating of fallacies, we do not reckon as such every error which, occurring in an argument, could vitiate the conclusion. To such a task, there

[1] *Soph. Elenchi*, c. 2, § 1.

[2] For a discriminating account of the Sophists, see Grant's *Ethics of Aristotle*, pp. 104–155.

could be no possible termination. Moreover most of such errors belong to some special science. Hence, even if we consider the Art of Discussion as part of Logic, it is not the function of Logic, but of the science to which the error belongs, to indicate its falsity. The consideration of erroneous premisses, is therefore excluded from the treatment of fallacies.

Another class of errors is similarly excluded, because their detection belongs to the special science, with the matter of which the syllogism deals. In these, the false conclusion occurs, not because the premisses are wrong, but because some method of solution inappropriate to the case in question is applied in the reasoning. Such, for instance, is the famous argument, relating to Achilles and the tortoise.[1]

Breaches also of syllogistic rule, such as undistributed middle and illicit process of the major or minor term, are further ruled out from the list of fallacies. A fallacy must present a semblance of truth. But a syllogism, in which these rules are violated, to men versed in the art of dispute, lacked the essential qualities necessary to recommend it. In these days, indeed, many of the false arguments propounded for our acceptance, err precisely in these respects. Perhaps the prevalence of these errors has arisen, as de Morgan suggests, because the study of Aristotelian Logic has been neglected, and men scarcely know which forms of argument are conclusive, and which are not. "The philosophers," he says, "who made the discovery (or what has been allowed

[1] In this fallacy, it was supposed that a race takes place between Achilles and a tortoise, and that while Achilles slept, the tortoise gained a small start. It was urged that Achilles could never overtake his rival. For if the tortoise had a start of, say five yards, then during the time that Achilles was advancing this distance, it would have made some further progress. In the same way, while he was covering the new ground gained upon him, the tortoise would still advance in front; and so on, *ad infinitum*. But no one, not even Achilles, can cover an infinite number of distances, however small. Therefore Achilles can never overtake the tortoise. Here the fallacy lies in an error relative to the nature of motion. It is throughout supposed that it is composed of a number of separate parts. It is in fact like space itself, *continuous*; thus the difficulty of traversing an infinite number of parts does not occur. (For another solution see Lewes, *Hist. of Philosophy*, 1st Epoch, c. 3.)

(to pass for one) that Bacon invented a new species of ' Logic, which was to supersede that of Aristotle, have ' succeeded by false history, and falser theory, in driving ' out from our system all study of the connexion between ' thought and language " (*Formal Logic*, p. 241).

The errors in argumentation, which are reckoned as fallacies, are thus limited to those cases, in which we have the appearance of a syllogism, whose premisses are undeniable, and whose conclusion follows in due form,—but in which nevertheless we have a false conclusion. A fallacy may then be defined as **a violation of logical principle, disguised under a show of validity.**

The following terms occasionally used as synonymous with fallacy, should be noted. **Sophism,** a false syllogism fabricated for the special purpose of deceiving others. **Paralogism** : this word is explained by Hamilton (following Kant) as a fallacy, of whose falsehood the employer is not conscious. It is more frequently employed to signify a breach of the formal rules of inference. **Paradox,** something contrary to received opinion.

§ 3. **Aristotle's List of Fallacies.** Aristotle divided fallacies into two groups.

(1) Those arising from the language (*fallaciae in dictione—παρὰ τὴν λέξιν.*)

(2) Those in which the error arises from some other source than the language (*fallaciae extra dictionem—ἔξω τῆς λέξεως.*)

These were called by some of the Schoolmen, ' Fallacies in the matter ' (*fallaciae in re*), since in them the source of the confusion is in the thing stated.

The following is the list of these two classes :—

A. Fallacies in the language.

1. Equivocation. (*Aequivocatio—παρὰ τὴν ὁμωνυμίαν.*)
2. Amphibology. (*Amphibologia—παρὰ τὴν ἀμφιβολίαν.*)
3. Composition. (*Compositio—παρὰ τὴν σύνθεσιν.*)
4. Division. (*Divisio—παρὰ τὴν διαίρεσιν.*)
5. Accent. (*Accentus—παρὰ τὴν προσωδίαν.*)
6. Figure of Speech. (*Figura dictionis—παρὰ τὸ σχῆμα τῆς λέξεως.*)

B. Fallacies in the matter.

1. Accident. (*Accidentis*—παρὰ τὸ συμβεβηκός.)

2. Confusion of absolute and qualified statement. (*A dicto secundum quid ad dictum simpliciter*—παρὰ τὸ ἁπλῶς ἢ πῇ λέγεσθαι.)

3. Refuting the wrong point. (*Ignoratio Elenchi*—παρὰ τὴν τοῦ ἐλέγχου ἄγνοιαν.)

4. Begging the question. (*Petitio Principii*—παρὰ τὸ ἐν ἀρχῇ λαμβάνειν.)

5. Consequent. (*Consequentis*—παρὰ τὸ ἑπόμενον.)

6. False Cause. (*Non Causa pro Causa*—παρὰ τὸ μὴ αἴτιον ὡς αἴτιον.)

7. Many questions. (*Plurium Interrogationum*—παρὰ τὸ τὰ δύο ἐρωτήματα ἓν ποιεῖν.)

* The list of the fallacies in the language, Aristotle tells us, is exhaustive, including all which can arise from the words employed in the argument.[1] St. Thomas proves that this is so in the following manner : There are, he says, three possible sources of misapprehension as to the meaning of the language. (*a*) The words employed may be really ambiguous. This gives us two cases, viz. : (1) when the ambiguity belongs to a single word (*Equivocation*), and (2) when it belongs to a phrase (*Amphibology*). (*b*) The second source of error occurs when the words are not really ambiguous, but may be rendered so by a change of pronunciation. This also gives us two cases, viz. : (1) when the quasi-ambiguity is found in a single word (*Accent*), and (2) when it belongs to a phrase (*Composition and Division*). (*c*) The ambiguity may be due merely to the misunderstanding on the part of the disputant (*Figure of Speech*).[2]

§ 4. **Equivocation.** Equivocation is the fallacy which arises from the employment of the same word in different senses. The premisses appear undeniable, for the context determines the word in question to an appropriate sense ; but we are brought to a false or unwelcome conclusion. Many examples given in illustration of the fallacy are simple enough. De Morgan supplies us with the following :—

All criminal actions ought to be punished by law.

[1] *Soph. Elenchi*, c. 4, § 1.
[2] St. Thomas, *Opusc.* 35, *de Fallaciis*, c. 3.

Prosecutions for theft are criminal actions.

∴ Prosecutions for theft ought to be punished by law (*op. cit.* p. 241).

But it should be remembered that Equivocation covers the case not merely of equivocal but of analogous terms. Indeed it would seem to have been these that Aristotle had specially in view.[1] Where the sophism is of this character it may sometimes cause grave perplexity to the student. Thus it might be argued:—

Nothing can be a real cause that does not possess real existence.

So-called final causes do not possess real existence.

∴ So-called final causes are not real causes.

Here the analogous word 'cause' brings us to a false conclusion. The major premiss is not true of all causes. A final cause exerts its causality, not in virtue of real existence, but in so far as an intelligent efficient cause sets it before him as his end-in-view. Yet the erroneous major may easily win acceptance, because when causes are spoken of, we ordinarily mean efficient causes.

§ 5. **Amphibology.** Amphibology, or as it is now often called, Amphiboly is identical with Equivocation, except that here the error is due not to an ambiguous word, but to the doubtful meaning of some phrase. The reply alleged to have been given by the oracle to Pyrrhus before his invasion of Italy, has appeared in almost all logical treatises from the time of Boethius to our own days as the typical example of this fallacy, and it can scarcely be omitted:

'*Aio te Aeacida Romanos vincere posse.*'

The form of the phrase is such that it may equally well signify, 'I tell thee, son of Aeacus, that thou canst conquer the Romans,' and 'I tell thee, son of Aeacus, that the Romans can conquer thee.'

When one noun is qualified by another in the genitive case, the relation between the two is left quite indeterminate, and can only be known from the context.

[1] *Soph. Elenchi*, c. 7, § 1.

Often the relation signified is that between the possessor and the thing possessed. But frequently some very different relation is in question, such as, e.g. that between an author and his book. Thus some of the old logicians give this example :—

This is Aristotle's book.
What is Aristotle's belongs to Aristotle.
∴ This book belongs to Aristotle.

The following instance may serve to show how amphibology may give rise to a philosophical difficulty.

It is impossible that anything can be made out of nothing (*Ex nihilo nihil fit*).

To create is defined as to make something out of nothing.

∴ Creation is impossible.

Here the fallacy arises from the ambiguity of the phrase 'to make out of nothing.' In the old axiom *Ex nihilo nihil fit*, we mean that 'nothing' cannot be a substratum, out of which something is made; in other words, no one can turn nothing into something. When, however, we define creation as 'to make out of nothing,' we mean 'to make without any substratum at all.' No impossibility is involved in this, as there is in the idea of employing nonentity as a substratum.

§ 6. **Composition and Division.** When one term of a proposition consists of several words, these members must sometimes be understood conjunctively, and sometimes disjunctively. Thus if we say, 'Five is two and three,' the words 'two and three' must be taken in conjunction as constituting a single predicate. But when it is stated that 'Bacon was a statesman and a philosopher,' we have two separate predicates; and the sentence can be analysed into two propositions. The fallacy of **Composition** occurs, when we join together the members of such a term, in a case where they should be kept separate. Thus if we argue,

Peers and ecclesiastics are excluded from membership of the House of Commons.

Two men only are peers and ecclesiastics.

∴ Two men only are excluded from membership of the House of Commons,

we commit this fallacy. For each of the two classes is excluded, and it is not necessary that a man should be alike a peer and an ecclesiastic, to be disqualified.

The fallacy of **Division** is the converse of the preceding. The argument is vitiated by this fallacy when we separate what should be taken together. Thus,

Heaven is the reward of all who love their friends and forgive their enemies.

Caius loves his friends.

∴ Heaven will be the reward of Caius.

An interesting example of the fallacy of Composition is furnished by Mr. Bradley. In treating of the Disjunctive proposition he lays it down as evident that the expression '*A* is *b* or *c*' "cannot possibly answer to 'real fact. No real fact can be 'either—or.'" On this he bases a long and intricate discussion as to the real meaning of the proposition. The alleged difficulty is simply a case of this fallacy. The words 'either *b* or *c*' have been read conjunctively instead of disjunctively (*Principles*, p. 122).

An instance of the fallacy to be found in one of Mill's works, has become famous. It occurs in his *Utilitarianism*, when he is seeking to establish that the *summum bonum*, after which all men should strive, is the 'greatest happiness of the greatest number.' He says, "Each person . . . desires his own happi-'ness. This being a fact, we have all the proof which the case 'admits of . . . that each person's happiness is a good to that 'person, and the general happiness, therefore, a good to the aggre-'gate of persons." The argument involves a syllogism of this character :—

The happiness of *a*, *b*, *c*, *d* . . . is the happiness of the aggregate of persons.

The individuals *a*, *b*, *c*, *d* . . . desire their own happiness (the happiness of *a*, *b*, *c*, *d* . . .).

∴ Each several individual [should] desire the happiness of the aggregate of persons.

Here in the minor premiss, as is evident, the predicate must be understood *in sensu diviso*. In the major premiss however the same term appears as subject *in sensu composito*. It is quite

untrue that the happiness of *a*, *b*, *c*, *d*, etc., is the happiness of the aggregate if they are taken severally and out of relation to all others, as they are when we say that each desires his own happiness. The conclusion is of course worthless. Moreover it is further vitiated by another illegitimate alteration, which we shall notice more particularly when we discuss the fallacy known as Figure of Speech. For he slips into this error as well; and when he should draw the conclusion that all men do as a fact aim at the general happiness, he concludes instead that it is the end at which they should aim.

§ 7. **Accent.** As to the fallacy of Accent, Aristotle says, "Accentuation in unwritten discourse can hardly 'furnish a fallacious reasoning, but only in written 'controversy and criticism on the poets." [1] It would appear that he only introduces it, in order to render exhaustive his list of the errors, which can arise from the language. Within the sphere to which he inclines to restrict it, the ambiguity arising in this way often makes it difficult to fix the precise thought, which an author intended to express. For example the words addressed by Cardinal Wolsey, when dying, to Sir William Kingston, are thus printed by Dr. Brewer, the historian of the reign of Henry VIII., "Had I but served my God with as much zeal as I have served my king, *He* would not have given me over in my old age to my enemies." [2] The stress on the word 'He' gives a new significance to the passage. If it be the true reading, Wolsey did not speak of his deliverance into the hands of the Boleyn faction as being a punishment inflicted upon him by God; he saw it as the act of an ungrateful master, and contrasted it with the treatment he would have received from God, had he made His service his chief aim.

§ 8. **Figure of Speech.** This fallacy occurs when the structure of a word or expression leads us erroneously to suppose that its meaning is analogous to that of words whose form is similar.[3] A term really belonging to one

[1] *Soph. Elenchi*, c. 4 (Poste's trans.).

[2] Brewer, *Hist. of Henry VIII.*, p. 444.

[3] We sometimes see it stated (cf. e.g. de Morgan, *op. cit.* p. 250) that the fallacy is simply the grammatical mistake of giving a feminine adjective to a masculine noun, whose termination happens to be one which is ordinarily

category may be understood as belonging to another: quantity may be taken for substance, agent for patient, and so on. No better example can be given than one found in Mill's *Utilitarianism*, to which reference has already been made in § 6. In the same passage from which our previous citation was taken, he says, "The only 'proof capable of being given that an object is visible, 'is that people actually see it. The only proof that a 'sound is audible, is that people hear it. . . . In like 'manner, I apprehend, the sole evidence it is possible 'to give that anything is desirable, is that people do 'actually desire it." Here the whole force of the argument lies in the similarity of the words 'visible,' 'audible,' 'desirable.' Yet 'visible' and 'audible' signify that the object *is* seen or *can be* seen, and *is* heard, or *can be* heard. But 'desirable,' though it implies that the object *can be* desired, means much more, and deals with the moral order; for it signifies that the object *ought to be* desired.[1]

On identical grounds exception may reasonably be taken to the use, by certain authors, of the term 'a perception,' to signify *an object perceived*. The form of the word 'perception' is similar to that of terms such as 'emotion,' 'passion,' etc., which denote subjective affections, and hence paves the way to the fallacious conclusion that the immediate objects of sense are likewise subjective.

Aristotle includes under this fallacy all cases, where there is an illegitimate transition from category to category. The following example, which he gives, well illustrates how a Sophist might succeed in putting an unskilled dialectician into a difficulty. The question is first proposed, Whether a man must not have lost that which he once had, and now has no longer. It is readily conceded that it is so. The Sophist then argues :—

He who has ten dice, and loses one, no longer has what he once had, viz.: ten dice.

feminine (e.g. *bona poeta*), and the like. Such an error is not logical at all. It is true that this case is mentioned by Aristotle; but it should be remembered that one of the objects of the Sophists was to make their opponents ridiculous by driving them into grammatical solecisms.

[1] In this argument, the fallacy carries us from one species of relation to another totally different in kind.

He who no longer has the ten dice he had once, has not necessarily lost ten dice.

∴ He who no longer has what he once had, has not necessarily lost it.[1]

The fallacy lies in the fact that in the original question the words 'to have no longer what he once had,' are understood of an object considered independently. As applied to the dice, 'what he once had' signifies the collective body. Objects collectively considered stand in mutual relation as members of a group of such and such extent. If one be removed, this relation is lost by each and all. Hence there is here an illegitimate transition from 'substance' to 'relation.'

§ 9. **Accident.** The meaning of the term 'accident' as here employed, is somewhat different both from that which it bears as applied to one of the Predicables, and from that it has when it denotes the nine last Categories. Here a predicate is said to be accidental to a subject, when, and in so far as, it has not the same definition. The form of the proposition '*S* is *P*' leads us to regard *S* and *P* as identical. They are so, if by this we mean that they denote the same individual; but considered in regard to what they signify, they are different unless their meaning, their definition, is the same. Viewed from this aspect, two terms may stand to each other in three relations.[2] They may (1) be synonymous, having precisely the same definition, e.g. 'A sheath is a scabbard.' Or (2) they may have definitions which are entirely different, e.g. 'The horse is black.' Or again, (3) the definitions may have something in common, e.g. 'Man is animal,' 'Man is risible.' We cannot conclude that what is true of the predicate, is true of the subject, unless what is asserted of the predicate, belongs to it, precisely in so far as its definition is identical with that of the subject.[3] The fallacy of accident results from neglecting this principle. Aristotle gives us various examples in illustration. Thus the Sophist is represented as asking, 'Do you know the man with his face muffled?' 'I do not.' 'Do you know Coriscus?' 'I do.' 'The man

[1] *Soph. Elenchi*, c. 22.
[2] S. Thomas, *Opusc.* 35, c. 10.
[3] *Soph. Elenchi*, c. 24, § 2, c. 5, § 1.

with his face muffled is Coriscus. You have both asserted and denied that you know Coriscus.' The solution of the difficulty, as he points out, is that it is one thing 'to be Coriscus,' and another 'to be with muffled face.' It is perfectly possible at the same time to know 'Coriscus,' and not to know 'the man with muffled face.' In the same way, we may dispose of the sophism, 'That dog is yours, and he is a father: therefore he is your father.' The two terms 'yours' and 'a father' cannot be treated as though they were one, merely because they refer to a single individual. Of a similar character is the argument,

Risible is a property.
Man is risible.
∴ Man is a property.

It is true that 'risible' contains 'man' as part of its definition, because it is a property of 'man'; but it does not follow that everything that can be affirmed of 'risible,' can likewise be affirmed of 'man.'

§ 10. **Confusion of Absolute and Qualified Statement.** This fallacy is commonly known as the fallacy *Secundum quid*, from its Latin name, *fallacia a dicto simpliciter ad dictum secundum quid*. A predicate is said to be affirmed *secundum quid*, i.e. only in some particular respect, when a word is added qualifying the application of the predicate to the subject. Qualification may be of two kinds. It may be such as to limit and to narrow the reference of predicate to subject, so that it would be false to affirm the proposition without mention of the qualification. Or it may be one which, so far from limiting the applicability of the predicate, presupposes the truth of the unqualified assertion. Thus the proposition, 'Gibbon is a great historian,' could not be true, were not the absolute statement 'Gibbon is an historian,' true.[1] But because it is the case that 'To offer up your aged parents in sacrifice, is a duty among the Triballi,' we cannot conclude that the same is a duty elsewhere.[2]

[1] See Ch. 6, § 8 (2) above 'Inference by the omission of determinants.'
[2] Cf. *Topics*, II. c. 11.

It is in regard to statements limited in this manner, that there is danger of the fallacy *secundum quid*. This limiting qualification may be such as to deny the reality of the predicate, e.g. 'This coin is a false shilling,' 'A centaur is an imaginary animal.' Or it may assign some precise place, time, or respect, in reference to which alone the assertion is made, e.g. 'Laudanum is conducive to health *in certain illnesses*,' 'It is lawful to take life *in a just war*,' etc., etc.[1] In none of these cases, can we from the limited, infer the truth of the absolute statement. A centaur is not an animal; nor is laudanum, speaking 'absolutely,' conducive to health.

This fallacy may be said to be connatural to the race of man. We are all apt to believe that what has been found useful within our own narrow experience, must needs be so to other men, and in other circumstances. It is hard, for instance, to persuade an inhabitant of this island that a nation can prosper and be happy, without representative government and trial by jury. Indeed, it is commonly said that we have in certain cases done positive harm by imposing English methods on races, to which they were unsuited. Our legislators, arguing too hastily *a dicto secundum quid ad dictum simpliciter*, judged that what was adapted to Englishmen would be equally serviceable to the Hindoo.

§ 11. **Ignoratio Elenchi.** The name *Ignoratio Elenchi* is now commonly applied to all cases, in which a man establishes, not the point which he has undertaken to prove, but some other conclusion. But the true significance of the term is 'ignorance of the nature of refutation.' It is the fault committed in controversy by the man who does not prove the true contradictory of the statement advanced by his opponent, but something which may be mistaken for it. A true refutation, as we have already seen (Ch. 4, § 2), only occurs when "the 'same predicate is denied of the same subject in the

[1] For a more detailed analysis of the various ways in which the predicate is limited, see St. Thomas, *Opusc.* 35, c. 11.

'identical respect, relation, manner and time in which 'it has been affirmed." [1] It is, for instance, not uncommon for the Catholic doctrine that the Pope is infallible regarding matters of faith and morals, to be attacked on the ground that it can be historically proved that some Popes have been guilty of grave sin. Here there is manifest *ignoratio elenchi:* for the statement brought as a refutation, viz.: 'The Popes are not *impeccable*,' does not deny the same predicate as was affirmed in the original proposition, 'The Popes are *infallible*.'

Within the sphere of philosophy, a good example is afforded in a well-known passage of Berkeley, contained in the Introduction to the *Principles of Human Knowledge*. He undertakes to prove that the mind is incapable of forming general concepts; and he does so as follows:

"I can imagine a man with two heads or the upper 'part of a man joined to the body of a horse. I can con-'sider the hand, the eye, the nose, each by itself abstracted 'or separated from the rest of the body. But then what-'ever hand or eye I imagine, it must have some particular 'shape or colour. . . . I cannot by any effort of thought 'conceive the abstract idea above described." It is plain that his reasons prove something quite different from what he is seeking to establish; for they only show that we cannot form a universal phantasm, not that we cannot conceive a universal idea.

Together with *ignoratio elenchi* we may class the *argumentum ad ignorantiam*, where the speaker trusts to the ignorance of his hearers for their acceptance of his statements: the *argumentum ad populum*, the appeal not to reason but to popular prejudices: the *argumentum ad verecundiam*, in which the speaker urges that the dignity of those who hold his opinion is such, that his hearer should feel himself constrained to yield his opinion to theirs: [2] the *argumentum ad hominem*, in which the previous

[1] *Soph. Elenchi*, c. 5, § 5.

[2] The *argumentum ad verecundiam* should be carefully distinguished from the perfectly legitimate *argumentum ad auctoritatem*. The latter relies, not on the dignity of those who hold the opinion, but on their supremacy in the particular matter under discussion. Thus in a question which concerns art, a man but little versed in the subject, will rightly regard his view as inferior to that of the great authorities on art. Similarly, in questions of political economy, statesmanship, religion, etc., etc.

history of a man is urged as rendering it impossible that he should with consistency hold certain views.

§ 12. **Petitio Principii.** The term employed by Aristotle to denote this fallacy, signifies literally 'to assume the very point proposed for debate at the outset.' Hence our English expression 'to beg the question,' renders the original sense with fidelity. For **the question** (*quaestio*) is the name used by the old logicians for the conclusion, which the disputant undertakes to prove, but for which he has not yet advanced his arguments. It is, of course, but rarely that it is possible for a man to assume without any circumlocution the point at issue. At the least, he must conceal the indecency of the proceeding by the employment of different words. Where this is done, he may escape detection. It would perhaps be possible at the present day to hear the thesis that the House of Lords is out of date, supported by the argument that an upper chamber in England is an anachronism. Aristotle enumerates five ways in which the question may be begged.[1] The first of these is the case in which the proposition itself is assumed. The example just given will illustrate this method. The other four are: (2) the assumption of a universal proposition which involves the conclusion to be established: (3) if the proposition needing proof is universal, the assumption of a particular case involving it: (4) the assumption of the truth of the conclusion in regard to each part of the subject, of which as a whole it is to be proved: (5) the assumption of a proposition, the truth of which carries with it the truth of the one to be established.

Of these various kinds perhaps the second is the one which affords the disputant the greatest facility for escaping detection.[2] A notable instance is afforded by the famous argument by which in the *Monadology* Leibniz

[1] *Topics*, viii. c. 13.

[2] Mr. Welton (*Manual*, § 187), speaking of this form of *Petitio Principii*, says: 'This was the sense in which the term was used by the Scholastics,' apparently implying that they did not recognize the other four forms. In this he is in error. Cf. e.g. St. Thomas, *Opusc.* 35, c. 13.

seeks to show that all material bodies are constituted of simple and indivisible monads. He argues that what is 'compound' must needs be an aggregate of simple substances; hence, since we know that there are 'compound' substances, we know that there are simple substances.[1] The argument relies on the supposed principle, 'What is not simple is *actually* compound.' But this is a mere assumption. For there is a third alternative, viz.: the continuous, which is not formed of *actual* parts, i.e. of parts already separated from each other, and yet is not a simple entity.

A well-known form of the fifth case is the *circulus in argumentando* or *vicious circle*, in which of two correlative propositions, each is made to depend for its proof upon the other. An instance may be drawn from the works of Mr. Grant Allen,—one of the popularizers of the evolutionary hypothesis in its more extreme forms. In his work *The Evolutionist at Large*, he accounts for the development of brilliant plumage in certain species of birds, on the ground that they have acquired an aesthetic taste, owing to the fact that their food has to be sought among bright fruits and flowers: while conversely, he accounts for the development of the bright colours of the edible berries, on the ground that they have 'assumed' these hues, because the birds which live upon them, have a taste for bright colours.[2]

In this connexion we may mention the *argument from silence*. This form of argument assumes that some event was unknown to a writer, because none of his existing works mention it. Many instances might be cited to show that conclusions thus based really amount to *petitio principii*. Thus Theophilus of Antioch, writing a defence of Christianity, does not once name Christ. Tacitus does not mention the tragedies of Seneca, and Martial and Statius never mention one another.

§ 13. **Fallacy of the Consequent.** This fallacy is ordinarily understood as denoting the illegitimate conclusion, which is drawn in a hypothetical syllogism from the assertion of the consequent or the denial of the antecedent: as for instance:—

[1] *Monadologie*, § 2. Il faut qu'il y ait des substances simples: puisqu'il y a des composés: car le composé n'est autre chose qu'un amas ou aggregatum des simples.

[2] Cf. Fr. J. Gerard, *Science and Scientists*, pp. 84, 94.

If a student is idle, he will have little acquaintance with his subject.

This student has little acquaintance with his subject.

∴ He is idle.

These errors have already been noticed in Ch. 13, § 1, and there is no need to deal with them afresh.

* This explanation of the fallacy has a long tradition behind it. It is not a novelty of the modern text-books, but has descended to them from the mediæval treatises. But it is not identical with the Aristotelian account. Aristotle gives no separate treatment in his logical works to the hypothetical syllogism. It is therefore unlikely that he should deal with that error in his enumeration of fallacies. As a matter of fact, the word *Consequent* (ἑπόμενον) in his logical treatises, where it is of frequent occurrence, has a different meaning from that now attributed to it. It denotes any predicate, which is necessarily involved in a subject.[1] Thus, in the propositions 'man is animal,' 'man is mortal,' both of the predicates are among the ἑπόμενα of 'man.' The fallacy of consequent occurs, when we treat the consequent as convertible with the antecedent. Rhetorical Enthymemes (of Fig. 3) are, he says, vitiated by this fault, and he illustrates this by the syllogism:

Men of loose character dress elaborately.
This man dresses elaborately.
∴ This man is of loose character.[2]

The fallacy, he says, is based upon a misapplication of the axiom, 'two things, which are identical with the same thing, are identical the one with the other.' Those who are unversed in Logic, erroneously imagine that this relation is that which obtains between the subjects and the predicate of their premisses. Indeed, fallacies of the consequent, he adds, are a subordinate species of fallacies of accident. The difference between them is merely that in the latter, we are concerned with a single subject: this we identify with its accident, and affirm of the accident what really belongs to the subject alone. In fallacies of the consequent we have two subjects, of which the same accident is affirmed: and on these grounds we identify the subjects.[3]

§ 14. **False Cause.** Both the fallacy of **non causa pro causa** and the following one of 'Many questions,' though incident to disputation, as it was carried on at ancient Athens, are not among those errors, into which

[1] Cf. Bonitz in Arist. *Met.* A. 981, a. 27. [2] *Soph. Elenchi*, c. 5, § 9.
[3] *Ibid.* c. 6, § 10.

the mind of man keeps falling, irrespective of time and place. They may therefore be briefly treated. The 'False Cause' was employed by the Sophist, when he desired to show that the statement of his opponent led to an absurd result. In his own argument, however, the absurd conclusion was not due to the statement of his adversary. For although he had feigned to employ this as one of his premisses, he had joined to it other assertions; and it was from these, and not from his opponent's doctrine, that the *reductio ad impossibile* was attained. Thus, if we suppose the Sophist's opponent to have affirmed that the death-penalty for murder is just, the Sophist might argue as follows: 'The position 'leads to an absurdity: for granting that the death-'penalty for murder is just, and that punishment is to 'be held just in so far as it is efficacious as a deterrent, 'then it follows that it would be equally just to inflict 'the death-penalty for pocket-picking.' Here the original statement has nothing to do with the conclusion obtained. This follows from the principle that the justice of a punishment is measured by its efficacy as a deterrent,—a principle which is in no way connected with the statement that the death penalty for murder is just. For those who hold this, would almost certainly base it on the very different principle, that the punishment must be proportioned to the crime. The opponent would therefore reply that the Sophist had alleged his statement as the cause of the conclusion, though in fact it was not so: that he was thus guilty of the fallacy *Non causa pro causa.*

§ 15. **Many Questions.** The fallacy of 'Many questions' was rendered possible by the fact that in the earlier days of dialectic disputation, a direct affirmative or negative was required in answer to the questions proposed by that one of the two disputants, who undertook the task of confuting the positions advanced by the other. The respondent might be put into a serious difficulty if the question, though apparently single,

was in fact complex, and required two answers, one affirmative and one negative. Thus a question, such as 'Do you not spend much of your time on the useless study of Logic?' contains two members, viz.: 'Do you spend your time on the study of Logic?' 'Is not that study useless?' It cannot be answered by a single reply. If the respondent attempts to do so, his opponent has a manifest advantage. For should the answer be affirmative, he assumes that it was intended as the reply to the member, which needed the negative answer, and should he receive a negative reply, he makes a similar misapplication of that. Such questions are still effective rhetorically, and many a schoolboy has felt himself aggrieved on being asked by his master whether he has not *wasted* his whole morning on cricket. Aristotle warns his disciples that they must insist on resolving the question into its constituent parts, and replying to each separately.

* It has been frequently asserted that the seven preceding fallacies have no point in common, and that a logical error was committed in referring them to a single group. It may be freely conceded that the resemblance is of the slightest. Yet it seems that there is a real justification for holding one and all to be due to a confusion as to the thing stated (*fallaciae in re*). In the fallacy of *Accident* we take things which are merely conjoined to be identical; in *Secundum quid*, we confuse the limited with the absolute; in *Ignoratio Elenchi*, we take what is not opposed to the original thesis to be opposed to it; in *Petitio principii*, we take the same fact asserted under two forms, to be two different truths; in *Consequent*, we confuse the condition and the conditioned; in *False Cause*, we imagine what is not the reason of the conclusion to be its reason; in *Many Questions*, we accept an interrogation referring to two facts, as though it had reference to one.[1]

§ 16. **Mill's Classification of Fallacies.** It is necessary to consider the classification of fallacies proposed by J. S. Mill. This scheme is based on a different system from that of Aristotle, and embraces a wider field of errors. With a new classification, he, as a natural

[1] St. Thomas, *Opusc.* 35, c. 9.

consequence, introduced a new terminology; and this has to a greater or less extent been adopted by many subsequent writers on Logic.

The Aristotelian scheme was, as we have seen, a classification of the principal cases, in which statements not in themselves untrue, are so presented to the mind, that it is misled into forming a wrong judgment as to their significance. Moreover, it only professes to enumerate those difficulties, which are of such a character as to occasion perplexity to a man of ordinary capacity. "The dialectician," says Aristotle, "must enumerate 'the sources of apparent proofs, apparent that is, not 'to any ignoramus, but to people of average intelligence; 'for it would be an endless work to enquire into the 'sources of every idiotic belief."[1] It was further pointed out, that the subject belongs properly, not to Logic, but to the theory of discussion, which is itself one of the subsidiary methods, by which we seek to arrive at truth. Mill's view as to the place of fallacies in Logic is different, as his view of Logic itself is different. Logic, he defines, is the science of Evidence; and precisely as the scientist who investigates into the nature of health is bound to treat of disease, so it is the duty of the writer on Evidence to discuss "what are the most dangerous varieties of 'Apparent Evidence, whereby persons are misled into 'opinions for which there does not exist evidence really 'conclusive" (*Logic*, Bk. V., c. 7, § 1). It follows that the treatment of fallacies should be coextensive with the science of Logic; and proceeding on this principle, he draws up a scheme of five different classes of fallacy, viz. :—

1. *Fallacies of Simple Inspection.* These errors are antecedent to all evidence. The essential character of the fallacy lies in the acceptance of erroneous propositions as self-evident.

The remaining four classes fall under two heads—that in which the evidence is clearly conceived, but the logical process is erroneous, and that in which the evi-

[1] *Soph. Elenchi*, c. 9, (Poste's trans.).

dence is confusedly apprehended. The former of these two categories embraces the 2nd, 3rd, and 4th classes : the latter gives us but one class, the fallacies of confusion.

Classes 2 and 3 are termed Inductive fallacies.

2. *Fallacies of Observation.* In these, the error lies in not sufficiently ascertaining the facts on which the theory is grounded.

3. *Fallacies of Generalization.* This class contains (1) Illicit Inductions, and (2) False Analogies.

Deductive fallacies.

4. *Fallacies of Ratiocination.* To this class are to be referred faults against the laws both of mediate and immediate inference, together with the Aristotelian fallacies of *Accident* and *Secundum quid.*

Errors due to confused apprehension of evidence.

5. *Fallacies of Confusion.* The remaining Aristotelian fallacies.

We shall deal briefly with such of these groups as seem to call for notice.

The Fallacies of Simple Inspection are those errors where " the proposition is embraced not as proved, ' but as requiring no proof ; as a self-evident truth." Some few of the instances cited by Mill are the vulgar errors of the uninstructed or the superstitious, such as the belief in pagan Rome that words of ill omen would bring disaster. Were this all, there would scarcely appear sufficient reason for the introduction of this class. But it assumes some importance, as Mill relegates to it a considerable number of philosophic principles, with which he did not agree, as e.g. that the same effect must be produced by the same cause.

Fallacies of Observation. The claim of errors in observation to a place in Logic, stands and falls with the Empiricist view of the science. This theory, as we have several times noticed, does not distinguish between sense-perception and intellectual cognition. It follows from this, that any science of knowledge must take account of sense-perception ; and thus the Empiricist Logic is bound to find room for some consideration of

errors arising from this source. These are distinguished by Mill into fallacies of **Non-observation** and fallacies of **Mal-observation.** In regard to Non-observation, the logician must give an answer to the question, "what 'sorts of instances, or of circumstances in any given 'instance, are most likely to escape the notice of observers 'generally?" (*Logic*, Bk. V., c. 4, § 1). The most frequent source of such neglect, Mill rightly finds in preconceived opinions. It is needless to point out how, when men approach a subject with their minds strongly biassed in one direction, they become almost incapable of judging the evidence on the other side. A strong partizan in politics will scarcely hold the balance even in estimating the character of King James II. or of William of Orange. Mal-observation differs from Non-observation, inasmuch as in it the error "does not lie in the fact that something 'is unseen, but that something is seen wrong." The origin of these mistakes lies in the confusion of our perceptions with the inferences which we draw from what we have perceived. The inability to discriminate between these is greatest among those of little mental cultivation. But it probably has happened to all of us to be convinced that we have seen and recognized some person whom we know, and afterwards to have discovered that we were deceived by certain points of resemblance, perhaps in themselves not particularly important. We had in fact inferred from these points, that the person we saw was our friend.

The most important among the Fallacies of Generalization are the illicit inductions arising from simple enumeration. These have been sufficiently adverted to in Ch. 14, § 6.

False Analogy provides us with a copious source of error. Few things carry conviction to the mind so much as a striking analogy, and few things can be more misleading. Whenever a doctrine becomes widely accepted, analogies are drawn from it to support views which are concerned with matters not even remotely connected with it. Of recent years, we have analogies

drawn from the theory of Evolution for every conceivable purpose. Thus for instance, in a valuable lecture on *Currency and Coinage*, given in 1905 by Sir R. Temple, the distinguished author throughout treats the history of coinage from the point of view of evolution, and at the close, writes as follows: "In very fact English 'sixpences, French francs, and pieces of any similar money 'one can mention . . . are all like their owners them-'selves, in obedience to *the natural law of heredity*, the 'heirs of the ages." [1] Any comparison between the law of natural heredity and the history of coinage, is the merest metaphor. Hence the analogical argument,

The race of man is ever advancing in perfection.
Shillings and sixpences are like men in being subject to the law of heredity.
∴ Shillings and sixpences are ever advancing in perfection,

is from a logical point of view, almost grotesque.

As another example of false analogy, we may cite the argument by which some of those interested in abnormal mental states, have sought to explain as a hypnotic phenomenon the power possessed by certain saints of the Catholic Church to read the secret thoughts of those with whom they spoke. The explanation enabled them to represent the alleged miracles as circumstances naturally incident to the condition of persons subject to hypnotic influences. The argument takes this form :—

Some persons manifesting abnormal phenomena are hypnotic.
The saints resemble the persons aforesaid in being able to read the thoughts of others.
∴ The saints were hypnotic.

M. Henri Joly in his work, *The Psychology of the Saints*, has well pointed out that the resemblance is entirely superficial, and that in consequence the analogy is worthless. "St. Catherine of Sienna," he says, "St. Vincent Ferrer, and St. Theresa divine the thoughts, *not of those*

[1] *Lectures on the Method of Science:* edited by T. B. Strong, p. 215.

who dominate them, but of those whom they dominate. They are rightly to be compared not to the hypnotized but to the hypnotizer. But the latter, even Charcot himself, . . . are the 'divined' and not the diviners." [1]

In regard to the *Fallacies of Ratiocination* and *Fallacies of Confusion*, it seems unnecessary to add to what has already been said on the subject. The various faults, which can be committed against the rules of immediate and mediate inference, are by this time familiar to our readers. And the foregoing sections of the present chapter have, as it is hoped, sufficiently explained the nature of *Secundum quid* and *Accident* and of the various *Fallacies of Confusion.*

[1] Joly, *Psychology of the Saints*, Eng. trans., p. 68.

PART II

APPLIED LOGIC

CHAPTER XVIII.

APPLIED LOGIC AND THE LOGIC OF THOUGHT.

§ 1. **Science and Philosophy.** In order to understand the precise meaning of the term 'Applied Logic,' and the connexion between the branch of knowledge thus designated, and the Logic of Thought, it will be necessary to explain with somewhat more detail than seemed advisable in Ch. I., the relation between the Logic of Thought and the sciences of the real order. The first three sections of the present chapter are devoted to this subject. In the present section, we treat of the distinction between Science and Philosophy, and of their mutual relations.

What is a science? We may define a science as **an organized body of truth regarding some special object of thought.**[1] By an object of thought, we do not signify a concrete individual thing. The various and manifold aspects of one and the same thing, which the mind can distinguish the one from the other, constitute so many different objects of thought. Thus, under its various aspects, the same living plant is the object of different sciences. It is studied by the chemist in regard to the organic products resulting from it; by the physiologist in regard to its vital processes: the botanist considers its structure, and its classification by genus and species. Wherever the mind can abstract an aspect of the know-

[1] "When we speak of 'the Sciences,' we mean what is sometimes more 'definitely expressed as 'the special sciences'—a group of organized bodies 'of general knowledge, each concerned with some aspect of the knowable 'world." H. Sidgwick, *Philosophy, its Scope and Relations*, p. 4.

able world, whose properties, principles, and causes provide an organized body of knowledge, there we have a science.[1]

It will be observed that scientific knowledge, in virtue of the fact that it is knowledge of the type considered in abstraction from the individual, is always universal. We consider, e.g. the characteristics of sulphur and of radium, of the lily and of the date-palm *in general*: we are not concerned with the individual peculiarities of the specimen under examination, save in so far as it illustrates a general law. *Scientia est de universalibus* was a dictum of the ancients. And Mr. Sidgwick expresses the same thought as follows: "To get a definition of science . . . we must, I think, take the characteristic of 'generality' as the essential distinction between scientific knowledge and merely 'historical' knowledge of particular facts. . . . It is true that we largely regard knowledge of particular facts—e.g. of the discovery of a new planet—as scientific knowledge: but only, I think, in view of its relation to general knowledge" (*op. cit.* p. 8).

Now even had every special aspect of things been made the object of a special science, the work of the mind would still be incomplete. A great province of knowledge would still be untouched. There are laws and principles, which relate not to some restricted area of the knowable, but which embrace in their reference the objects of all the special sciences. The mind can view the objects of all the sciences in common, can frame a system of the knowledge which relates to them as thus considered, and can thus attain to a universal science. It is this universal science which we term **Philosophy.**[2]

The special sciences depend for their very being on the universal science. It provides the foundations on which they rest. They can, it is true, develop many details within their own provinces, organizing knowledge

[1] Arist. *An. Post. I.*, c. 10, § 4.

[2] The word 'Philosophy' is used in various significations. The meaning here assigned is the traditional, and still, we believe, the most usual. For a somewhat different account see Erdmann, *History of Philosophy*, Introd.

and involving the facts in a rational system. They carry us far beyond the mere rule of common-sense. But they must postulate—for it is beyond their province to explain—many facts and principles, a denial or false explanation of which would ruin their whole system. In order even to make a commencement, they must accept as objectively real such facts as, e.g. the movement of bodies in space, their duration in time, the power of one material thing to act upon another, the unity of a substance notwithstanding its multiplicity of parts, the principles of mathematics, the law of causality. Thus the intellect finds itself forced to seek for a science having for its object these facts of universal import. This science is Philosophy. It is the part of Philosophy to deal with the universal conditions of things, and so to explain them that the account which it affords of them is consistent with the facts of experience. When it does this, the validity of the special sciences is vindicated. When in lieu of explaining them, it explains them away, the special sciences collapse. No science could lay claim to be true, if the principles of mathematics, or the objective reality of local motion or of temporal duration, were denied. Not one of its conclusions would possess any validity. The sciences can no more divorce themselves from Philosophy than they can from common-sense or experience. In the latter case they would have no data, in the former they would have no justification.

Philosophy then is **the science which treats of the principles and the characteristics which belong to the universe as a whole.**[1]

[1] Cf. Arist., *Met.*, I., c. 1. *τὴν ὀνομαζομένην σοφίαν περὶ τὰ πρῶτα αἴτια καὶ τὰς ἀρχὰς ὑπολαμβάνουσι πάντες.* 'All regard philosophy as the science, 'which treats of primary causes and principles.' St. Thomas, commenting on this passage of the Metaphysics, says, 'Sapientia est scientia quae considerat 'primas et universales causas.' Mr. Sidgwick expresses himself to the same effect, when he says (*op. cit.* p. 38), "As to Rational Theology, it seems to me 'that the questions with which it deals . . . are *primâ facie* philosophical 'questions . . . i.e. *they belong to the contemplation of the universe as a whole.*" Elsewhere, however, he explains philosophy somewhat differently.

Among recent philosophers, it is not uncommon to find Science distin-

* **The character of scientific knowledge.** We have in the present paragraph already noted one feature of scientific knowledge, viz.: its generality. Two other characteristics distinguish it from the majority of the assents we give. The assents of true Science are *certain*. At the present day, indeed, the term "science" is often used to signify the organized body of facts experimentally known regarding some aspect of reality, as expressed in terms of the theory most generally accepted by those who are experts in the matter in question. Thus, the body of truths known about the phenomena of light, when expressed in terms of the undulatory theory, is called the science of light. Such an expression of the facts is of course provisional: it lacks the full certainty of science strictly so called. There is no need to quarrel with the terminology, which has extended the signification of the word 'science' to this case also. But it should be noted that we have not here science in the fullest meaning of the term.

Science as defined by the ancients, was further the knowledge of things *by their causes—cognitio certa rei per propriam causam*. This demand has given occasion to much adverse comment. It has been asserted that the Schoolmen admitted no knowledge as scientific, except such truths as were capable of *a priori* demonstration. Some of the shallower spirits may perhaps have expressed themselves to this effect. But there is no reason whatever to attribute such an opinion to Aristotle or St. Thomas. It rests on an altogether too narrow interpretation of the word 'cause.' As we have seen, that term includes in its meaning all the four causes enumerated by Aristotle. Thus understood, such knowledge, as e.g. that sulphuric acid consists of hydrogen, oxygen, and sulphur, is scientific knowledge of its nature, though very incomplete and capable of immense extension. Indeed

guished from Philosophy, on the ground that Science is concerned with Appearance alone, Philosophy or Metaphysics with the Reality, which lies behind appearances. This distinction we owe to Kant, and by many writers it is accepted without question. There are however signs of a return to the traditional, and, as we hold, saner view. Thus, Mr. Sidgwick (*op. cit.* p. 93) declines to distinguish Physics from Metaphysics on this basis, "be-'cause Physics regards its object as real, and itself requires the distinction 'between Reality and Appearance." And somewhat similarly, Professor Pringle-Pattison writes, 'It is hardly possible to open a scientific or semi-'philosophical work, without meeting the complacent admission that our 'knowledge is 'only of phenomena.' Or the writer tells us that the science 'in question, so far as he is concerned, treats only of phenomena, the con-'sideration of the corresponding noumena being relegated to philosophy or 'metaphysics. . . . [These writers] may rest assured that the best result 'of this contemned metaphysics in modern times has been just this—to ex-'plode the conception of such duplicate entities as they still cannot help 'half believing in, and to repudiate in consequence the brand of that 'only' 'before phenomena' (*Scottish Philosophy*, pp. 175, 176).

it has been well observed, that according to this definition, science begins as soon as we have the generalized notion of the object.[1] For, as we have pointed out in Ch. 10, § 2, the generalized notion, i.e. definition of an object, is, logically considered, its formal cause.

§ 2. **The Subdivisions of Philosophy.** We have already explained (Ch. 1, § 3) that the Scholastic philosophers divided the sciences into the speculative sciences (*scientiae speculativae*) on the one hand, and the regulative or normative sciences (*scientiae practicae*) on the other. In the speculative sciences, our object is to know the order of things in the universe, as it offers itself to our contemplation. The normative sciences are concerned, not with the knowledge of an already existing order, but with the production of an order we desire to see realized. This order may be in our own acts, or in external things. The sciences, which deal with the production of order in our actions, are Logic and Ethics. Logic treats of order in the acts of the intellect; Ethics of order in acts of the will. Both of these sciences are rightly regarded as branches of philosophy; since, as we shall point out in the next section, both possess that note of supremacy, which is philosophy's distinctive characteristic.

In the present section, our purpose is to deal more

[1] The following passage from a little work, entitled *Qu'est-ce que la Science?* (Paris, 1906), by L. Baille, professor at the Leonine University of Anagni, throws light on this point; and further indicates with considerable insight, how it has come about that the acceptation of the term 'science' has varied so much at different periods. "En vertu même de la définition qu'ils [Aristote et S. Thomas] nous ont léguée, dès qu'une proposition s'appuie solidement sur l'observation en la dépassant par la généralisation, ou sur l'intuition généralisatrice en l'appliquant, elle commence à être scientifique: mais elle n'est jamais le dernier mot à dire sur le sujet. . . .

Le concept de science a toujours été, pour cette philosophie, un concept *analogique*, et par suite infiniment souple malgré l'apparente précision de sa définition, susceptible de développements variables, opposés même, et pourtant—et c'est là le mérite singulier de cette définition—toujours en continuité parfaite avec leur commune origine.

Il arrive toutefois qu'en poursuivant trop exclusivement l'un de ces développements, applicable seulement a l'une des branches de la science, on perd de vue l'origine de la notion et du concept primitif. . . . De l'arbre où l'on est monté on ne voit plus que la branche où l'on est, ou bien on regarde les autres rameaux comme des étrangers." *Op. cit.* p. 63.

particularly with the threefold division of speculative philosophy, which we owe to the profound insight of Aristotle.[1]

All science, as we have seen, considers primarily, not the individual, but the general type—the universal nature, which our intellect abstracts from the singulars, in which it is realized. By this, we do not intend to assert that science is concerned only with our abstract concepts, and not with things. This would be to fall into the error of the Conceptualists. Science is concerned with things, but solely in so far as the universal type is found in them.[2] If the scientist devotes himself to the study of the phenomena manifested by a particular electrical machine, or the motions of a particular spinning-top, it is that he may discover laws of universal import. Nor is his knowledge of the individual fact scientific knowledge, save in so far as he, to use St. Thomas's expression, 'applies' the abstracted universal to the particular case. There is not, nor can there be, a science of particulars *as such*; for the particular is in perpetual flux. It is ever changing. We make some statement about it; and before the words are well out of our mouth, they are no longer true. It was this, that led Plato to suppose his world of super-sensible entities, which were, he held, the true object of all scientific knowledge. The keener insight of Aristotle perceived that the solution of the difficulty lay in another direction. The object of science was not to be sought for in the super-sensible world of Ideas, but in those universal and stable types, which our intellect abstracts from the

[1] See Arist., *Met.*, V., c. 1. In this chapter Aristotle discusses the place of Metaphysics in relation to the other sciences.

[2] Cf. St. Thomas, *Opusc.* 63, *in lib. Boethii de Trinitate*, Q. 5, Art. 2, ad. 4. 'Scientia est de aliquo dupliciter. Uno modo primo et principaliter et sic scientia est de universalibus rationibus super quas fundatur. Alio modo est de aliquibus secundario, et quasi per reflexionem quandam: et sic de rebus illis est quarum sunt illae rationes, in quantum rationes illas applicat ad res etiam particulares quarum sunt, adminiculo inferiorum virium.'

The 'application' of the universal to the individual, which is here spoken of, is of course in the conceptual order. When we affirm 'Socrates is a man,' we mentally 'apply' the form 'humanity' to the individual Socrates.

ever-changing singulars, and in the singulars themselves so far as the type is realized in them.[1]

Abstraction is the basis of science, and to the different grades of abstraction must correspond distinct sciences. There are, as Aristotle has shewn us, three such grades. We may simply abstract from the individuating conditions. Such is the abstraction we employ in all the special sciences dealing with the material world. This abstraction, moreover, is the foundation of the science, which considers the general nature of material substance subject to motion and change. This science, whose object is the totality of material substances, may well be termed Natural Philosophy. The ancients, following Aristotle, called it **Physics.**

A further abstraction enables us to prescind altogether from those sensible qualities which are the conditions of all change and mutability, and to consider corporeal substance solely as the subject of extended quantity. The science which treats of bodies purely in so far as endowed with quantity, is **Mathematics.**[2]

[1] The following passage will show that this ancient difficulty is with us yet: "In strict fact nothing ever is; everything *becomes*, and turns our most conscientious predications into falsehoods. The real is here, there and everywhere, until we stop breathless in the chase, and point gasping. The 'eternal truths' unable to sustain the pace have long since ceased to reside with us, and have gone down or up, (one really cannot be precise about directions in these Copernican days,) into the *τόπος νοητός*, where it is possible to preserve one's dignity without doing any work." F. C. Schiller, *Axioms as Postulates*, § 34.

Mr. Schiller's lively and agreeable style should not blind us to the fact that he is drawing us back into a quagmire, from which Aristotle delivered philosophic thought more than two thousand years ago. The error contained here is identical with that which Heraclitus taught. Even in the early days of Greek philosophy, it was quickly recognized that such a doctrine does not account for the facts.

[2] It is not uncommon in the works of non-scholastic writers to find the assertion that Mathematics deals with merely mental creations, and not with the extended bodies of the real world. It is urged that the perfect circle and the perfect square, which are the objects of Geometry, are mere figments of the understanding to which nothing corresponds. To adopt this view is to introduce into science an error of the most serious and far-reaching kind. Geometry no less than the physical sciences deals with the real world. The solution of the difficulty mentioned, lies in the fact that the intellect in forming its concepts, finds its object not in the concrete individual—the object of sense-perception—but in the *type*. The intellect grasps *the principle of order and harmony* realized, however imperfectly, in the individual. The realization of the type is conditioned by the material receiving it,—a truth expressed by the Scholastics in the phrase *Quidquid recipitur, recipitur secundum modum*

A third degree of abstraction yet remains. By this we abstract from matter altogether, and consider those universal notions and principles, which dominate all Being; these must be true as well of the spiritual as of the material universe, since they hold good of things, not in so far as they are material bodies, nor yet in so far as they are endowed with quantity, but because they are *things*. This highest degree of abstraction gives us the science of *Ontology* or **Metaphysics.** It is this science whose function it is to treat of the principles of Contradiction and Causality, of the meaning of Being, Unity, Perfection, Substance, Accident, Potentiality, Actuality, and other such notions of absolutely universal reference. This science alone enables us to ascend by reason as distinct from revelation, from the contemplation of the visible universe to some knowledge of the First Cause, from whom all proceeds, from the creature to the Creator. Hence it was not undeservedly termed by Aristotle and his followers, the science of *Theology*.[1]

recipientis. Thus the realization of geometrical forms is, as far as we know, always more or less imperfect, according to the nature of the material. A plane surface as shewn on a black-board is very remote from the ideal. In polished metal the imperfection is in some measure removed: but perfection we cannot attain. Yet the intellect in the act of abstraction, seizes the type. Hence Geometry deals not with figments, but with the real world: and a proposition, e.g. in Trigonometry, gives us the real height of a real building.

A word may be added on the subject of discrete (arithmetical) quantity. This is reached by a similar process of abstraction. Arithmetical number is obtained by the division of continuous quantity into equal parts. The unit is a portion of continuous quantity viewed as an undivided whole: and number is produced by the repetition of the unit. Were the arithmetical unit not of this nature, there could be no such things as fractions. The expression e.g. '123 is intelligible as applied to a line or other continuum, but not so if applied to a quality or relation. On this subject see *Summa Totius Logicae*, Tract 3, c. 1.

[1] 'Quaedam vero sunt speculabilia, quae non dependent a materia secundum esse, quia sine materia esse possunt: sive nunquam sint in materia sicut Deus et Angelus, sive in quibusdam sint in materia, et in quibusdam non, ut substantia, qualitas, potentia et actus, unum et multa et hujusmodi: de quibus omnibus est *Theologia*, id est, divina scientia, quia praecipuum cognitorum in ea est Deus. Alio nomine dicitur *Metaphysica*, id est transphysica, quia post Physicam discenda occurrit nobis, quibus ex sensibilibus competit in insensibilia devenire. Dicitur etiam *Philosophia prima* in quantum scientiae aliae ab ea principia sua accipientes eam sequuntur.' St. Thomas, *Opusc.* 63, *in lib. Boethii de Trin.* Q. 5, Art. 1. It is a disputed point whether the name 'Metaphysics' was given for the reason here assigned, or simply because Aristotle's work of that name came next to his treatise on Physics.

In this hierarchy of sciences, the inferior are, as we pointed out in the preceding section, essentially dependent for their justification on the higher and more abstract. If the value of the higher science be impugned, nothing can save the lower. The conclusions of Mathematics are worthless, if Metaphysics cannot defend the principle of contradiction; and without the law of causality, it is idle to argue in the special sciences from effects to their causes. The long scientific calculations relative to the phenomena of heat and light, are a mere intellectual amusement, if the scientist be still doubtful whether the truths of Mathematics have reference to objective reality, or are fictions of the mind. And if Physics cannot establish the validity of the laws of motion, it is of no avail to look for certainty in the conclusions of Astronomy.

* In this paragraph, we have reckoned Logic with Ethics as a regulative science, in this following the division given by St. Thomas in *Ethics I.*, l. 1 (see p. 5, note). This arrangement differs slightly from that of Aristotle. He did not reckon Logic (*τὰ ἀναλυτικά*) as a science on a level with the other sciences, but as an introduction to, and an instrument of the sciences. His reasons are evident. That acquirement, which is requisite to the right ordering of all those organized bodies of knowledge, which we term sciences, seems to stand on such a different footing from them, that it might easily appear best to relegate it to a different category. Moreover, there was no place for it in Aristotle's famous division of the sciences into 'speculative,' 'practical' and 'productive.' For by 'practical,' he did not denote precisely what we have signified by the more general term 'regulative.' The function of a 'practical' science is to determine the *will* to choose the right course. The will being able to choose either the right or the wrong, it is essential to us that it should be guided by adequate knowledge. "In the practical sciences," he says, "the principle of action is the choice of the will (*ἡ προαίρεσις*)."[1] Logic provides us with rules; but it is not the choice of the will that is determined by them. As soon as we grasp them, the intellect spontaneously obeys them. Hence the Peripatetic school regarded Logic not as a constitutive part of philosophy, but as the instrument, by which we are able to attain science. Accordingly they gave to Aristotle's logical treatises the name of the **Organon** or **Instrument.**[2]

[1] *Met.*, V., c. 1, § 5.

[2] Ammonius the son of Hermias, in his comment on the Categories, tells us that the Peripatetic and Stoic schools were distinguished by the place

§ 3. **Logic and Metaphysics.** At the commencement of this work, two definitions of Logic were offered. It was explained to be *the science which directs the operations of the mind in the attainment of truth*, and also as *the science of the conceptual representation of the real order*. The former definition indicates its scope as the practical science, which regulates the operations of the intellect. The latter declares the subject-matter, with which it is conversant. It would be erroneous to term it, 'the science of the operations of the mind': for it does not in fact consider the actual operations—the processes of conceiving, judging, and reasoning,—but the result of these processes. It considers things as expressed in concepts and in judgments, and as made known to us in conclusions resulting from premisses. The subject of the science is, as we have seen in the course of the foregoing pages, the characteristics which things possess as thus expressed. It views things as possessed of those properties, in virtue of which we term them subject, predicate, genus, species, logical part, middle-term etc., etc.; and it is through the knowledge it gives us of reality thus expressed, that it enables us to tread the difficult path of reasoning securely. The characteristics, which things possess solely in virtue of their mental expression, were termed by the ancient logicians Logical Entities (*entia rationis*) or Second Intentions (Ch. 2, § 11). The former name contrasted them with Real Entities (*entia realia*), by which name were signified the actual substance and all those attributes, which it possesses in the order of existence. In these writers, Logic is frequently defined simply as the science which treats of the *ens rationis*.

When Logic is seen to be the science, which treats of

assigned to Logic in their respective systems. The Stoics held it to be a part of Philosophy, the Peripatetics regarded it as the instrument by which we reach it. Trendelenburg, *Elem. Log. Ar.*, p. 48.

St. Augustine *De Civ. Dei*, viii. reckons it as one of the speculative sciences. On this St. Thomas says, "Logica non continetur sub philosophia speculativa quasi principalis pars, sed quasi quoddam reductum ad eam, prout ministrat speculationi sua instrumenta." *Opusc.* 63, Q. 5, Art. 1, ad. 2.

Cf. also Boethius, *Comment. in Porph.*, Lib I. (Migne, P. L. t. 64, col. 74).

things as they exist in the conceptual order, it at once appears that there is a parallelism between Metaphysics and Logic. They are the universal sciences. Metaphysics is the universal science of the real order, the science which investigates principles of universal application, and the attributes common to all things which *are*. Logic is no less universal in its reference. For the range of the intellect is unlimited, and whatever can exist, can also be an object of thought, and can thus appear vested with the characteristics of the conceptual order. As Metaphysics is the universal science of the real order, so Logic is the universal science of the conceptual order: it deals with those conditions which are common to all things, in so far as they *are thought*.[1]

The very fact that the provinces of these two sciences are so different, and yet so closely parallel, renders it essential for all who enter on their study to be on their guard lest they should suppose that properties, which belong only to the thing as thought, must belong to it as it exists in the real order; and conversely, lest, because existence in the real order involves certain definite conditions, they hold that things must needs exist under the same conditions in the conceptual order. The history of philosophy is full of errors arising from this confusion of the real and the logical. Plato, recognizing the universality of the nature or essence as it is represented, affirmed the real existence of universal natures. The Empiricists, taking their stand on the fact that in the real order nothing can exist which is not individual, denied that even in thought one and the same nature could be repeated in a multiplicity of subjects. Kant, seeing that we possess our knowledge of things through

[1] Cf. St. Thomas, *Opusc.* 39, *De Natura Generis*, c. 4. "Sciendum est ergo quod sicut in 4 Metaph. dicitur, Logicus et Metaphysicus circa omnia operantur, differenter tamen. Sicut enim Primi Philosophi est loqui de Ente in communi, ita et Logici. . . . Ens dupliciter dicitur scilicet naturae et rationis. Ens autem rationis proprie dicitur de illis intentionibus quas ratio in rebus adinvenit: sicut est intentio generis et speciei quae non inveniuntur in rerum natura, sed sequuntur actiones intellectus et rationis: et hujusmodi ens est subjectum Logicae: et illud ens aequiparatur enti naturae, quia nihil est in rerum natura de quo ratio non negotietur." Cf. *in Met.*, IV., lect. 4; *in An. Post. I.*, lect. 20.

udgments, in which the predicate notion expresses the nature of the thing, taught that it is the act of judgment itself, which confers on the phenomenon the nature we attribute to it. That thinkers such as these confound one order with another, is a sufficient testimony to the difficulty involved in an accurate analysis of the mental processes in their relation to the real, and to the marvellous acumen of the great Stagirite who laid so securely the foundations both of Logic and Metaphysics.

If it is the distinctive mark of Philosophy to be supreme in some department, the claim of Logic to be designated a branch of Philosophy is sufficiently manifest. As soon as it is seen to be the science which deals with the representation of things in an intelligent subject, it is evident that it is not one of those sciences which are restricted in their range to some special sphere of being, but that it is concerned with attributes which belong to all things, both real and possible. The universality of Ethics is closely analogous to that of Logic. Here too we have to deal with an order of things, totally different from that with which we are concerned in the sciences of the real. The science which treats of the moral obligations of a free self-determining agent, cannot be brought into the same category as they. Ethics, in dealing with these obligations, deals with them not in regard of any particular circumstances, but as general laws of universal reference, valid in all circumstances in which the agent may be placed. It also therefore is rightly termed philosophy.

§ 4. **The Breach with the Past.** It must have seemed to philosophers in the golden age of Scholasticism, that the analysis of the sciences into their main divisions, was a permanent achievement of the human mind—a step not to be retraced. Yet, as all are well aware, between that age and this, a revolution has taken place in European thought. In all the great seats of learning, save where the influence of the Catholic Church is preponderant, the Aristotelian system no longer finds

acceptance. It might indeed appear at first sight, that the science of Logic had survived even in those places, where the rest of the philosophy had been forsaken: for to a large extent the form of the science popular in England and Germany during the nineteenth century, employs the same terminology, and treats of the same matters as the historical form has ever done from Aristotle through Aquinas, Scotus, Suarez to the modern Scholastics. But the similarity is deceptive. Logic is but one member of a whole, no part of which can stand without the others. The rise of the modern sects in philosophy was inevitably followed by a new theory of reasoning in which every feature of the ancient science was altered beyond recognition. Hence it seems advisable to say a little in regard to the history of the great change, in order to explain how it has come about that the subjects treated of in this second portion of our work, are by many now regarded as part of the same science, which treats of the concept, the judgment, and the syllogism.

The chief cause of the downfall of Scholasticism, is to be found in the intellectual movement of the Renaissance period. With the revival of classical studies, and the fresh interest aroused in the civilizations of Greece and Rome, a new set of subjects began to occupy men's minds. They were captivated by the beauties of the ancient literature, and by the living interest of the works now first placed at their command. In their enthusiasm for purity of style, they had nothing but contempt for the Schoolmen and their mediæval Latinity, which had been produced to meet the philosophers' need of a language at once clear, flexible and competent to express the finer operations and results of thought. And the scholastic discussions which emulous students too often reduced to mere word-fencing, seemed far less attractive than the myths of Plato.

While the zealots of the New Learning adopted an attitude of aggressive antagonism to the old order, many of the Scholastic doctors showed themselves equally

unconciliatory. They did not imitate their great predecessors of the thirteenth century, who when called to face the intellectual ferment occasioned by the introduction of the Arabian philosophy, had resolutely adopted all that was good in the novel ideas. They regarded the New Learning in the light of a connatural foe, and by their hostility to all innovation, drove many men of ability towards the opposite camp.

Amongst causes tending still further to weaken the position of Scholasticism two others call for mention. In the sixteenth century its defenders became too narrowly occupied with the traditional Metaphysics and Theology to the exclusion of other subjects. The earlier Scholastics had by no means neglected the physical sciences. The discoveries of Galileo owed a great debt to the pioneer work of Ockham and Buridanus in this direction. The classical revival on the other hand contributed nothing to the cause of physical investigation: its devotees had no interest to spare for such matters. Yet owing to the reaction occasioned by it, it came about that the majority of Scholastic writers cared for nothing save for the old paths. And a school which has ceased to press forward into new fields is bound to become decadent and ultimately to succumb. The second influence of which we have spoken, was the popularity enjoyed in the mediæval schools by Conceptualism.[1] The legitimate outcome of this doctrine is Scepticism. Men trained in a Scholasticism which offered them such a philosophy as this, might well think that such a cause was hardly worth defending.

When the beginning of the seventeenth century was reached, men scarcely knew what to believe. Some philosophy men must have: but in the northern universities the credit of Scholasticism, for the time at least, was gone. All confidence in it was lost.

Eventually, in the course of that century, two currents of thought arose, to one or other of which we may assign almost every thinker who has, since that period, exercised

[1] In 1425 the heads of the university of Cologne were called before the Elector to answer to the charge that they continued to teach the old-fashioned views of St. Thomas, to the neglect of the more modern Conceptualism.

commanding influence in Europe. These were Idealism and Empiricism. Idealism had its origin in Descartes (1596–1650). Though not himself an Idealist in the full sense, his system was based on suppositions which inevitably led to that doctrine. The philosophy of Locke (1632–1704) gave rise to the Empiricist school. Neither of these two doctrines is compatible with any theory of Logic, in the sense in which Logic was understood by the Aristotelian writers. With them, Logic was the science of the conceptual representation of the real. But Idealism denies the existence of the real, as Empiricism does that of the conceptual order. We cannot have a theory of the mode in which the mind represents the real order, if there be no reality outside thought. And such a science is equally impossible, if concepts are a figment, and our only form of knowledge is the perception by sense of concrete singulars. In the hands of writers belonging to either of these schools, the ancient science of Logic could hardly avoid being roughly handled. It was inevitable that its boundaries would become very uncertain, that, on the one hand, parts of it would be discarded as meaningless, and on the other, that to it would be assigned the treatment of topics, which the Scholastics would not reckon as falling within its province.

With Logic, as interpreted in an Idealist sense, we are not now concerned. What is called Applied Logic owes its incorporation into the treatises of the present day as an integral part of the science, to the influence of Mill, an empiricist. Though the subject is dealt with in idealist works, yet its treatment there is due to the influence exerted by Mill in establishing traditional limits for the science. Yet before dealing with Mill's theory, it is necessary to say something in regard to the views of Francis Bacon (1561–1626). For although, as Professor Minto has well said, it was by Mill that the attempt was first made "to incorporate scientific method with 'Logic, and add it as a new wing to the Aristotelian 'building," yet it was in the *Novum Organum* of Bacon

that this undertaking was suggested. That work has always been regarded as a landmark in the story of the revolt from Scholasticism. By its very title, the writer challenged comparison for his book with the *Organon* of Aristotle, and threw into strong relief his conviction that the Logic, which ever since the days of Cicero had been the groundwork of a liberal education, was of but little worth, and that the time had come to supersede it by something better.

§ 5. **Bacon.** Francis Bacon lived at the very time when the chaos of philosophical opinion was most complete. Endowed with an intelligence naturally both inquisitive and capacious, he turned it to every kind of problem both philosophical and scientific, and believed himself capable of solving all. In the fifth book of his work *De Augmentis Scientiarum* he exposes a completely new system of Logic. But by Logic he signifies something very different from the science his predecessors had known by that name, as may be gathered from the fact that he reckons Grammar, Rhetoric and the Art of Memory as parts of Logic. The Scholastic Logic he holds to be entirely useless ; he blames it especially for recognizing no other Induction save that by simple enumeration. This defect he undertakes to remedy by setting forth a new Inductive Method. For this method he makes the loftiest claims. Its discovery would, he averred, do for science what the discovery of the compass had done for navigation. Its efficacy was such, that it would put the intelligence of all men on an equality. Of this new science he does not treat in the *De Augmentis* : it was reserved for another volume, the famous *Novum Organum*.

The value of Bacon's Induction is intimately bound up with certain philosophical views of his own. He believed every substance to be an aggregate of *simple natures*. Thus, gold unites the following natures :—it is heavy, yellow, malleable to a certain extent, etc., etc. Each of these simple natures depends, he holds, on a

certain special disposition of the component particles. The disposition of the particles may thus be regarded as the constitutive principle of a nature, and is termed the *Form* of that nature. The aim of Induction is the discovery of these 'Forms.' When we know wherein the 'Form' consists, we shall be able to induce the simple nature corresponding to it on other substances. Thus an acquaintance with the 'Forms' of yellowness, heaviness, etc., will enable us to unite them in another substance, and so transform it into gold (II. v.).

In the *Novum Organum* Bacon prescribes what he conceives to be the most suitable method for directing our observations and our experiments to the discovery of the 'Form.' This is his Inductive Method. The first step is to draw up various tables containing respectively (*a*) instances of the presence of the nature which is the object of inquiry, (*b*) instances of its absence, (*c*) instances in which it is present in varying degrees. The method then proceeds by a series of exclusions. Thus if we are seeking to find the Form e.g. of 'yellowness,' and it be suggested that some particular internal character *x* be it, we forthwith examine our tables, and should we find any case in which *x* occurs in the absence of yellowness or in which a yellow object exists without *x*, we know that *x* may be excluded from the list of alternatives. Finally all forms but one are rejected, and we know that this is the object of our search.

In regard to this method, we may say first, that it has never been found of any service. It has been fruitful of no discoveries. No man of science has ever used it. Secondly, it is not in any way, as Bacon maintained it was, a new logical process. The method is little more than a direction to employ elimination in the discovery of laws of nature. Logically the process is the old disjunctive syllogism :—

The *form* in question is *a* or *b* or *c*.
It is not *a* or *b*.
∴ It is *c*.

The *Novum Organum* is not, in fact, a logical treatise

on Induction at all. There is in it no attempt to deal with the logical explanation of the mental act by which we pass from the individual instances to the universal principle.

Crucial Instance. This term employed by Bacon in describing his method, has acquired sufficient note to claim separate explanation. In the process of exclusion by which the Form is discovered, the following case, he says, will occasionally occur. Our choice will lie between two *Forms* only, to one or other of which the nature must be attributed. In these circumstances we may be fortunate enough to find an instance which absolutely excludes one alternative, thus establishing the other. This he calls an *Instantia Crucis*, taking the metaphor from the cross-posts, which stand where two roads meet (*Nov. Org.*, II. 36). The expression has passed into the language, and when an experiment shows one of two hypotheses to be false and thus establishes the truth of the other, it is termed a *Crucial Experiment.*

§ 6. **Mill.** Mill took up the suggestion contained in the *Novum Organum*. In his hands, Logic became a science having for its primary object the proof of natural laws. Like the older logicians, he tells us indeed, that Logic is concerned with 'the attainment of Truth' (*Exam.*, p. 397); but he makes use of this expression in a sense quite other than theirs. In his view, it contributes to this end, not because it deals with the general conditions of all intellectual representation, but because it prescribes the methods by which we obtain evidence for the laws of nature. The formal laws of the traditional Logic are indeed admitted into the science, but in a purely subordinate position. Their office is to secure us against inconsistency and self-contradiction. In this way, they fulfil a subsidiary, but not unimportant function in the search for truth.

This theory of Logic is clearly laid down in the *Examination of Hamilton*. "The doctrine [of Sir W. Hamilton 'assumes that with the exception of the rules of Formal, 'that is, of Syllogistic Logic, no other rules can be framed, 'which are applicable to thought generally abstractedly 'from particular matter, . . . that the problem which 'Bacon set before himself, and led the way towards

'resolving is an impossible one . . . that the study of 'nature, the search for objective truth, does not admit 'of any rules, nor its attainment of any general test '(p. 400)." And a little further on, Mill terms this enquiry "a Philosophy of Evidence and of the Investi-'gation of Nature," and says of it, that if such a theory be possible, "this must be Logic κατ' ἐξοχήν, and any-'thing else called by the name, can be only ancillary to 'it."

It is true that in the introductory chapter to his *Logic*, his account of the subject with which he is about to deal, is more in harmony with the traditional view. But though this be so, yet in the work itself, we find the science treated, not in accordance with the sketch there given, but with the theory set forth in the *Examination of Hamilton*, which renders it abundantly clear that it was thus in fact that he understood its scope. Professor Carveth Read well says, "It appears to me that the 'subject of those immortal volumes, is not the operations 'of the mind, but primarily the Laws of Nature and 'their Proof. . . . The highest Laws are the Axiom of 'the Syllogism, the Law of Causation with its derivative 'Canons of Experiment, the theory of Probabilities 'and perhaps the doctrine of kinds: all of which are 'plainly conceived by Mill to be Laws of Nature. Then 'in the First and Fourth Books there is much discussion 'of matters subsidiary to the discovery and proof of 'Laws, such as Names and Naming, Definition, Classifi-'cation, etc.: and here again facts and the order of 'Nature are the chief concern." [1]

It will be plain to those who have followed the foregoing paragraphs, that Mill was here combining two things, which are absolutely incompatible, namely, a science of the investigation of nature—a branch of knowledge which relates to the real order,—and the rules of the traditional Logic, which concern the conceptual order. No one can be blind to the importance of the subject with which Mill was principally concerned, namely

[1] Carveth Read, *Theory of Logic*, p. 7.

the methodology of the sciences. The subject was one which called for treatment, and had hitherto received inadequate attention at the hands of philosophers. His contributions to it were very great, and his work will probably long rank as a classic. His error lay in regarding this subject as one and the same science with the theory of our mental processes.

It may possibly be asked of us, to what science could we on the principles of Scholastic philosophy refer the general theory of the methods of Science. To this question, we reply that there is and can be no such general theory of investigation, prescribing the method to be followed in all Science. Each science proceeds upon the method which its peculiar subject-matter demands. Indeed, though Mill speaks of such a theory of scientific method as one, yet he can only do so by regarding it as concerned solely with the search for Physical Law. In his *Logic*, he devotes a separate treatise to the Logic of the Moral Sciences. And in the work of his disciple Bain, we have the Logic of Chemistry, the Logic of Medicine, the Logic of Mathematics. All this has no concern with Logic properly so called. The outline of the method to be followed in the various sciences, belongs in each case to the science to which it refers, and should be incorporated with it, not with Logic.[1]

Reference has been made in Ch. 1 to the novelty, entitled Symbolic Logic, in which mathematical methods are applied to terms and propositions. If the analysis of the sciences, which we have given, be correct, the pursuit can lead to no result of any value. For it consists in nothing else than in the application of principles belonging peculiarly to one science, to obtain the solution of problems belonging to another. Of the Symbolic Logician, as of the alchemist in *Faust*, it may be said that,

> He fused and fused by rule and recipé
> Things which by nature are antagonistic.

[1] Cf. S. Thomas, in *Met.*, II., lect. 5. Logica tradit communem modum procedendi in omnibus aliis scientiis. Modus autem proprius singularium scientiarum, in scientiis singulis circa principium tradi debet.

[2] 'Der in Gesellschaft von Adepten
Sich in die schwarze Küche schloss,
Und nach unendlichen Recepten
Das Widrige zusammengoss.' *Faust*, Act 1, Sc. 1.

Indeed there was a slight chance that an alchemist would obtain some solid result from his labours, though not perhaps in the direction in which he looked for it. It is difficult to see that Symbolic Logic can lead to anything. The root of the evil lies in the confusion on the part of many men of great ability as to the true character both of Mathematics and Logic. Thus, e.g. Professor Whetham writes in *Recent Developments of Physical Science*, p. 33, "The science of Mathematics has nothing to do 'with natural phenomena. . . . The mathematician lives in a 'purely conceptual sphere, and Mathematics is but the higher 'development of Symbolic Logic."

CHAPTER XIX.

OBSERVATION AND EXPERIMENT.

§ 1. **The Function of Observation and Experiment.** As we have already pointed out in the preceding chapter, we are now concerned, not with a science of the conceptual order, but with those of the real. We are not considering what it is that justifies us in passing from the premisses of a syllogism to its conclusion, or from the experience of one or two particular facts to a universal judgment. All that belongs to the sphere of Logic proper. We consider, with special reference to the subject we are studying, how we are to obtain our data, and how we must apply to these data our knowledge of Logic, in order to pass from them to the generalizations of science. This study is termed **Methodology.** It is in particular, with the methods of what are termed the Natural Sciences, that we shall be dealing in the following chapters.

The objects which Nature offers to our contemplation are of two kinds. They are either *static forms* or *dynamic activities*. The study of the former gives us the sciences of natural types, e.g. systematic botany and zoology, and in general what may be termed the classificatory sciences. The latter give us the sciences of physical law, e.g. chemistry, electricity, light, heat, etc. The discussion of the general methods employed in these two great groups of sciences, involves the consideration of the processes known as Observation and Experiment. For it is through them that we attain that knowledge of natural phenomena, which is the condition of scientific advance. They give us our data : it is by their means that we win from Nature a knowledge of her laws.

Nature does not reveal her secrets readily. Her processes are hidden and full of mysteries. We see enough to recognize from the first that we stand in the midst of an ordered cosmos, but what are the laws of that cosmos, we know not. Why does the rainbow appear in the sky after the shower? and why do the same prismatic colours play upon the waterfall? How is sound transmitted to the ear, and light to the eye? The threads of that mysterious web may be unravelled by diligent searching alone. Mere antecedence in time counts for little in establishing a relation of cause and effect; for to every event, there are many antecedents; and the difficulty lies in determining which of these antecedents is the true cause. For long it was believed that the cause of the malaria was the poisonous air of the marshes, since that was an antecedent of the illness. There was needed a closer examination among the various antecedents before the true cause was found. It is by the help of Observation and Experiment, that Nature's secrets are detected. We may at last find some case, in which Nature's process stands revealed to our rational insight, and we recognize the relation of cause and effect connecting two phenomena. Or we may find the data of an eliminative argument, and be able to assert that such and such an antecedent hitherto believed to be the cause, is not the cause, for we have observed a case where in its absence the phenomenon nevertheless occurred.

§ 2. **In what Observation consists.** Observation is the application of our faculties to the accurate determination of natural phenomena. The faculties we employ immediately are our external senses. But these do not act alone. Man is essentially rational. He cannot, even if he wishes, first apply his senses, as though they were unintelligent machines, and then set to work to use his reason. He cannot look out on Nature with the dull gaze of vacancy. It is the intelligence of man, that is the true observer; and the senses are but instruments of the observant mind.

It is for this reason that observation is always and necessarily *selective*. The mind cannot fix its attention on the myriad aspects of reality which sense-perception presents to it, but concentrates itself to take note of some one point. Nor is this the only part played by intelligence in Observation. Almost invariably, we observe in order to reply to some question which has proposed itself to our mind, and to see whether some particular answer is the true one or not. In other words, our observation is generally made in the light of some *hypothesis*. But the putting to ourselves of a question, and the suggestion of an answer, are alike the work of the intelligence.

One of the most essential conditions of observation is that the observer should most carefully distinguish between the facts which his faculties reveal to him, and those which he infers. Many of the facts which we ordinarily speak of as facts of observation, contain what may be loosely called inference, though it would be more accurate to style it imagination. Thus, for example, a man may assert with confidence that he has seen his brother in the street. Yet what he really saw was, it may be, a man resembling his brother in a few characteristic notes. Imagination filled in what was lacking, and he unhesitatingly judged that this was his brother. If his observation was hastily made, the judgment may well have been mistaken. Again men have often asserted that a thing does not exist, merely because under circumstances in which they believed they would have seen it if it did exist, they have failed to see it. Here, manifestly, the assertion is the conclusion of a hasty inference. It may be that the phenomenon was such as their unaided faculties were not capable of perceiving. Such errors are characteristic of those who are unversed in scientific observation. The scientific observer knows that true observation, though intelligent, must be free from inferential conclusions.

Though the external senses are the instrument given to us by nature to observe the natural order, yet man's inventive power

has found means to obtain auxiliary instruments, capable of registering facts outside the ken of immediate sense-perception. An interesting example is afforded in photography. The impression made by a ray of light on the retina of the eye, reaches its maximum in about $\frac{1}{10}$th of a second, and after that time ceases to increase, for the limits of sensibility have been reached. Hence when the action of the ray is too faint for it to become perceptible within that period, we are unable to see the object whence it proceeds. It is not so, however, with the sensibility of the photographic plate. Here the effect continues to accumulate as long as exposure continues. Thus many objects, which to the naked eye are invisible, leave a permanent impression on the photographic plate. This fact has proved of the utmost value in science, since it permits us to obtain records of numberless astronomical phenomena, which otherwise must have remained undiscovered. To the same category of auxiliary instruments belong microscopes, microphones, Röntgen-rays, etc.

§ 3. **Conditions of Observation.** It is impossible to draw up a set of practical rules to be followed in every case of observation. As Mill has well said, "The extent 'and minuteness of the observation which may be re-'quisite, and the degree of decomposition to which it 'may be necessary to carry the mental analysis, depend 'on the particular purpose in view" (III. c. 7, § 1). We can, however, indicate certain general conditions, which the work of observation demands in those who undertake it. These conditions are intellectual, physical and moral.

In the intellect, observation calls for the spirit of inquiry—the desire to know the reason of things, to have an explanation of what we see. This spirit of inquiry is as natural to the healthy mind, as is the appetite for food and exercise to the body. It is to this desire to find an explanation for things, that Aristotle rightly attributes the origin of science and philosophy.[1] The craving may, and often indeed does become atrophied. But when this occurs, no mere excellence of the sense-faculty will make a competent observer.

Physically, it is necessary that the sense or senses employed, should be sound. Those who suffer from

[1] *Met.*, I., c. 2. διὰ γὰρ τὸ θαυμάζειν οἱ ἄνθρωποι καὶ νῦν καὶ τὸ πρῶτον ἤρξαντο φιλοσοφεῖν.

colour-blindness cannot undertake observations, in which the discrimination of colours is in question, nor can the tone-deaf distinguish sounds. Yet if a man has once possessed the faculty, and has been deprived of it through illness or advancing years, he is not wholly debarred from the work of the observer. He may observe with the eyes of other men. For the faculty of imagination enables him to reconstruct what they communicate to him. Thus Arago, after blindness befell him, continued his researches into the polarization of light, employing the eyes of others to aid him in his work.[1] Where, however, the sense has never been possessed, there this mediate observation is impossible; for the imagination is powerless to reproduce phenomena unlike to anything of which we have had experience.[2]

The chief moral requisite, which observation demands, is impartiality. This condition is not one which it is easy to fulfil. Jevons truly remarks that "it is not 'easy to find persons who can with perfect fairness, 'register facts both for and against their own peculiar 'views" (*Principles*, p. 402). No one comes to the task of observation unbiassed. Each investigator has opinions and beliefs of his own; he desires that his beliefs may be confirmed, that he may not have to face the difficulty of seeking new solutions to problems he regards as solved. Indeed, in many cases the observation is actually prompted by such a belief; it is made because the observer is confident that it will confirm some cherished hypothesis. It requires great candour and openness of mind, for a man thus situated to be impartial in his study of facts, and to refrain from reading into them that which he wishes to see. Yet it is the first duty of the observer to accept the truth, whether it be welcome or unwelcome.

Some writers exaggerate the sphere, within which the observer is bound to preserve this openness of mind. They have urged that it is his duty to regard all his beliefs and convictions, metaphysical, religious and

[1] Rabier, *Logique*, p. 98.

[2] Cf. *An. Post. I.*, c. 18, § 1.

scientific, as liable to correction and revision as the results of his observations may prescribe.[1] To hold such a view as this, is to maintain that human knowledge is built upon the sand, and that neither in philosophy nor in religion have we found any foothold on the rock of certainty. Were this so, all intellectual effort would be futile indeed. But our case is not so desperate. Both in the sphere of religion and in that of science, we are in possession of irrefragable verities. Thus, to confine ourselves to the natural order, a man would lack intelligence, who should propose to hold the principle of contradiction or of causality as open to revision. We must distinguish between what is certain, and what, although probable and, it may be, supported by many facts, is not yet fully established. It is the latter alone, not the former, that the observer must be ready to relinquish. For truth is consistent with itself. And when once absolute certainty has been attained, then the supposition of contradictory evidence becomes an absurdity.

§ 4. **Experiment.** We have seen that the purpose both of Observation and of Experiment in the sciences of physical law, is to establish the existence of a causal relation between an antecedent and a consequent, or else to prove by elimination that some particular antecedent is not the cause. Experiment only differs from Observation, in so far as in Experiment we observe the phenomenon under conditions which have been artificially simplified. The necessity of this arises from the fact that the conditions, as they occur in Nature, are extremely complex. We should therefore be at a loss to distinguish which of the many antecedents was the true cause of the phenomenon, were it not possible to produce this latter in circumstances carefully determined, and thus

[1] " Le doute philosophique, dont il s'agit ici, ne consiste pas à douter de la science elle-même, ni de l'esprit humain en général, mais à tenir momentanément comme douteuses et susceptibles d'être redressées et corrigées par les leçons de la réalité toutes les opinions, soit réligieuses, soit métaphysiques, soit scientifiques, que nous considérons d'ailleurs comme les plus justifiées et les plus certaines."—Rabier, *Logique*, p. 103.

to exclude the supposition that any causes are operative, save those we are engaged in considering. A good example of such artificial simplification is afforded by the well-known 'guinea and feather' experiment, in which the two substances are placed in an exhausted receiver, and allowed to fall from the top together. They reach the bottom of the receiver at the same moment, and thus demonstrate to us that when the resistance of the air and other interfering influences are allowed for, all bodies tend to the earth with equal rapidity. It is plain that without some method of simplifying the conditions, it would have been difficult, if not impossible, to obtain conclusive evidence of this truth.

It will be seen that although Observation may take place without set purpose,—and indeed the history of science records not a few famous discoveries made by what may be called chance,—Experiment necessarily requires an hypothesis. Unless we frame some supposition, and make our experiment to the end that we may know whether this supposition is correct or not, no experiment can take place. To experiment is to question Nature; and our question must take some clearly defined form.

The mere use of a scientific instrument does not in itself constitute an experiment. We do not speak of experimenting, but of observing with a microscope. To make an experiment, it is not enough that the observer himself should be set in new and special circumstances. It is requisite that the object observed be placed under new conditions, and further that these new conditions should in some way modify its action.

In scientific investigation, it is not infrequently necessary to make use of what is termed the **Blind** or **Negative Experiment.** This is an attempt to show that not only is A always followed by α, but that α is always preceded by A: that a particular antecedent always involves a particular consequent, and that this consequent never occurs except when the antecedent in question has preceded it. An example is afforded by a series of

experiments which were undertaken in regard to the origin of the sleeping-sickness of Uganda. A series of careful observations were first made which appeared to establish that the disease was due to a certain microbe communicated to the blood by the bite of the tsetse fly. But a further series of experiments was undertaken, in which it was shown that even where all the other conditions, which usually accompany the disease, were present, yet if this microbe was not communicated to the blood, the disease never appeared. This last part of the investigation constituted the *Negative Experiment.*

§ 5. **Natural Experiments.** A Natural Experiment is the name employed to denote those events, in which the processes of Nature themselves produce special and determinate conditions, under which the phenomenon in question may be observed. Such events have proved of great importance in many sciences. Astronomers are indebted to these natural experiments for many of their discoveries. An eclipse of the moon, such as we have all of us witnessed, affords us a case in point. Here the shadow cast upon the lunar disc shows us the shape of the earth. Another interesting example may be given, relating to visual perception. It had long been disputed whether the defect to which colour-blindness was due, was in the eye itself or in the brain. The doubt was solved in the following manner. The affection is occasionally hereditary; and cases have been known to occur, in which the vision of one eye is normal, while the other suffered from 'red-green' blindness. Now, had the lesion been in the brain, this could not have occurred. For each of the eyes is supplied with nerves from either side of the brain, the nerves crossing each other in what is called the optic chiasma.[1]

§ 6. **Relative Advantages of Observation and Experiment.** In all cases, in which Experiment is possible, there can be no room for comparison between it and Observation, so complete is its superiority from the

[1] *Proc. Royal Society*, vol. 31, p. 302, 1881.

scientific investigator's point of view. Its limitations lie in the fact that a vast number of Nature's processes cannot be imitated. In these cases, we must be content to observe. Thus, we have no means of experimenting where geological and cosmic forces are concerned. We must be content to wait on Nature, and to learn from her, as by her own methods she achieves her work in the great laboratory. In the case of many phenomena, the scientist is fortunate, who has the opportunity of conducting an observation himself, and need not rely on the reports of others. Few astronomers have been able like Tycho Brahe to note the appearance of a temporary star at the very hour when it commenced to shine in the heavens. "This appearance of the star of 1572," writes Herschel, "was so sudden, that Tycho Brahe, 'returning one evening (11th November) from his labo-'ratory to his dwelling-house, was surprised to find a 'group of country-people gazing at a star, which he 'was sure did not exist half an hour before. It was 'then as bright as Sirius, and continued to increase till 'it surpassed Jupiter when brightest, and was visible 'at midday." [1]

The advantages of Experiment over Observation are to be found in the fact that we are enabled by it (1) to produce, under varying conditions, a number of instances of the phenomenon we desire to investigate, (2) to simplify the conditions, (3) to produce new phenomena of a similar kind.

(1) It is hardly necessary to dwell on the importance of varying the conditions of the phenomenon. The investigation, for instance, of the facts relating to the crystallization of bodies would have presented immense difficulties, had it not been in our power to reproduce such phenomena experimentally. The science of electricity would probably never have emerged from infancy, had it been possible to study it only during the thunderstorm.

A multitude of instances does much too to protect us

[1] Herschel, *Outlines of Astronomy* (11th edit.), p. 602.

against the tendency already mentioned (§ 3), which leads us to see our own beliefs confirmed by the facts we observe. When experiments are multiplied, in some one or other we find a crucial instance proving that we are in error.

(2) As experiment helps us to simplify the conditions of the phenomenon, it enables us to exclude any circumstances that may operate as interfering causes. We are thus able to control the phenomenon, and to obtain certain knowledge instead of mere conjecture. Thus a simplification, very similar to that of the guinea and feather experiment, provides us with a proof that the transmission of sound is due to vibration in the atmosphere. A bell is struck in a vacuum, and no sound is heard.

(3) Of the production of phenomena, similar to those which Nature shews us, we have striking examples in recent scientific investigation, in the famous experiments by which Professor Dewar succeeded in liquefying and freezing oxygen, hydrogen and air. These experiments revealed to us a series of facts analogous indeed to those, with which we are familiar, but which Nature uncontrolled affords us no opportunity of observing.

As we should be led to expect, having regard to the immense advantages possessed by experimental investigation, the sciences in which this has been possible, are those in which the greatest progress has been made. Mechanics, Physics and Chemistry all admit of experiment to a very large extent. In Anatomy, Physiology, Meteorology it can hardly be employed. It will not therefore surprise us that the former group of sciences has advanced far more rapidly than the latter.

CHAPTER XX.

METHODS OF INDUCTIVE ENQUIRY.

§ 1. **The Four Experimental Methods.** Our purpose in this chapter is to describe the principal ways, in which the evidence of a causal connexion manifests itself. We shall thus be describing the manner in which scientists are accustomed to direct their observations and experiments, in order to win from Nature the secret of her laws. For to know the various ways in which the evidence of universal laws appears, is to know the methods of scientific inquiry. As we have already said (Ch. 14, § 4), it is not possible to lay down fixed canons of evidence, prescribing the precise conditions under which we may be certain as to the existence of a general law. For what is satisfactory evidence in one case, is quite insufficient, when a subject of a different character is under consideration. If on two or three occasions I hear a wild bird of a certain kind give voice to a special song, I rightly conclude that there is a causal connexion between that species of bird and the song in question. But because it has occurred two or three times that a special sort of food has been hurtful to me, I should be rash in making an induction to a general law. In such a case, the effect may be due to accidental circumstances. Again, it is possible to test the evidence of a deductive argument by a canon; for in Deduction, the whole of the evidence is contained in the two premisses of the syllogism. In Induction, the evidence is not the mere singular fact from which I abstract the law, but a number of other facts recorded in memory, some of them concerning the phenomenon about which the law is

enunciated, and some concerning the surrounding circumstances.

The discussion of these Experimental Methods, as they are often termed, is in large measure due to the important place they hold in Mill's *Logic*. He doubtless took a mistaken view of them. He drew up elaborate canons for them, and believed that they were so many distinct processes of inductive reasoning. His treatment has been the object of much criticism. Yet it should be borne in mind, that though his analysis may be wrong, the methods he attempts to formulate are from a practical standpoint those employed in scientific investigation. We cannot therefore dismiss them from our consideration. We must endeavour to determine what is their true bearing on Induction.

The subject is usually discussed in relation to Mill's treatment, and we shall not deviate from that practice. We give here in tabulated form, his Methods with their respective canons. An example is given in each case, illustrating the application of the method in question :—

I. *Method of Agreement.*

Canon. "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree, is the cause (or effect) of the given phenomenon" (III. c. 8, § 1).

Mill represents this method by a formula in which the capital letters stand for antecedents, the small letters for consequents. Granted that the two instances may be symbolized respectively as ABC-*abc*, ADE-*ade*, then we may conclude that A is the cause of *a*.

The Method may be illustrated by the following example from Jevons. "Sir D. Brewster accidentally took an impression from a piece of mother-of-pearl in a cement of resin and bee's-wax, and finding the colours repeated upon the surface of the wax, he proceeded to take other impressions in balsam, fusible metal, lead, gum arabic, isinglass, etc., and always found that the iridescent colours are the same. He thus proved

'that the chemical nature of the substance is a matter 'of indifference, and that the form of the surface is the 'real condition of such colours" (*Principles*, p. 419).

II. *Method of Difference.*

Canon. "If an instance in which the phenomenon 'under investigation occurs, and an instance in which 'it does not occur, have every circumstance in common 'save one, that one occurring only in the former: the 'circumstance in which alone the two instances differ 'is the effect, or the cause, or an indispensable part of 'the cause of the phenomenon" (III. c. 8, § 2).

Formula. ABC-*abc*, BC-*bc*: whence it may be concluded that A is the cause of *a*.

A good example is provided by the experiment referred to above, in which a bell is struck in a vacuum, and no sound is heard. In the first instance, the air is present in the receiver, and the striking of the bell is clearly audible. Here the presence of the air is denoted by A, and the sound by *a*. The air is then removed by means of an air-pump, and the removal of this antecedent is seen to involve the removal of the consequent *a*. We are thus assured that the air must at least play an indispensable part in enabling us to hear.

Joint Method of Agreement and Difference.

This method is not reckoned by Mill as independent of the preceding ones, and as constituting a distinct method of proof. He holds it to be "a great extension 'and improvement of the method of Agreement, but 'not as participating in the more cogent nature of the 'method of Difference."

Canon. "If two or more instances in which the 'phenomenon occurs have only one circumstance in 'common, while two or more instances in which it 'does not occur, have nothing in common save the 'absence of that circumstance: the circumstance in 'which alone the two sets of instances differ, is the 'effect or cause or an indispensable part of the cause 'of the phenomenon" (III. c. 8, § 4).

It is of course to be understood that the instances in

which the phenomenon does not occur, are of such a character, that were it due to any other cause but the circumstance in question, there would be good reason to anticipate its appearance. Unless the instances were thus relevant to the subject under consideration, they would afford no grounds for a conclusion of any kind.

Formula. No formula is given by Mill. Mr. Welton suggests the following: ABC-*abc*, ADE-*ade*, BDM-*bdm*, CEO-*ceo*: whence we conclude that A is the cause of *a*.

In illustration of the method, we may take the investigations made by Darwin in regard to the influence of earthworms in the production of vegetable mould.[1] In these investigations, a large number of observations were made as to the formation of mould on soil of different kinds. The development of the mould and the apparent sinking of the objects on the surface, were carefully noted. Notwithstanding the variety of circumstances, the growth of the mould was a feature common to all these instances. And as far as could be detected, the one circumstance common to all alike was the presence of earthworms in great quantities in the surface soil.

A number of cases were also examined, in which there had been no increase of mould. The cases selected were thoroughly relevant, for the land observed was in the same districts as those which had provided the positive instances. It was invariably found that either from excessive dryness, or for some other reason, the soil here contained no earthworms.

These two classes of observations are those contemplated by the canon, and provide a large amount of evidence for the conclusion that the action of the earthworms is causally related to the development of the mould. To render the conclusion more certain, experiments were undertaken to test the adequacy of the cause to produce the effect. It was estimated that in cultivated soil, earthworms exist in sufficient number to give between seven-and-a-half to eighteen tons per acre of castings

[1] Vide Welton, *Manual*, § 154. Mr. Welton gives this example in a slightly different connexion.

every year. These results would yield a layer of mould from an inch to an inch-and-a-half in thickness every ten years. The cause is therefore fully adequate to produce the effect attributed to it.

III. *Method of Residues.*

Canon. "Subduct from any phenomenon such part 'as is known by previous inductions to be the effect of 'certain antecedents, and the residue of the phenomenon 'is the effect of the remaining antecedents" (III. c. 8, § 5).

Formula. If it be known that ABC-*abc* are causally connected, and further that A is the cause of *a*, B of *b*, then we may conclude that C is the cause of *c*.

The method may be illustrated by the discovery of the planet Neptune by Leverrier. Here the motions of Uranus provided the clue. While their general features were such as the position of the planet in the solar system would prescribe, they presented nevertheless certain residuary phenomena, which convinced the astronomer that yet another planet existed, to whose influence they must be attributed. This conclusion was shown to be justified by the subsequent discovery of Neptune.

In cases such as this we do not, strictly speaking, attribute the remaining consequent to the 'remaining antecedent.' We are led by the presence of an unexplained element in the phenomenon we are considering, to seek its unknown cause. Hence, it has been urged that Mill's canon is not applicable, and that these cases should be regarded as exemplifying another rule, viz., "*When any part of a complete phenomenon is still unexplained 'by the causes which have been assigned, a further cause for the 'remainder must be sought.*" The point need not detain us: for we are not here concerned to vindicate the validity of Mill's formula, but only to shew what modes of investigation his canons were intended to describe. In explaining his Method of Residues, he explicitly mentions as falling under it, those cases in which "the agent C is an obscure circumstance not likely to have been 'perceived unless sought for" (III. c. 8, § 5).

IV. *Method of Concomitant Variations.*

Canon. "Whatever phenomenon varies in any manner 'whenever another phenomenon varies in some par-

'ticular manner, is either a cause or an effect of that 'phenomenon, or is connected with it through some fact 'of causation" (III. c. 8, § 6).

Formula. The following expression is given by Mr. Welton: A_1BC-a_1bc, A_2BC-a_2bc, A_3BC-a_3bc: whence we conclude that A is causally related to *a*.

It is in this manner that Albert the Great proves the causal influx of the moon on the tides. The discovery of the connexion between the moon on the one hand and the ebb and flow of the tide on the other, had been made by the Arabs: and from them the knowledge had passed into Europe in the ninth century. Albert in one of his works, argues the point against certain opponents of the theory, and bases his conclusion on the principle that the concomitance of phenomena is indicative of causal connexion.[1]

In a similar way Pascal showed that the column of mercury in a barometer is sustained by the weight of the atmosphere. In 1648 at the suggestion of Descartes, he made a series of observations on the Puy-de-Dôme mountain, which conclusively established that the height of the mercury varied concomitantly with the different altitudes at which the observation was taken.[2]

§ 2. **Further Illustrations of the Methods.** The illustrations given in the last section were selected in each case because of their simplicity. In practice, however, it usually happens that the determination of a relation between cause and effect, involves the use of more than one of the methods, and is by no means so simple and straightforward a proceeding as these examples would

[1] Albertus, *De Proprietatibus Elementorum*, lib. I. tract 2, c. 8. "Contra secundam autem sectam est, quod si moveretur mare quia spirat naturaliter, sicut alia in quibus generantur venti et vapores spirituales; et tunc debet fieri accessus et recessus per unum modum semper; quia opera naturae sunt uno modo: . . . tamen ejus contrarium videmus, quod sequitur motum lunae, ad hoc quod *posito aliquo ponitur et destructo destruitur et hoc causatur ab ipso*: sic autem videmus semper accessum maris poni et fieri quando luna tangit circulum hemisphaerii: videbitur igitur causari ex motu lunae." Cf. Scotus, *Meteor.* lib. ii. q. 2.

[2] Haldane, *Descartes*, p. 310.

lead us to suppose. This point is of importance in considering the relation of the methods to Induction. In this section, we give two instances of such scientific investigation.

(1) *Effect of radio-active substances on gelatin media.* Our first example is drawn from an enquiry bearing on the alleged effect of radium in producing life. In 1906, considerable interest was aroused in some experiments made by Mr. Butler Burke to test the effect of radio-active substances on gelatin media. He found that as the result of their action certain 'bacteria-like' cells were obtained, containing a nucleus; that they appeared to be highly organized bodies; and that after growing to a certain point, they subdivided.

In order to test these results and discover whether Mr. Burke was justified in supposing that living cells had been produced, a series of experiments was undertaken by Mr. Douglas Rudge. It is these experiments which provide our example.[1]

(*a*) In the first place, several samples of radium salts were used. These differed in their degree of purity, and it was seen that the rate and amount of growth did not correspond to the amount of radium present in the sample. As radium salts contain a large admixture of barium salts, it was thought possible that the latter, and not the former, might be the agent which produced the cells. It was found on experiment that barium salts without radium, produced a growth which appeared identical with that caused by the radium.

These experiments were, as is easily seen, negative instances in the methods of Concomitant Variations and of Difference. They appear to render it certain that radium was not the cause of the cells.

(*b*) A systematic examination followed with all kinds of metallic salt. It was found that those of barium, lead and strontium were the only ones which resulted in the appearance of the growth. These three metals are those which form insoluble sulphates; and the con-

[1] Vide *Proc. Royal Soc.*, Dec. 20, 1906, vol. 78.

clusion suggested by the experiments, was that the cells developed round precipitates, which were formed owing to the presence of sulphur in the medium.

This group of experiments shows us the application of the Method of Agreement.

(*c*) Instead of gelatin, other forms of meat-culture were now tried, fixity being given to them by various mucilaginous substances, such as starch, gum, etc. It was found that if distilled water were employed, no growth appeared, and that the growth was in all cases proportionate to the amount of sulphur in the water. Further, between thirty and forty specimens of gelatin were tested, and with three exceptions, all proved to contain enough sulphuric acid to give a distinct precipitate.

In this case, the Method of Concomitant Variations was again used; but on this occasion, the result was not negative but positive.

(*d*) Finally, experiments were made in gelatin, from which all sulphuric acid had been removed, with the result that no sign of growth manifested itself.

Having regard to the evidence furnished by the previous experiments, this last may be considered as an application of the Method of Difference.

The final result of the whole investigation was entirely to discredit the view that the phenomenon was in any sense a vital process. It was shown (1) that the cells form round a precipitate of insoluble sulphate, the growth of the cell being dependent on the amount of sulphur present; and (2) that radium has no specific action in forming cells.

(2) *Possibility of spontaneous generation.* As our second instance, we give a short account of some experiments carried out by M. Pasteur in his famous investigations regarding this subject.[1] The spontaneous development of life in substances undergoing decomposition, was strenuously maintained by Pouchet and other scientists;

[1] Vide Guibert, *In the Beginning* (' *Les Origines,*' transl. by Whitmarsh), c. 12: and Rabier's *Logique*, pp. 138–141.

and these experiments were undertaken by Pasteur in order to shew that if sufficient precautions were taken to exclude the germs which are so plentiful in the atmosphere, no life would be developed, and no decomposition take place.

(*a*) He first filled a flask with a liquid known to be specially liable to decomposition. The long curved neck of this flask was connected with a platinum tube which passed over a flame. The liquid in the flask was boiled, and at the same time, the platinum tube was heated red-hot. The flask was then allowed to cool; but while it cooled the platinum tube was kept at its high temperature, so as to ensure that the air which entered the flask should be free from all germs. The flask was then closed; and it was found that no life whatever was developed in the liquid.

Here we appear to have an instance of the method of difference. For it seems that the sole circumstance distinguishing this case from one in which decomposition takes place, is the presence of germs in the air within the flask. Exception was however taken to the conclusiveness of the experiment. It was urged that spontaneous generation might depend on the presence of certain conditions in the atmosphere, with which the heating process had interfered.

(*b*) Recourse was therefore had to another experiment. The air admitted into the flask was not transmitted through a red-hot tube, but simply passed through a cotton-wool stopper. The effect was found to be the same. Further, in order to preclude any objections on the score that cotton-wool, being an organic substance, might have some peculiar effect in rendering the liquid sterile, a similar experiment was made, in which the stopper was not of wool, but of asbestos.

Here the method of agreement is exemplified. The three cases we have considered, agree in so far that life is produced in none of them, and they have but one material circumstance in common, namely, the exclusion of vital germs.

(*c*) Yet it is to be noted that in each case the liquid was first boiled. It might perhaps be said that this boiling had made spontaneous generation impossible. It was necessary to exclude this suggestion. M. Pasteur therefore made a fresh experiment, this time with blood obtained direct from the veins of healthy animals, so that it was as yet free from all contamination ; he exposed this blood to contact with air purified from vital germs. Though blood is a substance, which readily decomposes, no development of life took place.

This experiment may be regarded as completing the application of the Method of Agreement.

(*d*) Finally an especially interesting group of experiments was made in the following manner. Liquid was enclosed in a number of flasks. The necks of all these flasks were sealed over a blow-pipe, after all vital germs that might be contained in it, had been destroyed by boiling. The flasks were distributed in various places, their necks broken, and the liquid exposed to the air. It was found that the proportion of those in which putrefaction took place to those which escaped, was just what might have been anticipated, if, as was urged, decomposition was due to germs floating in the air. In a dwelling-room, in which the ordinary sweeping and dusting of a household made the distribution of dust general, not a flask escaped. In the open air of the country, decomposition took place in eight out of twenty flasks. On one of the lower spurs of Mount Jura, five only out of twenty, became infected. And on the Mer-de-glace, decomposition set in in one case, while nineteen remained free. Had the development of life been due, not to the chance distribution of germs, but to spontaneous generation, it is difficult to see how such results could have been obtained. Where the flasks contained the same liquid, and were exposed to the same conditions, either all would have developed life, or none.

This final experiment may legitimately be reckoned as falling under the Method of Concomitant Variations.

§ 3. **The Function of the Methods in proving a Law of Nature.** It was shewn in Ch. 14, that the act of Induction is immediately dependent on the recognition by the intellect of a causal relation connecting two facts. We pass to our universal conclusion, when the intellect, considering the facts revealed by sense, apprehends the precise characteristic, in virtue of which the agent produces such and such an effect. This may be otherwise expressed by saying that Induction is possible, when not only the complex elements which constitute the phenomenon are perceived by the senses, but when the mind also understands the relation uniting them.

The question now arises as to how the Experimental Methods, which we have been considering, assist us in this task. It is of course evident that in themselves the Methods are not logical processes. They are merely special ways of arranging our data. They constitute in each case a prerequisite to the mental act, by which we make an inference from the facts.

The Methods aid us in two ways. They provide the data for (1) Eliminative Syllogisms, (2) Induction.

Elimination must be carefully distinguished from Induction properly so called. It is of value as an ancillary process, but it is not Induction. Its importance is due to that complexity of Nature, which provides so many difficulties to the scientific investigator. When we are seeking to discover the cause of some fact, it is often the case that while we have good reason to suppose the cause is one of several antecedents, there is nothing to indicate with certainty, which of these it is. The application of the Methods enables us to eliminate many of these antecedents. For if it can be shewn (a) that some particular phenomenon is present when the fact in question is absent, or (b) that it is absent when the fact is present, or (c) that its variations bear no relation to variations observed in the fact, then we know that it is not the cause—at least not the complete and sufficient cause of the fact. The arguments by which we

reject these suggested causes, are deductive syllogisms in *Celarent* :—

No characteristic which is present when a is absent, is the cause of a.

β is a characteristic, which is present when a is absent.

$\therefore$ β is not the cause of a.

It has been maintained by some that the inductive process consists essentially in Elimination; that in a valid Induction, the antecedents among which the cause is to be looked for, are first tabulated and then successively eliminated until one only remains. This remaining antecedent is then declared to be the cause. "The 'essence of inductive reasoning," says Mr. Joseph, "lies 'in the use of our facts to disprove erroneous theories 'of causal connexion. It is . . . a process of elimina-'tion. The facts will never show directly that a is the 'cause of x : you can only draw that conclusion if they 'show that *nothing else is*" (*Introd.*, p. 395). This view we believe to be untenable. An argument drawn in this manner would remain for ever inconclusive. Where conclusions have been based on such reasoning, they have often proved to be incorrect. The list of antecedents was defective, and the true cause has been discovered in some factor that had been overlooked. We must not merely eliminate those antecedents which are not the cause; we must have positive grounds for asserting that the effect proceeds from such and such a fact and from no other.

The Methods provide us with the material not merely for Elimination, but for Induction properly so called. The object that we have in view in them, is so to simplify our data, that we shall no longer be bewildered by their complexity, but shall recognize the causal relation as it is exemplified in the individual case. Our aim is to see A producing a, and this so clearly that the mind shall affirm that a proceeds from A *quâ* A, that is from A in respect of its being A and nothing else. To quote some words of Aristotle already cited (Ch. 14, § 4) : "Not

'that the mere act of sight would give us scientific 'knowledge; but sight would be the means, through 'which we should attain the universal."[1] Such, to take a simple example, is the inductive process, when after employing the Method of Difference, we induce the general conclusion that gold is dissolved by *aqua regia*. Previous to the application of the *aqua regia*, there is no dissolution. It is applied, and the senses show us dissolution taking place wherever the liquid reaches. The intellect does not set to work eliminating, but abstracts a general conclusion that gold as such is dissolved by *aqua regia*.

In certain cases, the evidence of causation is provided not by one, but by a series of observations. Pascal's experiments as to the pressure of the atmosphere on the barometer furnish us with a case in point. Taken separately, each observation would have been insufficient for his conclusion. If, however, a series of such observations be viewed together, as they are in fact viewed when stored in the memory of the observer, the effect on the mind is convincing. It recognizes the combined testimony to the causal influx, and draws its universal conclusion.

Without doubt, a vast number of our inductions are only probable. We believe that we recognize one fact as determining another, whereas in truth we have merely some fallible signs of causality. Thus, when before the discovery of gravitation, the ancients judged that *All heavy substances tend downwards, all light substances upwards*,[2] it appeared to them that the facts showed a spontaneous tendency in light substances to rise above the earth; and they held that this motion must be caused by the 'lightness' inherent in them. In such cases, as we saw in Ch. 19, § 2, imagination outruns observation. We believe there is a relation of cause and effect, not

[1] *An. Post. I.*, c. 31. § 8. οὐχ ὡς εἰδότες τῷ ὁρᾶν, ἀλλ' ὡς ἔχοντες τὸ καθόλου ἐκ τοῦ ὁρᾶν.

[2] Cf. Arist. *Physics, IV.*, c. 4, § 21. ἐπεὶ τὸ μὲν κοῦφον τὸ ἄνω φερόμενόν ἐστι φύσει, τὸ δὲ βαρὺ τὸ κάτω.

because its existence is evident, nor because, strictly speaking, we infer it, but because at the suggestion of certain signs we imagine it. Of this character was the error described in the last section in regard to the activity of radium. The cells arose on the application of the radium salt. Here was a phenomenon which is one of the signs of causality, though not an infallible sign. Imagination did the rest.

It may be observed that the emendations suggested by various authors for the Method of Difference, harmonize with the account of the process we have offered. Thus Dr. Mellone proposes that the canon should run as follows—"*When the addition of the agent is followed by the appearance, or its subtraction by the disappearance of a certain event, the other circumstances remaining the same, that agent is the cause of the phenomenon.*" As we have stated, we do not believe that it is possible to make any canon, which can be rigorously applied as a test of causation. But though this may not be strictly speaking a canon, it is at least a description of the circumstances under which we are frequently justified in drawing a universal conclusion. And the circumstances presuppose an abstractive, not an eliminative process. Our conclusion is not reached by eliminating those circumstances which are not the cause. But when we see the agent *A* produce *a*, we abstract the characteristics in question, and say *A as such* is the cause of *a*.

§ 4. **Criticism of Mill's Canons.** Mill's view of the Four Methods is very different from that which we have sought to present. To him, they are not an analysis of the circumstances, which in the physical sciences best enable us to detect causal relations. They constitute so many independent canons of reasoning, comparable to the syllogism, but differentiated from it by the fact that syllogistic reasoning is deductive, proceeding from the general to the particular, while they are inductive, guiding us from the particular to the general. "The 'four methods,' he writes, . . . "are the only possible 'modes of experimental inquiry, of direct induction *à* 'posteriori, as distinguished from deduction. . . . These, 'with such assistance as can be obtained from deduction, 'compose the available resources of the human mind

'for ascertaining the laws of the succession of phenomena" (III. c. 8, § 7). The hostile criticism which his treatment of the subject has encountered, and of which we are about to recapitulate the chief points, is principally valid against this view of them.

In the first place, the reasoning of the methods as described by Mill, is deductive. A general principle is stated, and a particular case is brought under it. This may be seen on an analysis of an argument under the first of the canons :—

Wherever in two instances of the phenomenon under investigation, it is found that one circumstance alone besides the phenomenon is common to both instances, that circumstance is the cause of the phenomenon.

In the two instances *abc*, *ade*, in which the phenomenon *a* appears, it is found that the circumstance A alone is common to both instances.

∴ *A* is the cause of *a*.

All causal relations in Nature are constant.
A . . . *a* is a causal relation.
∴ The causal relation *A* . . . *a* is constant.

It may be objected secondly, that if there are really four canons of inductive reasoning, those canons should be independent of one another. It should not prove to be the case, that they are one and all based on the same principle. If the canons can be shewn to be all derived from one principle, they must be held to have forfeited their claim to be so many distinct principles of ratiocination. Now the inductive canons one and all are based on the same law, viz., that when of two facts the one can appear without determining the presence of the other, there is between these two no causal relation. This is the principle of the Method of Agreement. Here *a* is shown not to be causally related to *B* or *C* or *D* or *E*, because it can appear in their absence. Similarly, in the Method of Difference, we show that neither *B* nor *C* causes *a*; for they can be present without it. And

the reasoning of the other methods is of the same character. Mill himself frankly recognizes their intimate connexion. He tells us in so many words, that the "Method of Residues is in truth a peculiar modification of the Method of Difference" (III. c. 8, § 5); and in another passage informs us that the Method of Concomitant Variations may be identified either with the Method of Agreement or with that of Difference (III. c. 22, § 4). The two chief methods, those of Agreement and Difference are both, he tells us, methods of elimination (III. c. 8, § 3). Under these circumstances, there can be no justification for enumerating four canons of reasoning.

The canons further suppose an unreal state of things in Nature. For they apparently proceed upon the assumption that we have a number of antecedents all clearly determined, and a number of consequents of a similar character. In reality, so manifold are the links which unite natural agents, and so complex the process of action and reaction, that the representation of the antecedents by the symbols *A*, *B*, *C*, *D*, and of the consequents by *a*, *b*, *c*, *d* is quite deceptive. We have, for example, seen in the investigation as to the effects of radio-activity, how difficulties at once arose from the unexpected interaction of causes that had been overlooked. If Nature could really be represented by Mill's symbols, Induction would be a matter of little difficulty. Again, the employment of the large and small letters, would seem to indicate that we can detect at once which fact is antecedent, and which is consequent. This is frequently not the case. Thus, if a patient consults a doctor, the doctor may often have reason to doubt where to find the cause, and where the effect. If, for instance, it appears that the patient is suffering from nervous breakdown, and is also a prey to mental depression, it may be hard to say whether the nerves are responsible for the depression, or *vice versa*. Moreover, the use of the same series of letters both for antecedents and consequents certainly conveys the impression, that it can be

no very recondite problem to find the *a* which corresponds to *A*, the *b* which corresponds to *B*, etc.

From the point of view of Mill's own philosophical position, yet another objection may be raised. His doctrine of the Plurality of Causes renders it impossible for him to regard the canons as valid methods of inferring universal judgments. We may indeed conclude that in this instance *a* is caused by *A*, but if the same effect can follow from causes entirely different the one from the other, we have no guarantee that our judgment is true for any other case than the one under examination. In all other instances, *a* may be caused by *X*, not by *A*. A canon, which is always liable to frustration, is of little value as a criterion of valid reasoning.

CHAPTER XXI.

EXPLANATION.

§ 1. **Explanation.** A thing is said to be explained, when it is rendered intelligible—when the mind acquires a facility to grasp it, which it did not possess before. This may be understood in two ways. The explanation may have special reference to the particular mind to which it is addressed. It may not be intended to throw light on the nature of the thing itself, but merely by illustrations and analogies familiar to the hearer, to enable him to grasp something, of which he has no experience. Such are the explanations we give to children. But an explanation may be such as to give us a real insight into the nature of the fact explained. In this case, a new light is thrown on the thing itself, and not simply on this or that mind. It is with such explanations that we are concerned in this chapter.

A law of nature is said to be explained, when it is shewn to result from the operation of some wider law or laws, from which it can be deduced. Thus the motion of the moon round the earth was explained, when it was proved to be due to the force we call terrestrial gravity. Similarly, an individual fact is said to be explained, when we are able to point out the laws which have brought it about. Thus we explain the fall of a tree in a storm, by indicating the looseness of the soil, and the other circumstances, which in conjunction with the force of gravity, rendered it unable to withstand the stress to which it was exposed. There is indeed a sense in which, when we explain an individual fact, we are really explaining a law. For we are pointing out

how it is a necessity of nature that every such tree situated in similar circumstances should fall. In each case, we explain by assigning the reason of the law or of the event. For the mind regards a thing as intelligible when it knows the cause which produced it.[1]

It is manifest that this process must find a limit. We soon arrive at laws, which we cannot account for by bringing them under any law wider than themselves. These we must accept as *ultimate*. They are truths made known to us by the testimony of experience, but for which no reason, in the sense of which we have spoken, can be assigned.

* Logicians of the Idealist school, who take the view that individual entities consist solely of relations (Ch. 15, § 4), understand Explanation somewhat differently. By these writers no essential distinction is drawn between cause and condition, and Explanation is to them the "ascertainment of necessary conditions." A phenomenon is fully explained, "when the con-'ditions of every detail . . . are so fully and exactly known, 'that not only a phenomenon of that general character, but 'just this very phenomenon, with exactly these details, and in 'exactly this amount, must follow from these conditions." [2] The conditions in question are in fact the relations, in which the object is held to consist. Explanation is therefore the determination of the constitutive relations of the object or event.

Since every one of these relations is, as conceived by the mind, universal, the analysis of the event into its constitutive relations is described as a process of *Generalization*; for by Explanation, we bring to light, in each relation, some element which is common to other phenomena. Every relation present in a phenomenon, shows us some aspect, in which it agrees with many other individuals.

The Explanation of the Idealist philosophers differs fundamentally from that described in the body of this section. Their Explanation is not explanation of an effect by its cause, but of a part by its whole,—a very different thing. According to this school, Nature is an organism—a unit—of which individual things are but parts; it is not an organization formed of things, which are complete in themselves, though related one to the

[1] Cf. Arist., *An. Post. II.*, c. 11, § 1. ἐπίστασθαι οἰόμεθα ὅταν εἰδῶμεν τὴν αἰτίαν. "We believe ourselves to know a thing, when we are acquainted with its cause."

[2] Welton, *Manual*, § 159.

other. This difference is crucial. For it is impossible to understand a part save in its relation to a whole ; and we cannot have a correct idea of a part, unless we have a correct idea of the whole, of which it is a part. In an organism, the parts are what they are, because the organism is what it is. In an organization, on the other hand, the whole is what it is, because the units, out of which it is composed, are what they are. They must be understood, before the whole can be understood.

If Nature is an organism of relations, not an organization of related things, Explanation has no term or limit. All our explanations are provisional ; nor can we attain to the true explanation of any object or event, till we have grasped this scheme of things entire ;[1]—till we know alike the finite and the Infinite, nay, if there be no radical difference between thought and things, till our mind's vision surveys the whole of both orders. In other words, Science is impossible. But if Nature is an organized manifold, then Science begins with tasks that can be measured, and every step forward is a permanent acquisition of truth.

§ 2. **Explanation by Regressive Reasoning.** We have said that the task of Explanation is to assign the reason of the law, with which we are concerned. Logical principles indicate to us the way in which we must proceed. We must employ regressive argument from the effect to the cause (Ch. 12, § 4). We must show that as regards the antecedent and the consequent we are considering, their connexion is due to some cause of wider generality. We are given *S* is *P*, and our object is to prove that *S* is *P*, *because it is M*. This can only be done by showing that the phenomenon *P* proves the presence of the cause *M*. We must not merely establish the proposition 'If *M*, then *P*,' but the reciprocal of it, 'If *P* then *M*.' In other words, the proposition 'All *M* is *P*' must be simply convertible. We are thus enabled to form the syllogism :—

P is *M*.
S is *P*.
∴ *S* is *M*.

[1] "Ultimately, you may imagine, nothing can be known rightly, without knowing all else rightly." Bosanquet, *Logic*, I. p. 393.

In this way we demonstrate that the agent *M* is present in every *S*, and so explain how it comes about that 'All *S* is *P*.' To take a simple case, we may thus assign the reason of the law that potassium floats in water.

Substances which float in water, have a specific gravity less than that of water.

Potassium floats in water.

∴ Potassium has a specific gravity less than that of water.

When the conclusion '*S* is *M*,' is thus established, the syllogism, by converting the major, may be restated in the form:—

M is *P*.
S is *M*.
∴ *S* is *P*.

In this form, the cause, and not the effect, is used as the middle term. The order of nature is followed: we prove the effect by the cause.

Substances, whose specific gravity is less than that of water, float in water.

Potassium has a specific gravity less than that of water.

∴ Potassium floats in water.

These two methods of reasoning were known to the ancients as the *via resolutionis*, i.e. the *analysis* of a fact into its principles, and the *via compositionis*, i.e. the *synthesis* of principles which determine the fact. In this connexion we may conveniently notice two other expressions employed, *via inventionis* and *via doctrinæ*. The former is used to signify all reasoning in which there is an advance from the known to the unknown, whether by way of progressive or regressive argument (S. Thomas, *Summ. Theol. I.* q. 79, art. 8, 9) The *via doctrinæ* is the reasoned exposition of truth already attained. For this purpose we naturally proceed by the progressive method from cause to effect.

Science owes many of her greatest triumphs to regressive argument from effect to cause. Professor Case (*Lecture on Scientific Method*, p. 14) instances what is perhaps the most famous of all—Kepler's discovery that the path of the planet Mars is an ellipse with the

sun in one of the foci. The observations, which formed the basis of the argument, Kepler owed in large measure to Tycho Brahe. These observations revealed certain regular movements of the planet, but what law governed these movements was not apparent. Many were the hypotheses that he tested, only to find that they did not agree with the facts. At last the clue was found, and he was able to deduce his conclusions in the following manner :—

Such and such positions are the properties of an ellipse.

The orbit of the planet Mars has such and such positions.

∴ The orbit of the planet Mars is an ellipse.

This argument deserves careful attention for two reasons. In the first place, it has been misunderstood by recent logicians, and several erroneous accounts have been given of the reasoning involved. Whewell, for example, has described it as an induction ; though it is clear that the task before Kepler was to explain a law, not to generalize from particular facts. But a yet greater interest attaches to the argument in so far as it signalizes for us the intimate connexion between the science of the ancients and that Copernican astronomy, which is regarded as characterizing in an especial manner the modern era. For the major premiss in the reasoning of Kepler was a conclusion drawn from the theorems of Conic Sections : and the science of Conics was an inheritance from the days of classical antiquity. Science did not spring suddenly into being at the close of the middle ages, as some writers would have us believe. The discoveries of the great scientists of the sixteenth and seventeenth centuries were rendered possible by the labours of their predecessors.

§ 3. **Explanation by Hypothetical Deduction.** A different account of Explanation is given by some logicians. It will be necessary for us to say something of this other view, as it has exercised great influence on recent logical

speculation, especially in England. According to this theory, Explanation is achieved, not by regressive reasoning from effects to causes, but by deduction from a cause hypothetically suggested. Thus, if it is desired to explain the law *S* is *P*, an hypothesis is ventured that *P* may be due to *M*. The consequences of *M* are then calculated by deduction, and they are compared with *P* as manifested in experience. If it is found that they agree, the hypothesis is thereby confirmed. The process therefore consists of :—

(1) The formation of an hypothesis to explain the law or fact under consideration.

(2) Deduction : the hypothesis being treated as a general principle from which conclusions are drawn.

(3) Verification : or comparison of these conclusions with the facts of Nature.

This view of Explanation is that held by Mill. In his account, it appears as a special case of what he terms the **Deductive Method.** This Deductive Method is a way, by which we are enabled in certain cases to determine the laws governing complex phenomena, in regard to which we cannot apply Observation and Experiment. These are all cases in which a Composition of Causes has resulted in Intermixture of effects (Ch. 15. § 2). The application of the method supposes that we are acquainted with the various simple laws which combine to produce the complex result. Mill cites as cases in which the method is employed, such problems as that of three bodies gravitating together, and that of the path of a projectile, when the causes affecting its velocity and range are known. Here Observation and Experiment are of no avail. But solutions, at least approximately correct, may be obtained for these problems by Deduction.

Hypothetical Deduction he regards as differing from this process only in so far that our deductive calculation is made, not from a known law, but from an hypothesis provisionally assumed. The conclusion in the Deductive Method, no less than that obtained from the hypothetical

cause, must be verified by comparison with facts. Without this step, we may have overlooked some contributory cause, and so vitiated our calculations.

Mill quotes as one of the most notable discoveries we owe to the method of hypothetical deduction, Newton's conclusions in regard to the system of the universe. We shall show in a subsequent section that Newton's discoveries were not made by this method, but by regressive reasoning. But it can scarcely be doubted that we owe this theory of Explanation to an erroneous analysis of Newton's argument.

The mere fact that the conclusions drawn from our hypothesis agree with the law we desire to explain, does not, it is evident, constitute a proof that we have discovered the cause of the law. We cannot argue; If *M* then *P*; but *P* is true, ∴ *M* is true. Hence Mill rightly insists that not only must we shew that the hypothesis accounts for the facts, but we must shew that there is no other hypothesis which will do so. But he fails to observe that if we can attain certainty on this point, our argument, logically analysed, is no longer a hypothetical deduction, but a strict regressive syllogism. Our major premiss is no longer, '*M* is cause sufficient to account for *P*'; it is, '*P* is an effect which proves the presence of the cause *M*.'

Unfortunately, not all subsequent logicians have insisted on the absolute necessity of proving that the hypothesis proposed is the only one which will account for the facts. Yet without this, the conclusions obtained are logically worthless.

* § 4. **Hypothetical Deduction and Induction.** We have seen that it is impossible to accept Hypothetical Deduction as a true account of Scientific Explanation. It can only be regarded as an error on Mill's part. A worse mistake was to come. Jevons identified Mill's Hypothetical Deduction with Induction. He terms Induction "the inverse operation of deduction" [*Princ.*, p. 122]. Deduction is the process, which starts from laws that govern the combination of qualities, and infers the combinations agreeing with these laws. In Induction we have the contrary

tions of qualities as our data, and our task is to determine the laws governing the combinations. This is accomplished by hypothetical deduction, in the three stages of (1) forming the hypothesis, (2) deducing consequences, and (3) verification. The value of inductions regarding laws of nature is, he holds, never more than probable. To begin with it is ordinarily possible to form rival hypotheses as to the laws governing the combinations, in such a way that each hypothesis will give results approximating in some degree to the facts. It would be necessary to entertain all these rival hypotheses, and hold each of them to be of value in proportion to their probability. Hence the inductive inference, as regards its validity, is based on the mathematical theory of probability. Moreover, even had we accurately determined the law in question, we have no ground for certainty as to the uniformity of nature. As far as we can see, natural laws are constant; but reason cannot justify our conviction that it must be so. Belief should never amount to certainty till the experiment has been tried.

Jevons's theory that in Hypothetical Deduction we have the true account of Induction, has, strange to say, found a wide acceptance. Sigwart, Bosanquet and Welton may be mentioned among the adherents of this view. They part company however with Jevons, in so far as he bases inductive argument on the theory of probability. Certainty, they maintain, can be reached by hypothetical deduction. For this view, there can be no logical justification.

§ 5. **Explanation as employed by Newton.** In this section, we propose to consider the method of proof by which Newton succeeded in establishing the universal gravitation of matter according to the law of the inverse square. In most recent works on Logic, some attention has been devoted to this subject. Nor need we feel surprised at this. It has been justly said by Whewell that it was "indisputably and incomparably the greatest 'scientific discovery ever made. . . . Any one of the 'five steps . . . would of itself have been considered 'an important advance;—would have conferred dis-'tinction on the persons who made it, and the time to 'which it belonged. All the five steps taken at once 'formed not a leap but a flight—not an improvement 'but a metamorphosis—not an epoch but a termination." [1]

[1] Whewell, *Hist. of the Inductive Sciences* (3rd ed.), vol. 2, p. 136.

Moreover, this great discovery was precisely of the kind with which we are now concerned. It did not consist in the detection of some natural phenomenon never yet observed, in the sense in which we speak of the discovery of galvanic electricity, of argon, of the Röntgen-rays. The work was essentially explanatory. A treatment of Explanation might justly be held to be incomplete, which did not show how the account given of the process, was borne out by this greatest of all explanatory reasonings.

There is another reason, as we have already noted, for dealing with Newton's arguments. Several eminent logicians have misunderstood the nature of the reasoning, and on their erroneous interpretation have based a false theory of Explanation. Our object here is to shew that the method employed by Newton is simply the regressive method, as we have described it in § 2.

Of the five points into which Whewell divides the discoveries connected with gravitation, we shall restrict our attention to the two which may be regarded as the principal, viz. :—

(1) that the force by which the planets are attracted to the sun, is in the inverse proportion to the square of their distances :

(2) that the earth also exerts such a force on the moon, and that this force is identical with terrestrial gravity.

In each case, it will appear that the method employed was regressive reasoning from facts to their causes. Newton takes the known motions of the heavenly bodies, and argues directly from these to the nature of the force which is necessary to produce them.

(1) The facts in the first of the cases we are about to consider, were those which are contained in Kepler's laws, viz. :—

First law. The planets move in elliptical orbits round the sun in one of the foci.

Second law. The areas swept by lines drawn from the

sun to the planet, are proportional to the times employed in the motion.[1]

Third law. The squares of the periodic times are as the cubes of the mean distances from the sun.[2]

It had been surmised by several mathematicians that an attractive force situated in the sun, and acting according to a law of inverse squares, would account for these motions. But the complexity of the problem baffled their attempts at solution. Newton's genius triumphed over the obstacles. His proofs rest upon the three laws of motion. These he regarded as securely established by induction. Basing his demonstrations on these laws, he proved deductively two famous theorems in mechanics: (*a*) Every body which moves in a curve, and by a radius drawn to a point describes about that point areas proportional to the times of description, is urged by a centripetal force to that point (*Princ., Lib.* 1, *prop.* 2): (*b*) When the squares of the periodic times are as the cubes of the radii (i.e. the mean distance from the central body), the centripetal forces by which the bodies tend to the centre are inversely as the squares of the radii; and the converse of this proposition is also true (*Princ., Lib.* 1, *prop.* 4, *cor.* 6).

These conclusions provided him with all that was required to prove that the law of inverse squares governs the planets in regard of their motion towards the sun. The proof is given in the second proposition of Bk. III. of the *Principia*. The proposition is as follows:—

The forces, by which the planets are withheld from recti-

[1] The accompanying figure may be taken to represent the course of a planet round the sun; and it may be supposed that the areas ABS and CDS are equal. Then according to the law, the time employed by the planet to cover the distance between A and B, will be the same as that employed in covering the shorter distance between C and D. For the times are proportional, not to the distance covered, but to the areas swept by the imaginary lines.

[2] Thus the period of the earth's revolution round the sun is 1 year: that of Saturn 30 years. The squares of these two periodic times are respectively 1 and 900. The mean distance of the earth from the sun will therefore stand to the mean distance of Saturn from the sun as the cube root of 1 to the cube root of 900. In other words, the mean distance of Saturn is a little more than nine times the mean distance of the earth.

linear motion and retained in their orbits, tend to the sun; and these forces vary inversely as the squares of the distances from the sun.

Proof of part 1. Every body, which moves in a curve, and by a radius drawn to a point describes about that point areas proportional to the times of description, is urged by a centripetal force to that point (*Princ., Lib.* 1, *prop.* 2).

The planets, by radii drawn to the sun, describe areas proportional to the times of description (Kepler's 2nd law).

∴ The planets are urged by a centripetal force to the sun.

Proof of part 2. When the squares of the periodic times are as the cubes of the radii, i.e. the distances, the centripetal forces of the bodies will be inversely as the squares of the radii, i.e. the distances (*Princ., Lib.* 1, *prop.* 4, *Cor.* 6).

The squares of the periodic times of the planets are as the cubes of their mean distances from the sun (Kepler's 3rd law).

∴ The centripetal forces of the planets tending to the sun are inversely as the squares of their distances from the sun.[1]

(2) The demonstration we have just considered may be applied to the case of the moon in its relation to the earth. Our data regarding its motion prove that it also is subject to the law of inverse squares. But we must now consider the further step which was taken, when Newton shewed that the force which retains the moon in her orbit is one and the same with the force known to us as terrestrial gravity. It was this identification that caused us to see in the courses of the stars and the planets, a manifestation of the very same phenomenon which we witness when a stone falls to the earth.

[1] On this demonstration, see the treatment in Professor Case's lecture on *Scientific Method as a Mental Operation*, to which I am under great obligations. The translation of the proposition is that there given.

In addition to what is required in the proofs with which we have already dealt, this new argument involves further presuppositions. These are stated in the first two of Newton's *Rules of Philosophizing*,[1] which run as follows.

First Rule. No more causes of natural things are to be admitted than such as are both true (*verae*), and sufficient to explain the phenomena of these things.

Second Rule. And therefore natural effects of the same kind are to be referred as far as possible to the same causes.

The importance of these rules to his argument will appear in what follows.

The supposition that the force retaining the moon in her orbit was none other than terrestrial gravity, presented itself to Newton as an hypothesis. In order to test the worth of that hypothesis, it was necessary to know the precise amount of the force exerted on the moon by the earth. This was easily susceptible of determination. Without the influence of the earth, the moon, if the first law of motion be true, would leave her orbit, and pass away into space at a tangent. Hence, the attractive force exerted by the earth on the moon in one minute, is represented by the amount of deflection from the tangent during that period of time. Newton calculated that this amounted to very nearly 16 feet. Allowing for the law of inverse squares, it could be shewn that, if such be the attraction at the distance at which the moon stands from the earth, the ' pull ' which would be exerted, were the moon close to the surface of the earth, would be precisely 16 feet in one second. This is identical with the rate at which bodies are drawn to the earth by terrestrial gravity.[2] Here then Newton

[1] These rules will be dealt with in the next section.

[2] The proof, by which Newton shews this, may be thus summarized. The distance of the moon from the centre of the earth may be taken as equal to sixty of the earth's radii. Since bodies at the surface of the earth are at a distance of one radius from the centre, it follows by the law of inverse squares, that the fall of the moon in one minute, were it close to the surface of the earth, would not be 16 feet, but 16 feet $\times 60^2$. What then would it be in one second? Galileo's discovery that the distances fallen by bodies vary as the

applies his two rules to establish his proof. We have in the fall of the moon a phenomenon exactly similar to the fall of bodies at the earth's surface. It must therefore (*Rule* 2) be attributed to the same cause, viz., gravity. For in thus explaining it, we are not introducing any merely hypothetical agent: we account for it by a *true cause*, one that we know to be really operative in nature, and to be adequate to the effect (*Rule* 1).[1] This gives us the following regressive syllogism :—

Any material body which, on commencing to fall to the earth, passes through a space equivalent to a fall of 16 feet in one second at the earth's surface, is urged to the earth by gravity.

The moon is a material body whose fall to the earth is of this character.

∴ The moon is urged to the earth by gravity.

Such is the proof by which Newton establishes the second of those great discoveries to which we referred above, namely that the force which the earth exerts upon the moon is none other than terrestrial gravity.

§ 6. **Newton's Rules of Philosophizing.** The 'Rules of Philosophizing' preface the third part of Newton's *Principia*—the part in which he unfolds his conclusions as to the system of the universe. In them, he evidently intends to lay down the fundamental principles which have governed the reasoning of that treatise. They are, in fact, a statement of what he held to be the true method to follow in Natural Philosophy. When we reflect that these rules were drawn up by one of the greatest geniuses who ever lived, and in reference to a series of reasonings which have influenced the thought of the whole human race, we may be confident that

squares of the times, shows that to find the distance fallen in one-sixtieth part of a minute, we must divide the distance fallen in a minute by 60^2. This gives us 16 feet as the space, through which the moon would fall in one second, were it close to the surface of the earth (*Principia, Lib.* III, *prop.* 4).

[1] "Propterea vis qua luna in orbe sua retinetur, si descendatur in superficiem terrae, aequaliter evadit vi gravitatis apud nos, *ideoque* (*per regulam I et II*) est illa ipsa vis quam nos gravitatem dicere solemus" (*ibid.*).

they merit the most careful consideration. They are as follows :—

Rule I. "No more causes of natural things are to be admitted than such as are both true, and sufficient to explain the phenomena of those things.

It is a saying of the philosophers that Nature does nothing in vain, and to employ more means when less will serve would be to act in vain : for Nature is simple, and is not prodigal in unnecessary causes."

Rule II. "And therefore natural effects of the same kind are to be referred as far as possible to the same causes.

As for instance, respiration in man and beast : the fall of stones in Europe and America : light in the fire on our hearth and in the sun : the reflexion of light on the earth and in the planets."

Rule III. "Those qualities of bodies that can neither be increased nor diminished in intensity, and which are found to belong to all bodies within the reach of our experiments, are to be regarded as belonging to all bodies whatever."

(In a somewhat lengthy comment, Newton points out that it is on these grounds that we infer the extension, hardness, impenetrability, and mobility of all bodies, and the laws of motion themselves. 'And this,' he adds, 'is the foundation of all [Natural] Philosophy.' Further, an induction of this kind will enable us to conclude that gravity belongs to every particle of matter ; though, as it is capable of increase and diminution, it cannot be regarded as an *essential* attribute of material substance.)

Rule IV. "In experimental philosophy, propositions collected by Induction from phenomena are to be regarded as exactly true or as very nearly true, notwithstanding any contrary hypotheses, until other phenomena occur, by which they are made more accurate or are rendered subject to exceptions.

This must be granted, lest conclusions based on induction be denied in favour of hypotheses."

The first two rules stand in close connexion with each other. They may be considered together.

By what must be considered one of the strangest of mistakes, these rules are frequently described as stating the conditions requisite to a valid hypothesis. They are in fact rules to ensure that the doctrines advanced and the conclusions attained in Natural Philosophy, should be free from the least suspicion of being based on hypotheses. To ensure this, says Newton, we must never assert that a phenomenon proceeds from any cause, unless we know that this cause is a true cause, actually existing in nature, and is such as would produce the effect. Then, indeed, we may do so, for *Nature is not prodigal in unnecessary causes.* It would be contrary to sound philosophy to suppose that there are two natural forces each destined to produce precisely the same effect. Hence the second rule follows as a corollary of the first, and the examples indicate the use which Newton intends to make of it as to the motions of the moon in regard to the earth, and those of the planets in regard to the sun.

The principle, to which he makes appeal as the basis of the first rule, is derived from Aristotle, and is of frequent occurrence in Scholastic philosophy.[1] It is grounded on the evident fact that reason is everywhere apparent in the world, that finality and purpose rule the whole natural order. Hence, it is inadmissible to attribute to Nature what would be a mark not of reason but of unreason. The principle is fundamental in Newton's argument. If its truth be denied, his reasoning loses all conclusiveness. We can no longer assert that such and such motions on the part of the moon furnish a proof that it is swayed by gravity. It becomes perfectly

[1] Arist., *De Anima*, III. c. 9, § 6. *εἰ οὖν μήτε μηδὲν ἡ φύσις ποιεῖ μάτην, μήτε ἀπολείπει τῶν ἀναγκαίων.* 'If then Nature does nothing in vain, and omits nothing that is necessary . . .' Cf. S. Thomas, *Opusc.* 13, *De Verbo Intellectus.* 'Natura non agit aliquid superfluè.' It would seem to have been the perception of this truth which led another great scientist, Clerk Maxwell, to say that it would be "poor scientific taste" to choose the more complicated of two equally well balanced scientific conceptions.

reasonable to attribute them to another cause. All we can say is that gravitation would be capable of accounting for them. And with this, the keystone of the whole proof is taken away. The reasoning becomes hypothetical, and Newton's boast that his philosophy does not consist of hypotheses becomes empty verbiage. It is on the foundation afforded by this Aristotelian principle that the cogency of the great argument depends.

The third and fourth rules summarize Newton's view as to the true starting point in all our reasonings on physical science, with special reference to the matter dealt with in the treatise he is beginning. In the third rule, he claims the right of inferring universal propositions as to corporeal substance by induction from experience. Against such inductive conclusions, no objections based on mere hypotheses, can, he urges, have any weight.

These two rules should be compared with the following passage from the *Optics*. "In Natural Philosophy the 'investigation of difficult things by the method of Ana-'lysis, ought ever to precede the method of Composition.[1] 'This Analysis consists in making Experiments and 'Observation, and in drawing general conclusions from 'them by way of Induction, and admitting of no objec-'tion against the conclusions, but such as are taken 'from experience or other certain truths. For hypo-'theses are not to be regarded in experimental philo-'sophy. . . . And although the arguing from Experi-'ments and Observation be no demonstration of general 'conclusions:[2] yet it is the best way of arguing that 'the nature of things admits of, and may be looked 'upon as so much the stronger, by how much the induc-'tion is the more general. And if no exception occur 'from phenomena, the conclusion may be pronounced 'generally. But if at any time afterwards any excep-

[1] By Analysis, Newton signifies all argument from effects to causes whether by induction or regressive deduction: by Synthesis, the argument from causes to their effects. The terms are more fully explained below, Ch. 25, § 1.

[2] Demonstration is here used in its technical sense of syllogistic proof from first principles of admitted certainty.

'tion shall occur from experiments, it may then begin 'to be pronounced with such exceptions as occur."[1]

The only difference between this passage on the one hand and the rules on the other, lies in the fact that in the *Optics* Newton is dealing with physical science in general, while in the rules he speaks only of the inductions which shew that some attribute is to be reckoned among the properties of all corporeal substance. It is by such inductions that the laws of motion are established, on whose truth depends the value of his whole physical system. The legitimacy of the principles expressed in the rules, he regards as evident. And if these be granted, a valid science is possible, grounded on rigorous reasoning from facts to their causes. To labour at this is, he holds, the true work of the Natural Philosopher. "Later 'philosophers," he says, "feign hypotheses . . . for 'explaining all things mechanically . . . whereas the 'main business of Natural Philosophy is to argue from 'phenomena without feigning hypotheses, and to deduce 'causes from effects till we come to the very first cause, 'which certainly is not mechanical." [2]

It will be evident from these citations that the method employed by this greatest of scientific discoverers was none other than that recognized by the Scholastic philosophers. Most assuredly it was not the hypothetical deduction attributed to him by recent logicians.[3]

[1] Newton, *Optics*, p. 380 (3rd edit. 1721).
[2] Newton, *Optics*. Qy. 28.
[3] On Newton's Rules of Philosophizing, see Pemberton's *View of Newton's Philosophy* (1728), Introduction, §§ 22-25.

CHAPTER XXII.

HYPOTHESIS.

§ 1. **Hypothesis.**

An hypothesis is a supposition made with evidence recognized as insufficient, in order to account for some fact or some law known to be real. In other words, an hypothesis is a provisional or tentative explanation.

Discovery is almost entirely dependent on hypothesis. No man is likely to distinguish the law which lies hidden behind the complex phenomena of experience, unless he hazards an hypothesis, and then by observation and experiment forces Nature to declare whether his hypothesis is correct or not. The first task of the investigator is to ask himself what various hypotheses, in themselves possible, may be imagined, which are capable of accounting for the phenomena. Logic can teach no art of making suitable hypotheses. The peculiar insight, which enables a man to seize on the clue to Nature's secrets, is a gift,—a gift which may be cultivated, but not one which can be communicated in a code of rules.[1]

The method of investigation advocated by Francis Bacon dispensed with hypotheses. He believed, as we have seen, that to employ tables of presence, absence, and presence in varying degrees, would provide a royal road to discovery. The laws of nature would on the application of this method reveal themselves, and all men's minds stand on the same level.[2] Hypotheses

[1] Cf. Aristotle's account of *εὐστοχία*, *An. Post. I.*, c. 34, *Ethics*, VI., c. 10.

[2] *Nov. Org. I.*, 61. "Our method of discovering the sciences is such as to leave little to the acuteness and strength of wit, and indeed rather to level wit and intellect."

have no place in his system ; and in this opinion, he has been followed by certain extreme empiricists. But it need scarcely be pointed out that to suppose a registration of facts will enable us to dispense with hypotheses, is as though one were to suppose that the invention of the rule and compasses would enable us to dispense with genius in the architect.

Yet notwithstanding the important rôle played by hypotheses in discovery, no part of the scientist's task is more important than the rigid testing of suggested hypotheses. The aim of all investigation is the discovery of truth. To this hypotheses are but means and instruments. The scientist must not be so captivated by the ingenuity of his hypothesis, and the accuracy with which it appears to explain the phenomena, as to accept it as true before he can say with certainty that his evidence is such as to exclude every other supposition. The greatest discoverers have insisted forcibly on this truth. Pasteur in his address to his colleagues at the inauguration of the *Institut Pasteur* writes as follows :—" For ' the investigator it is the hardest ordeal which he can ' be asked to face, to believe that he has discovered a ' great scientific truth, to be possessed with a feverish ' desire to make it known, and yet to impose silence on ' himself for days, for weeks, sometimes for years, whilst ' seeking to destroy those very conclusions, and only ' permitting himself to proclaim his discovery when all ' the adverse hypotheses have been exhausted."

Descriptive Hypotheses. Hypotheses are employed in science for another purpose, besides that we have just considered. When a number of facts relating to a certain subject have been gathered together, and the true explanation of the facts is still to seek, it is often useful to group them round some hypothesis, even though it should hardly seem likely that this hypothesis will be found to represent the actual truth. Such a mode of *colligating the facts*, as it has been termed, enables us to describe them in terms of a coherent system, and

may afford us a basis for deductions relating to future phenomena of the order in question. A good instance is afforded by the Ptolemaic hypothesis in regard to the heavenly bodies. The ancients were well aware that the system of cycles and epicycles was but an hypothesis. St. Thomas expressly calls attention to the fact that at some future time it may be discovered that some other supposition is better able to account for the facts.[1] Yet it served a useful purpose, assisting many generations of astronomers in framing their calculations. It may be noted also that Andreas Osiander, in the letter which he wrote by way of preface to the work of Copernicus, reminds the reader that the hypotheses of astronomers are not necessarily asserted to be true by those who propose them, but only to be ways of representing facts.[2] Many theories in modern physics are of this character. Such, for instance, is the Atomic theory of Dalton. Such is the Electron theory of matter, which in the last few years has won for itself considerable popularity.

The view discussed in § 3 and § 4 of the preceding chapter tends not a little to confuse hypotheses of this kind with established truths of science. Mill justly pointed out that there is no reason why a false hypothesis should not afford deductions agreeing in a surprising way with facts (*Logic*, III., c. 14, § 6). Yet because the deductions from Fresnel's undulatory theory of light are in accord with facts, Jevons tells us that we must needs accept it and all that it involves, as true. "We are asked by 'physical philosophers,'" he writes, "to give up our pre-'possessions, and to believe that interstellar space is

[1] St. Thomas, *Summa Theol.* I., Q. 32, Art. 1, ad. 2. Ad aliquam rem dupliciter inducitur ratio. Uno modo ad probandum sufficienter aliquam radicem: . . . Alio modo inducitur ratio non quae sufficienter probet radicem, sed quae radici jam positae ostendat congruere consequentes effectus. Sicut in astrologia ponitur ratio excenticorum et epicyclorum ex hoc quod, hac positione factâ, possunt salvari apparentia sensibilia circa motus coelestes: non tamen ratio haec est sufficienter probans, quia etiam forte aliâ positione factâ salvari possunt.' The expression *salvare apparentia sensibilia* is the Latin equivalent for σώζειν τὰ φαινόμενα. Cf. Ueberweg, § 134.

[2] "Neque vero necesse est eas hypotheses esse veras, immo ne verisimiles quidem, sed sufficit hoc unum, si calculum observationibus congruentem exhibeant." Copernicus, *De Revolutionibus*, p. 1.

'filled with something immensely more solid and harder 'than steel. . . . We live and move without appreci- 'able resistance through this medium, immensely harder 'and more solid than adamant. . . . It is no more 'than the observed phenomena of heat and light force 'us to accept" (*Principles*, p. 515). The observed phenomena of heat and light force us to nothing of the kind. All that can be said, is that they are such as to render this a convenient, though hypothetical, method of representing the facts.

Newton again and again points out the importance of distinguishing between the established truths of science and mere hypotheses. Hypotheses, he says in a famous passage, have no place among the conclusions he proposes for our acceptance—*hypotheses non fingo*. And for this reason, he refuses to hazard any conjecture as to the cause of gravity, and bids us not regard the term 'attraction' as indicating any theory as to the manner in which bodies are urged one towards another; for he has not been able to discover from the phenomena, the cause of their gravitation. He employs the word as a mathematician to signify the mechanical law, in accordance with which bodies act.[1]

§ 2. **Origin of Hypothesis.** The source of an hypothesis is to be found either in an induction based on grounds which are recognized as only probable, or in an analogy. This may be illustrated by one or two instances of hypothesis, which we have already had

[1] See the *Scholium generale* at the end of the *Principia*, "Rationem vero harum gravitatis proprietatum ex phaenomenis nondum potui deducere, et hypotheses non fingo. Quid enim ex phaenomenis non deducitur, *hypothesis* vocanda est: et hypotheses, seu metaphysicae, seu physicae, seu qualitatum occultarum, seu mechanicae, in philosophia experimentali locum non habent." With this passage, the following words from *Definition VIII.* in Part I. of the *Principia* may be compared. "Voces autem attractionis, impulsus vel propensionis cujuscunque in centrum, indifferenter et pro se mutuo usurpo: has vires non physicé sed mathematicé tantum considerando. Unde caveat lector ne per hujusmodi voces cogitet me speciem vel modum actionis causamve aut rationem physicam alicubi definire vel centris (quae sunt puncta mathematica) vires vere et physicé tribuere: si forte aut centra trahere aut vires centrorum esse dixero."

occasion to consider. When Sir D. Brewster observed the colours of mother-of-pearl on the bee's-wax, his hypothesis that iridescence is caused by striation, was clearly a rapid induction. The data before him were two objects, both striated and both iridescent. The hypothesis, which presented itself, was that possibly striated objects *as such* are iridescent, that striation is the cause of the phenomenon.

Analogical hypotheses are no less frequent. Such in fact was Newton's hypothesis in regard to the moon :—

Bodies near the surface of the earth are under the influence of gravity.

The moon resembles bodies near the surface of the earth, in so far as it tends towards the earth.

The moon *may be* under the influence of gravity.

Such too was James Watt's hypothesis as to the motive force of steam :—

Ordinary agencies employed to raise weights do so by the exercise of motive power.

Steam resembles these agencies in its power to raise a kettle-lid.

Steam may be possessed of motive power.

There is no essential difference between the two modes of arriving at hypotheses ; for, as we have seen (Ch. 16, § 6), the basis of every analogy is a probable induction, by which the two classes S_1 and S_2 are brought together in some universal concept.

An analogical hypothesis will prove valueless, unless the character on which it is based is *important* in relation to the attribute to whose presence we conclude. The power, however, to distinguish between what is important and unimportant in any regard, depends not on a knowledge of rules, but on native intelligence, and on acquaintance with the matter in hand. The same remark also applies to hypotheses based on induction. Their value must depend on the perspicacity and insight of the investigator, in detecting which attributes are likely to be causally connected.

§ 3. **Conditions of a Legitimate Hypothesis.** The requisites of a valid hypothesis have been variously formulated. The following are the conditions as proposed by Jevons (*Princ.*, p. 511).

(1) The hypothesis must allow of the application of deductive reasoning, and the inference of consequences capable of comparison with the results of observation.

(2) It must not conflict with any laws of nature or of mind, which we hold to be true.

(3) The consequences inferred must agree with the facts of observation.

The first of these conditions needs no comment. The sole object of an hypothesis is to suggest a cause capable of accounting for the facts before us. Hence, unless it is such that we are able to deduce the phenomena from it, it fails to fulfil the very purpose for which it exists.

The second rule that the hypothesis must not conflict with any known law of nature or of mind, must not be understood to mean that the cause supposed must always act in accordance with laws, with which we are familiar. All that is necessary is that it should not violate a law, of whose existence we have positive evidence. We may be forced sometimes to posit a cause, which acts in a manner to which we have no known parallel. Thus, to take an instance in point, it has seemed to some thinkers that the Atomic theory is beset with so many difficulties, that it would be preferable to adopt the view that matter is continuous, and that though continuous, it is capable of contraction and expansion. Here, the hypothesis proposes for our acceptance a phenomenon subject to a law quite unlike those with which we are familiar. Yet it cannot on that ground, be dismissed as an impossible supposition. Professor Poynting well remarks on this subject :—

"If we believed that a piece of matter is as continuous as it 'seems to the eye, we should have to admit that contraction 'and expansion are simple facts, facts like any others. This

'supposition was characterized by Principal Sir A. Rücker in 'his British Association address at Glasgow, as unintelligible 'and absurd, in that it leaves contraction and expansion unex- 'plained. This appears to me to be carrying the passion for 'explanation to excess. To say that any simple fact, any fact 'which so far stands by itself and is unlike others, must have a 'hidden likeness, must be explicable, and that the contrary is 'absurd, is an *à priori* way of dealing with Nature, which she 'may at any time resent and refute, by bringing our so-called 'explanations to nought."[1]

In the same way, the third rule must not be so strictly interpreted as to signify that the consequences inferred from the proposed hypothesis must so agree with all the facts, as to leave no perplexities. It may well be that we are justified in accepting an hypothesis on the ground of its evident reasonableness, even though some facts remain, which *prima facie* appear to contradict it, and of which no explanation can as yet be suggested. Such was the case in regard to the heliocentric theory, when first proposed by Copernicus. It seemed to demand that the planet Venus, when nearest to the earth, should appear to be of sixteen times the size, which it has when most remote. This, of course, is not the case. Yet this apparently inexplicable difficulty did not prevent the acceptance of the Copernican theory by many of the most cultivated minds.[2] The seeming contradiction was removed, when Galileo through his telescope saw that Venus passes through moon-like phases, and that what we see is not the whole area of the planet, but its luminous part only. The portion which is luminous when the planet is near the earth, is less than that which is luminous when the planet is more remote.

It is evident that an hypothesis acquires more and more probability in proportion to the number of facts which it explains; and that if it leads to the prediction of facts, which subsequently prove to be true, this tends in an especial manner to confirm it. An hypothesis in regard to natural law, is further greatly con-

[1] '*Physical Law and Life*' in the *Hibbert Journal*, Vol. I., p. 728 (1903).

[2] Andreas Osiander in the prefatory letter before alluded to, states that this is an insuperable objection to the acceptance of Copernicus's theory regarded as an account of the actual facts.

mended by its *simplicity*, since all recognize it is characteristic of the complex processes of Nature, that they are governed by wide and simple laws,—*simplex veri sigillum*. On this point something has already been said in reference to Newton's first Rule of Philosophizing (Ch. 22, § 6). If an hypothesis suffices to explain a vast variety of phenomena by a single simple law, that alone affords an argument that we have detected the plan followed by the Supreme Reason to Whom the order of the universe is due. Attention has been called (Ch. 18, § 5) to the expression *Experimentum Crucis*, employed when we are able by the test of a decisive experiment, to exclude one of two rival hypotheses, and so to demonstrate the truth of the other.

§ 4. **Various Kinds of Hypotheses.** Different terms are employed to designate different kinds of hypotheses. The most important of these are as follows :—

(1) *Hypothesis of Cause.* This expression is used when the hypothesis relates to the agent, or to what we regard as the agent, producing the phenomenon under consideration. Thus when Pasteur was led to the belief that fermentation was caused by yeast-cells, his supposition was an hypothesis of cause. Such too was Pascal's hypothesis referring the rise of the mercury column to atmospheric pressure. In this category we may also reckon the ancient hypothesis that meteoric stones were due to the activity of terrestrial volcanoes.

(2) *Hypothesis of Law.* Often our supposition does not relate to the cause of the phenomenon, but to the law governing its operation,—the formula, so to speak, which expresses it. These are termed hypotheses of law. Until Newton established his proofs, the conjecture that the courses of the planets' orbits were determined by a force varying according to the law of inverse squares, was an hypothesis of law. Such were also the nineteen different hypotheses made by Kepler as to the orbit of Mars, before he was fortunate enough to light on the true explanation. Mendelejeff's famous hypothesis that the properties of the elements would be found to vary periodically as their atomic weights, belongs to the same class.

(3) *Working Hypothesis.* We have already explained at length the meaning of a *Descriptive Hypothesis.* The

expression 'Working Hypothesis' is used when it is desired to lay special stress on the provisional character of the assumption, and on the conviction of the investigator that it will need large modifications, before it will be found satisfactorily to account for all the facts, for which it should provide an explanation.

Theory : Hypothesis : Fact. In physical science the terms *theory* and *hypothesis* are employed in contrast to each other. The term *theory* is ordinarily understood to signify that the suggested explanation is held to have been satisfactorily proved, and to be no longer open to question. Thus we find writers use the expression 'Theory of Gravitation,' not 'Hypothesis of Gravitation.' So too the divergence of opinion between two schools may be marked by the fact that the same doctrine is by the one termed a theory, by the other an hypothesis. Such expressions as 'the theory of Evolution' and 'the evolutionary hypothesis,' indicate clearly enough the tendency of the work in which they occur. There are however certain scientific writers who do not adhere to this distinction, but assign other significations to the terms in question.

The term *Fact* is sometimes restricted to signify the particular concrete facts of experience. Sometimes it is extended to include whatever has been proved to be real, and in this sense can be used of a theory whose truth has been established.

CHAPTER XXIII.

QUANTITATIVE DETERMINATION : ELIMINATION OF CHANCE.

§ 1. **Measurement.** In the ascertainment of many natural laws, precise quantitative determination is required. "Accurate and minute measurement," says Lord Kelvin, "seems to the non-scientific imagination, 'a less lofty and dignified work than the looking for 'something new. But nearly all the grandest dis- 'coveries of science have been the reward of accurate 'measurement, and patient, long-continued labour in 'the minute sifting of numerical results." It is not, of course, the case that all laws are capable of quantitative expression. It is, for instance, as much a law of nature that dogs are carnivorous, as that the wave-length of the red line in the spectrum of cadmium is the 1,553,164th part of a metre. The laws of light demand quantitative expression : those of zoology are purely qualitative.

It should be noted that in those sciences in which measurement is employed, the stage of accurate quantitative determination is always an advance on an earlier period in which investigation was concerned with quality alone. Thus in chemistry the nature of the elements composing a compound is first discovered : the accurate determination of the precise amount of each element is a subsequent stage. Somewhat similarly in the science of astronomy, the qualitative character of the earth's motion as an ellipse round the sun was first established, and then by exact quantitative methods the precise form of that ellipse was ascertained.

In a treatise which has for its subject the general methods employed in the discovery of physical laws,

something must be said as to the nature of measurement, and as to the conditions affecting it.

All measurement is an act of comparison. To take the measure of a magnitude is to determine the relation it bears to some other quantity, which we have adopted as a standard. As regards spatial measurement, the standard quantity for scientific purposes is the metre and its subdivisions, the centimetre, millimetre, etc.[1] For temporal measurement the standard is the period constituted by the earth's revolution on its axis,—the day : this being subdivided into hours, minutes, seconds.

The act of comparison is beset by great difficulties. The conditions under which it is carried out, forbid us to hope for perfect accuracy. In the first place it depends in the last resort on sense-perception. The senses are trustworthy to a certain point : but they demand that their object should be proportioned to their powers. When we force them to ascertain something which transcends these limits, they decline the task : and we find that the reports as to an object, given by the same sense on different occasions, are at variance with each other. To ensure accuracy, instruments have been devised, which enable us to determine magnitudes many thousand times finer than the finest sense can perceive.[2] But this does but push the difficulty one stage further back. The discrepancies, it is true, relate now, not to millimetres, but to millionths of a millimetre. But they still exist. Moreover, the instruments themselves, though capable of far more accurate measurement than the unaided senses, involve imperfections of their own.

[1] What is the metre ? The standard metre is a piece of metal preserved at Paris, which originally was intended to represent precisely one ten-millionth part of the earth's quadrant. An artificial standard of this kind has however certain inconveniences. We have e.g., no guarantee that it will not change by age through the settling down of its component molecules. Hence the scientific world has adopted a natural standard. By a metre is understood a piece of metal whose length at 0° C. at the epoch A.D. 1906 = 1,553,164 times the wave-length of the red line in the spectrum of cadmium, when the latter is observed in dry air at the temperature of 15° C. of the normal hydrogen-scale at a pressure of 760 mm. of mercury at 0° C.

[2] Thus Nichol's radiometer can detect the heat received from Vega or Arcturus.

Metal contracts and expands with variations in the temperature. It is also impossible so to fit the parts of the mechanism together, as to ensure the perfect working of the instrument. And, as we shall shortly show, these are not the only sources, whence errors in our observation arise.

In most cases, the enquiry into a law of nature has relation to some special purpose ; and when an element of error is very minute, it ceases to be of any importance in regard to the purpose in hand, and may be safely disregarded. Thus, to the engineer who is constructing an iron bridge, it is of vital moment to know the laws as to the expansion of metals. It is said that the longitudinal expansion in the Forth bridge is such as to amount to a difference of seven feet between the summer and the winter measurement. Unless space is allowed for such expansion, the strain involved would rapidly prove fatal to the most solid structure. But it is sufficient for the bridge-builder to attain a certain degree of accuracy. To determine the law with greater precision, would for him be but lost labour. For other kinds of work, greater exactitude may be required. The introduction of compensations for temperature into a naval chronometer demands a delicacy of adjustment very different from that which is supposed in building a bridge. The law of expansion must be determined far more exactly. Yet even here there is a point, at which the accuracy attained suffices for the needs of the navigating officer ; and that point is far short of absolute perfection.

For the practical needs of life, nothing is lacking to us. It is when man seeks to satisfy his desire to know, that he discovers how endless is the task of unravelling the secrets of Nature. To enumerate but a few among the sources of error with which he has to reckon, we may mention not only the limitations of sense and the problem presented by changes of temperature, but further, the impossibility of securing perfect rigidity in the instrument, and in optical investigations the difficulties which arise through refraction. There may also be some

unsuspected electrical or magnetic influence exercising a disturbing effect. Finally, there is always the ' personal equation,' by which name we denote the deviations from accuracy caused by anything in the personality of the observer.

The effect of such disturbing causes is well illustrated by the case, in which after various observations have been taken as to the position of a star, and every effort has been made to ensure absolute exactitude in each, the calculated results are found to be at variance. They may approximate closely, but each places the star in a position slightly different from that indicated by the others.

This may fairly be said to be the normal case when delicate quantitative determination is in question. What course then is the investigator to adopt ? His first step must be to eliminate by calculation all errors of which he has cognizance. Thus allowance may at once be made for any slight idiosyncrasies of a particular instrument ; and in the case of those observers whose personal equation is constant, this can similarly be corrected. When this has been done, reason prescribes that he should take the mean of his observations, and adopt this as a nearer approximation to the truth than any of the separate measurements. By the word ' mean ' we do not here signify simply the ' average ' or ' arithmetic mean.' It is not always practicable to avail ourselves of this. In the next section, we shall discuss the way in which the mean must be determined. But we must now show why the mean of a number of experiments is to be regarded as equivalently their combined testimony.

The reason for this will be seen, if, in illustration, we examine the case of a particular cause of error, namely sense-perception. If we consider the nature of sense-perception in the abstract, it is clear that, viewed as a cause of error in the measurement of a magnitude, it is indifferent as to excess or defect. It tends equally to mistakes on either side. The inaccuracy, which

characterizes its testimony, arises not from any special tendency to exaggerate or to diminish, but from the inability of the senses to attain perfect exactitude. Hence it follows that as the number of observations is increased, the mean error will grow less and less, excess and defect—at least at the end of an infinite series,—tending to a ratio of equality. To suppose otherwise—to imagine that the term of the whole process would be inequality—is to suppose an effect without a cause. But this involves that when very many observations of a magnitude have been taken, *if there is good reason to regard them as equally reliable*, the mean of these observations will give us as near an approximation to the true magnitude as it is possible for us to obtain.

Now we may safely hold that the various sources of error, which we are unable to eliminate independently, are of this character. They are indifferent to excess or defect. The observations taken in a particular state of the temperature may perhaps tend to exaggeration: but there will also be days on which the error will be in the opposite direction. In proportion as the number of competent observers increases, and reliable observations are multiplied, so there should be a corresponding approximation to absolute accuracy.

It is found in practice that the experimental error of measurements, varies inversely as the square root of the number of observations, whose mean is taken. Thus the error contained in the mean of 81 observations will be 3 times less than the error of 9 observations.

In the collation of experiments, investigators at once reject any measurement which differs widely from the testimony of the others. Small discrepancies are explicable. But a wide divergence cannot arise in the manner indicated. Either the observation was carelessly made, or in some way the conditions must have been different.

* § 2. **Methods of Approximation.** There are two ways in which the mean of a number of observations may be found. These are (1) the method of means, (2) the method of least squares. As the term 'mean' has by custom been assigned to one of these two methods, it will be more convenient to speak of the two as

Methods of Approximation rather than as Methods of determining the mean.

We shall describe these two methods separately. They are distinguished for practical purposes; but it will appear that the first of the two is really a simple case of the second.

(i) **Method of Means.** This method—the simple plan of taking the average,—is only applicable when we are dealing with a single magnitude, as for instance the length of a line. The sum of the observed measurements is taken, and is divided by the number of the observations. Thus if four observations have been made as to the length of a line P, and these are denoted respectively by P_1, P_2, P_3 and P_4, the mean will be represented by the formula, $\frac{P_1+P_2+P_3+P_4}{4}$.

The method is susceptible of various modifications. Thus, investigators are accustomed to attribute *weight* to their observations, in proportion to their assumed value. If, for instance, one observation has been made under especially favourable circumstances, another under less favourable, and a third under somewhat unsatisfactory conditions, they would adopt the plan of attributing to the first of these the weight of three observations, and to the second the weight of two. This would give as a formula $\frac{3P_1+2P_2+P_3}{6}$.

(ii) **Method of Least Squares.** The Method of Means, as we have said, can only be applied when we are dealing with a single magnitude. Such a case is infrequent. As a rule, greater complexity is involved. For instance, we may suppose a case where we are engaged in determining a magnitude P, and our observations consist of two series, one relating to P, and a second relating to a magnitude q, which is some function of P. That our illustration may be a simple one, we will suppose q to be the square root of P. The two series may be represented respectively by P_1, P_2, P_3 . . . P_n and q_1, q_2, q_3 . . . q_n; and we have reason to regard each series as of equal reliability. How are we to deal with such a case? One way undoubtedly would be to take the squares of the values of q, and then find the arithmetic mean of the P and q series in combination. But there is a serious objection to this rough-and-ready way of calculating. There is, according to our supposition, a parity between the errors of the P series and those of the q series. By squaring the values of q, we have greatly increased the relative amount of error in that series, and thus have destroyed the parity between the two sets of values. We have not in this way secured a body of equally reliable values, which justify us in supposing that as the number of our observations increases, so the mean error must constantly decrease. We no longer have the same right to disregard the

residuals, as those quantities are called by which our measurements differ from the mean. Hence, wherever calculations demanding great precision are in question, it is advisable to employ another method which is mathematically superior, viz. : the Method of Least Squares. This consists in discovering that value *which makes the sum of the squares of the residuals, the least possible*. An algebraic process enables us to assign the particular value of *P*, which stands in that relation to the two series. It will be observed that this characteristic belongs to the arithmetic mean itself. In the average or arithmetic mean of two or more numbers, we have the value which makes the sum of the squares on the residuals the least possible. If we choose any other value than the mean, be it above or below, and take the squares of the quantities by which that value differs from the original numbers, the sum of these squares will be greater than the sum of the squares of the residuals given by the mean.[1] But it is not a property of the arithmetic mean as such. It is a property of the mean generically ; and even when the arithmetic mean would be of no service, we can adopt this more general method. The Method of Means is rightly termed a special case of the Method of Least Squares.

§ 3. **Chance.** Another method sometimes usefully employed in discovering and determining a law of nature, now claims our attention. It is termed the *Elimination of Chance*. The scope of this method is limited. We cannot by its means establish a relation of cause and effect between two phenomena. It merely serves to show the presence of an **Empirical Law.** This term 'empirical law,' is used with some variety of meaning. It is sometimes employed to signify any law of nature, which is with good reason believed to be a *derivative law*, capable of explanation by being brought under some more universal law, though as yet it has not been so explained. But more usually, as in this section, a restricted signification is given to it, and it denotes an observed uniformity between phenomena, which we see to be connected, though how they are connected we

[1] Thus, if we take the three numbers 8, 10, 12, whose mean is 10, the sum of the squares on the residuals (2^2+2^2), is 8. If instead of taking the mean, we select a number either above or below it, we shall find the sum of the squares on the new residuals will exceed 8. Thus if we take the number 9, they will be $3^2+1^2=10$.

know not. They are connected for us empirically, that is, in our experience : but they are not connected for us rationally, that is, in our intelligence. We observe that under certain conditions X is followed by Y. But whether X is the cause of Y, or whether they are both effects proceeding from A, or whether X is a necessary condition enabling A to produce Y—of all this we are ignorant. Knowledge of these empirical laws, imperfect as it is, is of great importance to us ; for it is not infrequently the first stage in discovering a true law of nature.

Before proceeding to the discussion of the method in question, it will be well to state clearly what is meant by Chance.

The term Chance is accurately defined by Mill. "We 'may say," he writes, "that two or more phenomena 'are conjoined by chance . . . meaning that they are 'in no way related by causation." And again, "Facts 'conjoined by chance are separately the effect of causes, 'and therefore of laws : but of different causes, and of 'facts not connected by any law" (*Logic*, III., c. 17, § 2).

Thus if I visit London, and there unexpectedly meet an old friend, who was as ignorant of my presence in the metropolis, as I of his, I attribute the meeting to chance. In doing so, I do not regard Chance as a positive entity, to whose causality the meeting was due. I simply signify that the event was a coincidence, that it was outside the scope of either of the two causal series needed to bring it about.

If we consider the causal series in this universe separately, then many of the events that take place are, in relation to the separate series, *casual*. They are not such effects as the agent of its own nature tends to produce ; but are, in fact, conjunctures. Hence, chance corresponds to a real element in our experience ; and nothing is gained by saying that since everything has a cause, there can be no such thing as chance.

It should, however, be borne in mind that what is casual in regard of this or that particular cause, may be a result directly intended by a superior cause, to which

the two causes in question are subordinate. This point has been illustrated by the case of a master, who despatches two servants on different errands, each without the knowledge of the other, to the same place. Their meeting is, as far as they are concerned, casual. But to the master, it is the result in view of which the action was planned, and to which its various stages were all directed. This suggests how in our lives coincidences may, if considered in relation to an overruling Providence, be directly intended, and constitute the true term of a chain of causality. Bain, when dealing with this subject, remarks that none but the superstitious could have seen any connexion between the death of Oliver Cromwell and the storm which, when he was expiring, burst over London. 'Each event,' he urges, 'grew out of its own independent series of causes. . . . 'They concurred in time, and that is all that should be 'said concerning them' (*Logic*, III., c. 9, § i.). The particular instance selected is a matter of indifference, and does not concern us. But the objection involves a *petitio principii*; for it implicitly denies the existence of a First Cause, who disposes and orders all finite and subordinate causes.[1]

The meaning we attach to Chance, when we attribute to it, for example, the result of a throw at dice, is but very little removed from the sense, which we have been discussing. In this case, it is true, the result is not due to a coincidence. The physical action of the player may in a true sense be held to be the reason not only that dice fell from the box, but that on coming to rest, they present such and such faces. Yet the player does not attribute this latter effect to himself, because his action in throwing was not designed to produce that particular result. In general, we regard as our own only those effects which we designed our action to produce. Here, the action was solely directed to ejecting the dice from the box; it bore, and could bear, no relation to any particular face turned up. Hence, we do not attri-

[1] On the subject of Chance, cf. St. Thomas, *Summa Theol.* I., Q. 116, Art. 1.

bute the good or ill success of the throw to ourselves. We regard it as casual, just as we regard the unlooked for meeting with a friend as casual.

§ 4. **Elimination of Chance.** The Elimination of Chance is a method by which we show that the conjunction between two circumstances is not casual, that they must be connected in virtue of some law. It is based on the principle that if two events are connected by chance alone, their connexion will not be stable. The cases in which X is followed by Y, will be balanced by other cases in which it appears without Y. On the other hand, if one event accompanies the other in a way that cannot be attributed to mere chance, then we are justified in supposing that there is valid evidence for the existence of a causal connexion. The principle, it will be seen, is identical with that of the Method of Agreement. But it is peculiar to the Elimination of Chance, that it enables us to allow for a number of cases in which, owing to what are termed *counteracting causes*, the law is not verified, the antecedent appearing without the consequent.

The method is well illustrated by the case, in which it is employed for the detection of loaded dice. If a die shows a marked tendency to turn up the number six, the question naturally suggests itself whether the appearance of that face, is after all a purely casual result of the throw. Is it not a direct and natural result of the action, due to the presence of a bias within the die? We know that if the die is perfectly normal the figure six should appear on the average but once in six throws. If then it turns up, not once in every six throws, but three times in every four, we may justly conclude that there is something amiss.

In a case like this, the reason which convinces us that the result is not casual, is the same as that on which we relied to shew that we are justified in adopting the mean of a number of measurements as the nearest approach to the true determination of a magnitude. For, since

in the case of a normal die, our action in throwing has no natural tendency to turn up one face more than another, to suppose a great preponderance in favour of a special result, would be to suppose an effect with no adequate cause.

Yet here we are met with a difficulty. It is only after an infinite series that we can count on attaining a ratio of equality between the number of times that each face has appeared. No impossibility is involved in the supposition that in the first four throws with a perfectly normal die, the six should appear three times, and that after twenty or thirty throws, the proportion of sixes should be greatly above the due average. Now every series of experiments, which we can adduce as evidence, is finite, not infinite. How are we to decide that the excess over the proper average, is such as to postulate a cause? Must this decision be left to the rough estimates of common-sense, or is it possible to obtain some scientific determination of the question?

This problem has been dealt with in the mathematical theory of Probability. To that subject, our next section must be devoted.

It is evident that the point we have raised has an important bearing on the question of the adoption of the mean measurement. There also we calculate from a limited number of measurements, and not from the infinite series, which would render us certain that excess and defect must have attained equality. Hence, in that case too, the investigator employs the theory of Probability, and determines by the aid of its calculus how far it is in any degree probable that his results may be erroneous.

§ 5. **Probability.** Even when a proposition is not certain, there may be valid motives for giving a definite measure of rational credence to it. In this case we say that the proposition is more or less probable: and by the probability of the proposition we understand **the measure of rational credence due to it.**

In the mathematical theory of probability, numerals

are employed to denote the measure of credence. Certitude as to the truth of a proposition is denoted by unity, certitude as to its falsehood by zero. Where the motives are such as to lead us to give it a *probable* assent, our credence is denoted by a fraction corresponding to the weight of the motive.[1] The question may indeed suggest itself, whether it is possible thus to measure motives. It is perhaps, for the present best answered by our old illustration of the dice. Here the motives which influence our expectation of the six, are 5 against its appearance and but 1 in favour. Hence the probability of the proposition that the six will appear, is $\frac{1}{6}$.

What has been said will have shown that the meaning of the term probability as here employed, differs somewhat from its customary use. As it is ordinarily used, we only term probable that which happens more often than it does not. What happens but seldom, is designated as possible, not as probable. But in mathematics even that which is barely possible, is reckoned as probable, its probability being expressed by a fraction corresponding to such motives as it can claim.

What is the proper subject-matter of the theory of Probability ?

There are certain classes of events, such that, though we are able to affirm with confidence that a particular proposition will be true in regard to a definite proportion of them, yet we cannot affirm this proposition of any member of the class taken separately. Let us suppose that in regard to the population of England at the present epoch, reliable statistics, extending over many years, shew us that from every thousand men of twenty-five years of age, ninety per cent. live to be thirty-five. In such a case, we have no doubts that this will hold good of the population as a whole in the near future ; but we should not venture to assert of any particular individual who is now twenty-five years old, that he will live another decade. Similarly, I may have good grounds

[1] ' La probabilité est donc une fraction toujours inférieure à 1.' M. Victor Baudot in the article *Probabilité* in the *Nouveau Dictionnaire des Sciences*.

for believing that in the coming autumn five days out of every seven will, in the part of England where I live, be wet. But I cannot venture to predict of any special day, whether it will be among the fine or the rainy ones.

It is where our knowledge regarding events is of this character, that probability has its place. Now it will be noticed, that our knowledge here has two special features. In the first place, it is based on the existence of law. If we were not convinced that the proportion of deaths to survivals between the ages of twenty-five and thirty-five was the result of definite laws, which will continue to produce the same result under similar conditions, we should have no ground for believing that this proportion would be maintained. We cannot argue from past to future, from the known to the unknown, except on the ground of some law. Even when we hold that, in the long run, each face of the die will appear an equal number of times, we do so relying on the principle that no event can take place without a sufficient reason. Secondly, our knowledge is of a very restricted kind. We know that causes exist producing certain results according to a definite law, but these causes are so complex as to elude any attempt to track their operation in detail. Our only resource is to take some class, within which they operate, and see what is the proportion of the cases, in which the event occurs, to those in which it does not occur. Thus, we take the class Englishmen between twenty-five and thirty-five, and estimate what is the proportion of cases, in which the causes that are hostile to human life, actually destroy it. The class we select, though it be one within which the cause operates, must, of course, be such that in a certain number of cases the effect is not produced. Otherwise, we should have certainty, not probability.

Although probability is thus based on definite data, it supposes a certain ignorance as its condition. To an omniscient Being, there can be no probability; for to such a one the law governing each cause would be known.

It will be observed that the probability of an event

to one person, is often not the same as its probability to another. Thus, of a man concerning whom I only know that he is an Englishman twenty-five years of age, I may say that the chances of his living to be thirty-five, are 9 to 1. Some one who is better informed, and knows that he is a worker in a factory, where the proportion of those who survive to be thirty-five, is only 80 per cent., may say that his chances are but 4 to 1. The reason of this divergence lies simply in the fact that the data are gathered in relation to different classes. The former case deals with the chances of the man, in so far as he is an Englishman ; the latter with his chances, in so far as he is a worker at such and such a trade.

It may perhaps be asked, in what sense we regard the probability of an event as the *chances* of that event. This question will be answered if we consider what is involved in the expression of probability as a fraction. When, for instance, we say that the probability an Englishman of twenty-five will survive to be thirty-five, is $\frac{9}{10}$, we mean that we view the class in question as divided into ten parts, nine of which belong to the survivors, and one to those who die. We consider these ten parts as representing ten alternatives of equal likelihood, just as when the die is thrown, we regard it as equally likely that any one of the faces may appear. But as nine of the alternatives are the same, the chance in favour of that event is $\frac{9}{10}$. In considering the alternatives as equally likely, we in no sense deny that in every case the event is determined by its own causes. But in relation to our principle of enumeration, there is an equal likelihood of each alternative. As regards the fact that a person is an Englishman of five and twenty, his survival to the age of thirty-five or his death previous to it, is a casual result.

We have now explained in outline the meaning of probability, and shewn what are the cases in which we can discover by a mathematical calculus, the measure of credence which a proposition claims. It remains to

point out how this calculus can assist us to determine with accuracy the point at which the repeated conjunction of two events affords conclusive evidence of causal connexion.

Let us take a case, in which we have to draw a ball from a bag containing three white balls and two black, replacing after each trial the ball chosen. The chances of drawing a white ball are $\frac{3}{5}$; and if a large number N trials be made, it is probable that the white ball will appear $\frac{3N}{5}$ times. Yet it must be owned that though this number is more probable than any other, the probability that it will be exactly realized is but remote. The famous mathematician James Bernouilli (1654–1705) proposed to himself to determine within what limits in a large number of trials, the result actually obtained will approximate to the ideally most probable result. This difficult problem occupied his thoughts during twenty years, and his solution of it is known as *Bernouilli's theorem*. In a case such as the above, where the probability of the event in each trial is $\frac{3}{5}$, he showed that the odds are 1,000 to 1 that in 25,500 trials, the event shall occur not more than 15,841 times, and not less than 14,819,—that is that the deviation from 15,300, the ideally probable number, shall not exceed $\frac{1}{50}$ of the number of trials.[1]

It need hardly be said that such a probability amounts to a moral certainty.

Further, as the number of trials is multiplied, the ratio of the number of deviations to the number of occasions on which the occurrence agrees with the laws of probability, will grow less and less.[2]

[1] Vide article on *Probability* in *Encycl. Britan.* (9th edit. vol. 19, p. 769) by Prof. Crofton.

[2] The following is the account of Bernouilli's theorem in M. Victor Baudot's article *Probabilité* in the *Nouveau Dictionnaire des Sciences.*

"Théorème de Jacques Bernouilli.—Définissons d'abord ce qu'on nomme 'écart : supposons que la probabilité d'un événement soit p et qu'on fasse μ 'épreuves successives, pendant lesquelles l'événement se produit m fois : l'écart 'est la différence positive ou négative $\mu p - m$. Ainsi, on lance 4040 fois une 'pièce en l'air, $\mu = 4040$; la probabilité p qu'on ait face est $\frac{1}{2}$, donc $\mu p = 2020$;

The bearing of this upon the elimination of chance is evident. If we are able to say that in a definite number of trials, certain limits will not be overstepped, and if it is discovered that the result is totally at variance with our mathematical estimate, then it becomes clear that we were mistaken in our view as to the nature of the case. We are manifestly not dealing with events conjoined only by chance, the relative number of whose conjunctures is determinable on mathematical principles, but there is a law of connexion (or of repugnance) at work. A loaded die gives, it is true, very irregular results. Were it not so, it would be detected at once, and so defeat its own purpose. But there comes a point at which after a certain number of trials, the mathematician is able to say that the appearances of the six exceed the limits of mere chance, and that the very act of throwing must tend to turn that face uppermost.

Philosophical Probability. We have been dealing in this chapter with Mathematical Probability alone. It remains to add a few words, in order to distinguish it clearly from Philosophical Probability—the probability with which we are concerned when judging as to the value of various hypotheses (Ch. 22, § 3). Where this is in question we estimate the *relative degree* of rational credence to be attached to two suggested explanations. But we cannot apply mathematical methods to determine the quantitative measure that expresses that relation. The comparison is qualitative, not quantitative,

'si face se montre 2,048 fois, on dira que l'écart est − 28 ; si elle se montrait '2,000 fois, l'écart serait 20. Le théorème de Bernouilli revient à ceci : 'Si 'on fait un nombre indéfiniment croissant d'épreuves l'écart est infiniment 'petit par rapport au nombre des épreuves.' Il faut bien remarquer que c'est 'l'écart relatif qui tend vers zéro, l'écart absolu devient au contraire de plus 'en grand. Cette dernière remarque fait comprendre certains résultats qui 'peuvent paraître, au premier abord, contraires à l'évidence : Pierre et Paul 'jouent, sans jamais se lasser, à pile ou face, l'enjeu est chaque fois de 1 franc ; 'il arrivera un moment ou l'un des deux adversaires sera ruiné ; il est vrai 'que le rapport du nombre des parties gagnées par chaque joueur au nombre 'des parties perdues par le même tend vers 1 quand on joue indéfiniment, 'mais la différence de ces deux nombres, et c'est là ce qui ruine l'un des joueurs, 'grandit aussi de plus en plus."

and it is only where quantity is concerned that numerical calculation is possible.

Philosophical probability resembles mathematical probability in that it rests on the principle that similar conjunctions of events demand a cause or law governing them and accounting for them. To suppose a series of independent causes, when the resemblance of the events is real and not imaginary, is as much out of the question in the one case as in the other. But it differs from mathematical probability in that its degree is estimated by the character of the explanation suggested: and this is judged by the ability of the hypothesis to account for the facts, by the analogy it bears to other known causes, by its intrinsic reasonableness. We have indeed no means of measuring values such as these numerically. We cannot determine the number of possible laws which might account for the facts, nor range them in a quantitative scale according to the degree of reason and harmony they manifest.[1] But although numerical calculation is impossible, the scientist can affirm without hesitation that of two alternative explanations the one is more probable than the other.

[1] "Mais d'autre part, la probabilité philosophique diffère essentiellement de 'la probabilité mathématique, en ce qu'elle n'est pas réductible en nombres: 'non point à cause de l'imperfection actuelle de nos connaissances dans la science 'de nombres, mais en soi et par sa nature propre. Il n'y a lieu ni de nombrer 'les lois possibles, ni de les échelonner comme des grandeurs, par rapport a 'cette propriété de forme qui constitue leur degré de simplicité, et qui donne, 'dans les degrés divers, à la conception théorique des phénomènes, la symmétrie, 'l'élégance et la beauté." A. Cournot in Franck's *Dictionnaire des Sciences Philosophiques*.

CHAPTER XXIV.

CLASSIFICATION.

§ 1. **Classification.** We have already (Ch. 19, § 1) distinguished the natural sciences into two groups—those which deal primarily with the static forms of Nature, and those which deal with her dynamic activities. To the first group belong, for instance, Systematic Botany and Systematic Zoology, the portion, that is, of the sciences of Botany and Zoology, which deals with the distribution of animals and plants into their genera and species. Here, just as in the physical sciences, the object which the investigator sets before himself is the knowledge of natural laws. He seeks to establish necessary and invariable connexions between attribute and attribute. Thus it is the science of Systematic Zoology which teaches us that each of the general types, mammals, birds, reptiles, amphibians and fishes is characterized by certain definite properties peculiar to the class in question, and found in each and every member of the class without exception. The methods, however, employed in this group of sciences, differ in some measure from those we make use of in physical science. The relative importance of Observation and Experiment in the one group, is almost the direct converse of what it is in the other. In the physical sciences, Observation unaided by Experiment can do comparatively little. The successful investigator is he who can best devise and best conduct his experiments. In the sciences dealing with Nature's stable forms, Experiment, hitherto at least, has played a smaller part.[1] It is on Observa-

[1] The discoveries of Abbot Gregor Mendel, and the application of the principles called after him 'Mendelism,' seem to point to a far greater employment of Experimental Methods in these sciences in the future.

tion, that investigators have had principally to rely. Further, the method of **Classification,** with which we are concerned in this chapter, is altogether proper to those sciences. Thus they are not infrequently simply termed the *Classificatory Sciences.*

The work of Classification is to group individuals according to their natural species, and to group the species according to their natural co-ordination and subordination. Hence the object which we propose to ourselves in this Scientific—or as it is usually termed, Natural Classification, is the knowledge of the order which Nature offers to our contemplation. In the natural grouping of the classes, the mind contemplates the unity, the harmony, the reason, which pervades the whole creation. It recognizes that there is working in Nature an intelligence, to which its own is akin. Classification is far from being, as some describe it, merely an aid to the attainment of practical ends. It is a method subservient to speculative science.[1]

By many writers, as we have already noticed, Classification has been confused with Logical Division. They are correlative, the one to the other : they are not identical. In Classification, we deal with the real order : it is the concrete individuals whom we classify. To this classification of the individuals, there corresponds in the conceptual order a logical division. The concept of the genus—the logical whole—is divided by the addition of differentiae into its species, and these again into subordinate species, until the logical unit, the *ultima species,* is reached.

The term Classification is, however, not confined to the Natural Classifications of which we have spoken. The exigencies of practical life often demand that we should form Artificial Classifications. Whenever we

[1] Cf. Franck, *Dictionnaire des Sciences Philosophiques,* Art. *Classification.* ' Les classifications dites naturelles ne nous laissent pas le choix entre plusieurs points de vue ; il n'y en a qu'un seul qui soit vrai, et pour le découvrir, il faut préalablement évaluer, avec le concours de l'expérience et du raisonnement, l'importance relative des diverses parties des objets. Tel est le principe de la subordination des caractères, que M. de Jussieu a le premier dégagé, et qui généralisé par M. Cuvier a renouvelé la face des sciences naturelles.'

have to deal with a great number of individual objects, if we are to avoid complete bewilderment, we must find some means of classifying them. The arrangement may be wholly arbitrary, as for instance, that of a biographical dictionary, in which men of different races and colours, some of them famous for their virtue, and some for their crimes, are distributed into twenty-six groups according to the place which the first letter of their name holds in the alphabet. A more artificial system could hardly have been selected. Yet it is well adapted to its purpose; for our aim is not speculative, but purely practical. It is that we may without difficulty be able to find any particular biographical notice, which we desire to consult.

Before dealing with Natural Classifications, it will be well to consider shortly these Artificial Classifications, noting how completely they differ from those which form the proper subject of this chapter.

§ 2. **Artificial Classification.** Any attribute whatever may be chosen to serve as the basis of an Artificial Classification, provided that it fulfil two conditions. It must be common to the whole group of objects to be classified, and its different modifications must be sufficiently distinct to enable us to assign each individual to one and one only of the subordinate classes. Hence, according to the purpose which a man has in hand, he will select a different attribute as the ground of his classification; and the same group of objects will be found variously classified by different men. Thus the same collection of engravings might be classified by an historian according to the events represented, by an artist according to the various schools of art or according to the individual artists whose work was found in the collection, by a print-dealer according to market value.

The cases, in which we employ Artificial Classifications, are of two kinds, which need to be carefully distinguished.

We may be concerned (1) with a limited number of individuals. The classification of the words in a par-

ticular language, of the pictures in a gallery, of the automobiles in a given region, will serve as instances. Or (2) the number of individuals to be classified may be unlimited. This occurs, when we are dealing with all the members of some natural class.

In the first of these cases, we often find it convenient not to select a characteristic which the objects already possess, as the basis of classification, but ourselves to confer a characteristic for this very purpose. We affix an artificial mark to each member of the class, and then wherever we may chance to come across any individual, we recognize its place in our classification. The method is familiar to us in its application to the army, police force, automobiles, etc.

In the second case of which we have spoken, we must perforce rely on some natural characteristic. The famous Linnaean system of botany affords us the classical instance of such classification. In this system, the various species of plants are arranged in genera according to the number of stamens and pistils they possess, and irrespective of any other similarity or dissimilarity than is afforded by this single characteristic. Such a distribution is rightly termed artificial, even though it is based, as we have stated, on a natural characteristic. It brings together into a common genus, species which are connected by the sole fact that they have the same number of stamens and pistils, but which in all else are remote from each other : and it relegates to widely divergent classes, species which in the natural order are closely allied.

Investigators have had recourse to these classifications, when the perplexities of the natural system have been such as to baffle their efforts to arrange the classes. In such cases, they have judged an artificial system better than chaos. It at least enables us to assign a definite place to each member of the group in virtue of distinctive and easily recognizable marks. Yet such a classification is only of temporary value. Even for the practical end in question, the natural classification

is far the most convenient, and in the long run the artificial system is discarded in its favour. What artificial arrangement, for instance, could assign such distinctive class-marks as are afforded by the classification of phanerogamous plants into dicotyledons, monocotyledons, conifers and cycads, or of vertebrates into mammals, birds, reptiles, amphibians, fishes? It is true that the riddles of natural classification are many. To these we shall advert in a subsequent section. But it is nevertheless the case that natural classes display conspicuous and characteristic features, by which, better than by any other attribute we might select, we may assign to each object a determinate place in the group to which it belongs.

§ 3. **The Doctrine of Natural Species.** The aim of Scientific Classification is, as we have already noted, to group individuals according to their natural species, and to group the species according to their natural co-ordination and subordination.

It is evident that Classification, thus understood, depends entirely on the existence of species. Yet it is a fundamental tenet with many men of science that the doctrine of natural species is destitute of all solid foundation. We propose to consider this question in the present section. It would be idle to discuss how species should be classified, were it dubious whether they exist or not.

It is a manifest fact of experience, that a vast number of the members of the animal and vegetable kingdoms are, at the present epoch of the world's history, distributed in types. They are not a mere assemblage of individuals, all of them so different the one from the other, that it is impossible to affirm a universal proposition about any determinate group. On the contrary, the types in question involve, not only certain broad notes of resemblance, but a multitude of the most detailed characteristics, common to every member of the class. The minuteness of these resemblances is often extraordinary.

This point is well brought out in the following passage :—

"Not so mysterious as the similarity of character, 'but equally wonderful, is that of the outward form. '. . . . I will take a bird which every one must know '—the Chaffinch. Every cock Chaffinch has a black 'forehead, and a bluish grey head and nape, with a 'narrow half-collar of oil-green between this and the 'chestnut of his back : the quill feathers of his wing 'have each a narrow edging of greyish-white : of the 'wing coverts, some are always black, and some white, 'and one row is black at the base with a white tip to 'each feather : the inner primaries have each a white 'patch at the base of the outer web, while the pair of 'tail feathers next to the outer ones, have each a narrow 'white outer margin and a triangular patch on the inner 'web. . . . Never assuredly did human skill more 'accurately reproduce the simplest of designs. It makes 'the matter not less but more astonishing that the bird 'exhibits unmistakeable tendencies to vary. . . . What 'is it that holds such tendencies in check ? " [1]

The similarities are not merely external. In every type they manifest themselves through the whole of the organism. The conclusion seems forced upon us that the complicated system of characteristics, of which the type consists, constitutes in each case a *morphological unity*. The type is not merely an average standard, to which the members of the class approximate more or less roughly, some of them lacking some of the characteristics, and some lacking others. These characteristics unite to form a whole, and each such whole is a closed system in itself.

Moreover, not merely is it the case that morphological structure is identical in all the members of the class ; but the process of development, of metamorphosis when such occurs, and of reproduction, are the same in each individual, and as far as our experience reaches, are repeated generation after generation without change.

[1] J. Gerard, S.J., *Science and Scientists*, p. 70.

Biologically as well as morphologically the type is a unity.

Notwithstanding the weight of these facts, the existence of separate species is denied, as a view now entirely superseded by the doctrine of Evolution as enunciated by Darwin and so widely accepted since his day. At the present time, the theory of Evolution has been developed along several different lines. Hence it is impossible to give an account of it which shall be both brief, and at the same time complete. According to the teaching of the founder of the school, the manifold forms of animal and plant life now existing owe their origin to the principle of Natural Selection. Animal and plant life is susceptible of indefinite variation, and in the struggle for existence, the weaker and less efficient forms have been extirpated, the stronger and better-equipped alone have survived. The characteristics which mark the different birds and beasts, are not, as had been thought, due to the impress of different types. They are merely the record of this long battle. There is no such thing as type. Class merges into class ; and there are no abrupt transitions in the scale of living beings.

The more recent followers of Darwin admit that Natural Selection alone is insufficient to account for the phenomena. Each of the more eminent among them has a different theory as to the true explanation of our present classes. On one point, however, they are agreed : they are unanimous in holding that what seem to us to be distinct types, have all arisen from a common stem, through an accumulation of small changes : that there are no breaks in Nature's chain : that species are a figment of the imagination. Professor Ray Lankester thus describes the effect as he conceives it, produced by Darwin's *Origin of Species* :—

" Universal opinion has been brought to the position ' that species as well as genera, orders and classes, are ' the subjective expressions of a vast ramifying pedigree, ' in which the only existences are individuals, the apparent ' species being marked out, not by any distributive

'law, but by the purely non-significant operation of 'human experience." [1]

Now those who maintain the existence of distinct species, are in no way concerned with the question as to the manner in which they originated. The point at issue is a different one, namely whether at the present time there do not exist types, each of which is a morphological and biological unity in the sense explained: whether there is not a real objective principle present in each member of the type, in virtue of which it possesses certain characteristics common to every individual of that species. Notwithstanding the popularity which the Darwinian hypothesis still enjoys in certain schools of thought, it may be said without hesitation that the weight of evidence is overwhelmingly in favour of the existence of such types. To this effect we may cite some words of Fr. Wasmann, one of the most competent German authorities :—

"The world of organic life, at the present day as also 'in the past, certainly does not consist—as the Dar-'winian form of the theory of evolution would have us 'believe . . . of a chaos of minute variations. . . . 'It constitutes on the contrary a well ordered system 'of species, genera, families, orders, classes, phyla." [2]

Fr. Wasmann's conclusion is the more interesting inasmuch as he adheres to the view that our present species originated by evolution, though not in the manner suggested by Darwin.

Another investigator of the first rank, the Dutch botanist H. de Vries, has recently shown that such experience as we possess regarding the origin of classes by descent, tends to prove that the transition need not take place by the accumulation of numberless small differences, but may be sudden and abrupt, involving the production of a new type in the full sense of the word.[3] He himself discovered among some 'escape'

[1] *Encycl. Brit.* (9th ed.), Art. *Zoology*, vol. 24, p. 805.
[2] *Die moderne Biologie und die Entwicklungstheorie* (3rd ed.), p. 316.
[3] *Die Mutationstheorie*, 1903.

specimens of *Oenothera Lamarckiana* (the large-flowered evening primrose) several novel forms which displayed that persistency in their characteristics, which is the mark of the true species as distinguished from the mere variety.

It is urged as a conclusive objection to the doctrine we are now defending, that in some genera there is found a series of transitional forms so complex and various that the number of species into which they are divided in any system, depends on the personal predilection of the systematist. As instances in point, our attention is called to the fact that the number of German species of *hieracium* (hawk-weed) has been fixed by one author at 300, by another at 106, by a third at 52, while a fourth is content with only 20. The willows, brambles, and calcareous sponges, are said to afford examples of the same state of things.

The difficulty indicated has been met in more than one way. Wasmann contends that in the animal and plant world there are classes in which the formation of new species is not yet at an end. This he believes is fairly clear in the case of many plants such as *hieracium* and *rubus* : and he claims that his own investigations regarding the *dinarda* show that it is true of some at least among the lower animals. He considers moreover that in certain cases, the development of new types produces a greater variety of transitional forms, than appear in the instances observed by de Vries.

Others meet the objection in a different manner, and reply that it proves no more than that hitherto the species constituting the genera in question, have not yet been distinguished. When the species of any genus are closely allied, and there is moreover considerable variability within the limits of the different species, their separation is necessarily attended with great difficulty. In such cases, it must be a matter of prolonged labour to distinguish the types, and determine the definition of each. It has yet to be shewn that in the genera in question this task is impossible, and that we may

not eventually succeed in determining a precise and definite boundary between species and species.

The solution of this problem may be left to scientists. We are not concerned with the question whether natural species are universal. It is enough for our purpose, if it be admitted that they exist even within a limited range, and that we have thus a ground for classifications based not on the 'non-significant operations of human experience,' but on Nature's own 'well-ordered system of species and genera.'

§ 4. **Natural Classification.** The systematic ordering of species according to their natural relationships is rendered possible by the fact that the numerous characteristics which constitute the type, have a hierarchical order among themselves. If for instance, we enumerate the characteristics present in a *bos taurus*—one of our ordinary domestic cattle,—it is patent that some are proper to the species *taurus*; they are not found in the other *bovinae* but in the *bos taurus* alone. Others belong to all the *bovinae*: others again are qualities of every ruminant. There are some, which are distinctive of the mammal as such; and some which are common to all vertebrates.

The series of properties which distinguish every such grade are connected one with the other in such wise, that from one of these properties we may argue to the presence of the others. "Hoofs on the feet," says Cuvier, "indicate molar teeth with flat crowns, a very 'long alimentary canal, a large or multipled stomach, 'and a great number of relations of the same kind." [1] Similarly, just as the characteristics of the *ungulata* are thus determined by their mutual relation, the general features of the *carnivora* and other orders are no less rigidly connected.

It is not, of course, the case that an animal is a mere agglutination of notes; that, e.g. all ruminants are identically the same up to a certain point, but that while

[1] Cuvier, *Leçons d'Anatomie comparée*, t. I, IIre Leçon, Art. iv.

the *oves* have a few additional notes of one description, the *bovinae* possess corresponding peculiarities of another kind. All ruminants are doubtless distinguished by characters that are *generically* alike. But though the general type of an ox and a sheep, in so far as they are ruminants, is the same, the actualization of that general type is different in every detail. The stomach of a sheep is not like the stomach of an ox. Yet both are expressed in the common concept of ruminant; for the concept admits of, and demands, ulterior determination. It is generic, and does not inform us in what specific form the general type is realized.

The hierarchical order described, is one of subordination between the grades. If, for example, the investigator, who is examining some fossil, can detect a feature proper to the *mammalia* as such, he knows that the digestive system of the animal must have been realized in one out of a few clearly distinct modes. The general character of mammal is compatible with any of these modes. They, on the other hand, are incompatible with any other animal form than that of the mammalia. Each of them is found only among the mammals, but none of them embraces the whole of that class. Hence, the characteristics of the mammal are more fundamental in the structure of the animal, than those which belong to it as ruminant: those which mark it as a ruminant, are more fundamental than those which it possesses as a *bos*. The properties which distinguish the lower grades are subordinate to those which mark the higher. This will explain what is meant by the term **important** as applied to some characteristic. It is often said that our classifications must be based on the relative importance of properties. The importance of a property depends on the position it holds among the characteristics, considered in their natural subordination. Those properties are the most important, which are the most fundamental in the structure of the animal, and which thus exercise a determining influence on the greatest number of characteristics.

The different grades of properties will enable us to frame the definitions, on which the attainment of the classification depends. When we have, by a careful process of observation, and so far as experiment is possible, by experiment, secured the true definitions, the classification is achieved : we know how Nature has distributed the real order into its classes.

In regard to natural classes, we must, as we have already noticed (Ch. 10, § 2) define by properties. Our definitions are *Distinctive Definitions*. Both genus and differentia contain an indefinitely large series of notes ; nor have we any means of knowing when we have exhausted the properties of any class.

The preceding considerations will justify the following rules, which are those usually given for the work of natural classification.

(1) All groups should be so constituted as to differ from each other by a multitude of attributes.

(2) The higher the group, the more important should be the attributes by which it is constituted.

(3) The Classification should be graduated, so that the groups with most affinity should be nearest together, and so that the distance of one group from another may be an indication of the amount of their dissimilarity.[1]

It must not be supposed that the classification of the natural orders is a simple matter. While certain main outlines are easily recognizable, there is much that is profoundly obscure, and which seems destined to baffle the most earnest seekers after truth. This is especially the case at both extremities of the great scale. The classification of the lower animals presents manifold riddles. And the great principle which determines the whole hierarchical subordination, is still in dispute. The school of Linnaeus sought it in an ascending scale of greater complexity of organization : de Jussieu and

[1] Cf. Welton, *Manual*, § 61.

Cuvier believed it to lie in the different types of anatomical structure, which characterize certain great divisions of the animal kingdom : Darwin, in the laws of descent.

Moreover, even where most progress has been made, Nature refuses to be bound by our hard and fast lines. The natural order is too complex and too various to fit into our schemata. Our system may be admirably suited for a large part of the animal or vegetable world ; but there will always be some classes, which seem to demand a totally different arrangement. It may be well, to indicate one or two of the difficulties, which foil the systematizer.[1]

(1) In co-ordinate classes, the same organ is often found to possess very different values. Thus, in mammals, the dental formula is an important property, determining the character of many other attributes. In the parallel class of fishes, it is relatively of no value, while the nature of the integument is of high importance among fishes, but of little or none among mammals.

(2) The same property is of different value at different stages of the animal's life-history. Sometimes, organs which are indispensable during the embryonic stage, lose their value when it attains maturity. Further, some animals, if judged by their embryonic characters, have a claim to be ranked in a class from which in their maturity they would be far distant. On grounds such as these, the Ascidians or Sea-squirts are now frequently assimilated to the Vertebrates under the common denomination of Cordata.

(3) The same animal may possess organs, which are characteristic of different species. Thus, the Sloth has a dental formula, which would place it among the lowest mammals, while many of its properties would entitle it to a place among the Primates.

[1] 'On ne peut pas demander à la science de nous montrer dans la plan de la Nature une netteté de divisions, une inflexibilité de lignes, dont nous nous accomoderions fort sans doute, mais dont la Nature, qui a mieux à faire apparement, ne s'accomode pas.' Rabier, p. 211. The examples which follow are drawn from M. Rabier's work.

This inability of ours to make a final scheme coinciding exactly with Nature's plan, led Whewell to advance his theory of **Classification by Type.** The term 'type,' as will be seen, is here used in a different sense from that in which we have been employing it. We cannot, he urges, classify by definition; for there will always be classes which fall outside our definition of the genus, and yet must be reckoned as belonging to it. We must classify by type. "A type is an example of any class, for instance a 'species of a genus, which is considered as eminently possessing 'the character of that class. All the species which have a greater 'affinity with this type-species than with any other, form the 'genus, and are arranged about it, deviating from it in various 'directions, and in various degrees. Thus a genus may consist 'of several species which approach very near the type, and of 'which the claim to a place with it is obvious: while there may 'be other species which straggle further from this central knot, 'and which are yet more closely connected with it than any 'other" (see Mill, IV., c. 7, § 3).

To this theory Mill objects that if classifications were framed not by definitions but by these exemplar-types, no general proposition of any kind could be asserted about the class: whereas classifications are chiefly valuable in so far as in them we attain to general truths and laws of Nature. "The truth is," he says, "that every genus or family is framed with distinct reference 'to certain characters, and is composed first and principally, of 'species which agree in possessing all these characters." It is these characters, as he rightly holds, which give us the definition of the class. "To these [species] are added as a sort of appendix, 'such other species, generally in small number, as possess *nearly* 'all the properties selected: . . . and which, while they agree 'with the rest *almost* as much as they agree with one another, do 'not resemble in an equal degree any other group" (IV., c. 7, § 4).

* § 5. **Classification by Series.** Mill devotes a chapter to what he entitles Classification by Series: he holds that the Natural Classifications of the animal and vegetable kingdoms should conform to this type. This Classification by Series is, however, something quite distinct from Natural Classification, as we have understood it. We have regarded the attainment of a Natural Classification as an end desirable in itself; since in such a classification we possess speculative knowledge of the real order. To Mill, classification is merely a means: it is an operation subsidiary to induction, which provides, "that things shall be 'thought of in such groups . . . as will best conduce to the 'remembrance and ascertainment of their laws" (IV., c. 7, § 1).[1]

[1] This view of Classification finds expression in the terms *Classifications for General Purposes* and *Classifications for Special Purposes*, by which Mill desig-

Thus, the object of a systematic classification of animals, is to assist us in discovering the laws of animal life. The two requisites of a classification, intended to aid in the study of any phenomenon, are, he tells us, (1) that it should embrace every class of things in which that phenomenon is found, and (2) that it should "arrange those kinds in a series, according to the 'degree in which they exhibit it, beginning with those that 'exhibit most of it, and terminating with those that exhibit 'least" (IV., c. 8, § 1). This method of arrangement is required, because it is in this way that we are enabled to apply the Method of Concomitant Variations, and observe what phenomena vary in the same ratio as the one under consideration. It follows that a Natural Classification of animals will place man as the example of life in its highest degree, and will arrange the other species in a descending scale. It should not however be thought such a classification involves that all the species are to be drawn up in a linear progression. Many of the subordinate classes would probably be equidistant from the leading type, so that the arrangement would be best compared to the disposition of objects at various distances from one centre.

Mill attributes this portion of his theory to Comte, and regards it as of considerable importance. But it may be doubted whether it possesses the value he attaches to it.

nates Natural and Artificial Classifications respectively. These terms are preferred by many logicians who, on the ground that there are no fixed species, deny that any classification is, properly speaking, Natural.

CHAPTER XXV.

METHOD.

§ 1. **Scientific Method.** Some treatment of Method forms part of most works on Logic. Two separate subjects are, however, included under the common designation of Method : (1) the different ways in which scientific knowledge can be attained and presented to the mind, and (2) the general rules which should guide us in the arrangement of our arguments.

In the present section we are concerned solely with the first of these. It is a question, which, strictly speaking, belongs to the first part of this work ; and some portion of the matter proper to the section, has been anticipated. Yet we have reserved the more formal consideration for the present chapter.

There are two ways in which the mind can acquire scientific knowledge. It may attain it by passing from the cause to the effect, from the law to the fact exemplifying the law, from the essence to the property ; or on the other hand, it may pass from the property to the essence, from particular events to universal laws. One of these ways, it must needs use. For it is characteristic of science, that in it we know not merely the fact, but the reason of the fact. We view the fact as the expression of a law, the effect of a cause. In order to possess knowledge of this kind, we must either have passed from the antecedent to its consequent, or from consequent to antecedent.

These two ways are known respectively as **Synthesis** and **Analysis.**[1] The reader will see that the distinction

[1] The Scholastic logicians employ the synonymous terms *methodus compositionis* and *methodus resolutionis*. Vide St. Thomas, *Opusc.* 63 *in Boeth*, Q. 6, Art. 1, ad. 3.

between them is based on the same principle as that between Progressive and Regressive Reasoning (Ch. 12, § 4, Ch. 21, § 2). The scope of Synthesis and Analysis is however somewhat wider than that between the two forms of deductive argument. Analysis embraces not merely Regressive Deduction but further, Induction : for by Induction we argue from facts to laws : from particular events to the universal principle. Every act of Natural Classification is thus an analytic process. For in classifying, we bring to light a law : we pass from the complex individuals to the simple universal type.

There is, it is true, a perfection in the method of Synthesis, which is lacking in that of Analysis. Knowledge finds its ideal, when the mind grasps first the truth of principles, and from them reasons to their results. In such reasoning, we trace the same path as Nature. The logical process is in full harmony with the ontological, the conceptual order with the real.

But human science inevitably depends more on Analysis than on Synthesis. For our knowledge begins from the concrete particular. It is from singular objects perceived by sense, that we must discover universal laws : from effects, we must pass to their causes. Even when we employ the synthetic method in exposition, this is not because we have grasped the truth of the principle first : we are merely retracing the steps we have previously travelled along the *via resolutionis* (Ch. 21, § 2).

This will explain the well-known Scholastic distinction between *things which are better known to us* (*notiora quoad nos*) and *things which are better known naturally* (*notiora secundum naturam, notiora simpliciter*).[1] To us concrete particulars—the object of sense-perception—are better known than universals ; while the universals which are

[1] The distinction is Aristotle's. *An. Post. I.*, c. 2, § 4. Elsewhere (*Met. II.* c. 1, § 2) he compares man's intellect to the eye of a bat, which sees in the dark, but is dazzled and blinded by the light of day. For our faculties find their connatural object in the particular ; yet, if considered not in relation to our faculties but in its own nature, the more universal is any principle and the wider its range of application, the ampler must be the knowledge which it really affords.

intelligible to reason, are 'better known naturally.' Analysis is the method, in which starting from things which are *nobis notiora*, we reach the principles which are *notiora secunduam naturam*. In Synthesis, as we have already said, we follow the path of Nature herself.

There is one science, and one alone, viz., Mathematics, in which Synthesis is the native process of the mind. Our faculties give us an insight into the nature of spatial extension and figure, such that in this case it is connatural to the intellect to reason from principles to their necessary consequences. Here, what is better known to Nature, is at the same time also better known to us.[1] Hence, just in the measure that we can apply Mathematics to the solution of problems in the other sciences, those sciences can be treated synthetically.

In some measure indeed, Mathematics employs the analytical method; and the other sciences make use of synthesis. Geometrical problems are not infrequently dealt with by analytic reasoning. The problem is supposed solved, and we trace step by step what principles are involved in its successful solution. If this process leads us at last to principles known to be true, we are able to reverse the reasoning, and deduce the solution from these principles. This regressive method in Mathematics is the original sense of the word Analysis. It was employed by the Greek mathematicians with this signification.

We make use of Synthesis in the natural sciences in certain cases, where the composition of causes is involved. Jevons supplies a good example in regard to the nature of storms. Here the complexity of the operative conditions is such as to render analytic reasoning almost out of the question. We can however employ the converse method, and argue from experiments of the labo-

[1] Cf. St. Thomas *in An. Post. I.*, lect. 4. "Non enim aliquid potest fieri nobis notum nisi per id quod est magis notum nobis. Quandoque autem id quod est magis notum quoad nos, est etiam magis notum simpliciter et secundum naturam: sicut accidit in mathematicis. . . . Item quandoque id quod est notius quoad nos, non est notius simpliciter, sicut accidit in naturalibus."

ratory to what takes place on a larger scale in the atmosphere. " Putting together synthetically from the sciences ' of chemistry, mechanics and electricity, all that we ' know of air, wind, cloud and lightning, we are able ' to explain what takes place in a thunderstorm far more ' completely than we could do by merely observing ' what takes place in a storm " (*Elem. Lessons*, p. 207).

The student will be careful not to confuse the logical analysis, of which we have been treating, with analysis as practised e.g. in chemistry. We have been dealing with the mental process, by which we pass from a result to its principles. Chemical analysis deals with the real order.

Though the concrete particular is better known to us than is the abstract universal, yet in our knowledge of the concrete particular itself, there is first a transition from the more general to the less general. Thus we see an object at a distance, and recognize it first as substance, then as animal, then man, lastly as such and such an individual.[1]

§ 2. **The Methodic Pursuit of Truth.** Method has been defined as *a systematic manner of carrying on the search for truth.* Such a definition might indeed be applied to Method as explained in the last section : it might signify the two systems of Analysis and Synthesis.[2] But it may also be taken to signify the general rules, which should guide us in the arrangement of our reasonings ; and in more recent times the term has often been understood in that sense. Various lists of such rules have been drawn up by different authors.

Jevons has well remarked that it is quite impossible to frame prescriptions of this kind. No general directions can possibly be given for the arrangement of things

[1] This point is treated fully by Albertus Magnus, *Phys.* I., Tr. I. c. 6. ¶ ' Ex istis est advertendum.' Cf. also St. Thomas, *An. Post. I.*, lect. 4. ' In omni enim generatione, quod est in potentia est prius tempore. . . . Cognitio autem generis est quasi potentialis in comparatione ad cognitionem speciei. . . . Unde in generatione nostrae scientiae prius est cognoscere magis commune quam minus commune.'

[2] ' Modus ordinatus procedendi in veritatis inquisitione.' Goudin, *l.c.* He however understands Method as signifying solely Analysis and Synthesis. The Port Royal definition is as follows : *The art of disposing well a series of many thoughts, either for the discovering of truth when we are ignorant of it, or for proving it to others when it is already known* (Part IV. c 2, *Baynes's trans.*)

which require an infinite variety of arrangement. No useful purpose, as he rightly urges, can be served by such rules as we find in Aldrich's *Logic*, where we are told that in every argument we propound :—

1. Nothing should be wanting or redundant.
2. The separate parts should agree with each other.
3. Nothing should be treated save what is suitable to the subject or purpose.
4. The separate parts should be connected by suitable transitions. In brief, completeness, harmony, relevancy, and continuity must be observed.

The student will observe that these so-called rules of Method are merely what common-sense prescribes in the case of any end which must be attained by the employment of various means. They would be as appropriate in the case of a watch-maker as they are to the logician. What we need, if it were possible to obtain it, is guidance to distinguish what is wanting and what is redundant : and this would depend on the particular matter in hand, of which it supposes expert knowledge. The Cartesians brought it as a reproach against the Scholastics, that they had omitted this part of Logic. In fact, nothing was to be gained by attempting to treat it.

Since Mill included the general theory of scientific research in Logic, Method has ceased to signify simply the general rules of argument. It has been taken to mean the method of successful scientific research. As an example of the treatment accorded to the subject from this point of view, we may notice the article on *Scientific Method* in *Baldwin's Dictionary of Philosophy and Psychology*. The author of the article—Prof. C. S. Peirce—in regard to those sciences which have reached the nomological stage in which universal laws are known, distinguishes five steps in the work of scientific research. These steps are, in brief, as follows :—

(1) To form a perfectly definite and consistent idea of what the problem really is : then to develop the Mathematics of the subject in hand as far as possible : and to establish a mathe-

matical method appropriate to the particular problem, if it be one which admits of exact treatment.

(2) To consider the methodeutic of the research in hand, and the manner in which analogous subjects have been treated.

(3) In the third place, if the subject be a broad one, the student should reform his metaphysics.

(4) The enquiry into the laws of the phenomena constitutes the fourth stage. This is commenced by a careful observation of the phenomena themselves. Observation is followed by the framing of an hypothesis, the deduction of the results which would follow from the hypothesis suggested, and a verification of the hypothesis by a comparison of its results with the facts.

(5) Finally the law of the phenomena once made out, it remains to measure with precision the values of the coefficients in the equation, which expresses it.

The steps here prescribed cannot, we think, be regarded as satisfactory. Metaphysics are, without doubt, of essential importance in all scientific questions. But a student, who has embarked on a scientific problem, and equipped himself for the task by the requisite knowledge of the question, can hardly be advised at that stage to embark on the somewhat vague task of 'reforming his Metaphysics.' Logically Metaphysics must take the first place, as the basis of all science. Then follows Mathematics as a requisite for every science in which quantity is concerned. But these, together with the particular branch of Mathematics applicable to the subject in hand, should be supposed known. They are an essential preliminary to the methodic enquiry into the phenomenon under investigation. As regards this essential part, it will be noticed that Prof. Peirce's method is none other than that explanation by hypothetical deduction, which we have considered in Ch. 21, § 3.

§ 3. **Philosophic Terminology.** Although it is impossible to prescribe precisely for the systematic ordering of our thoughts in the attainment of truth, yet some such general rules may be given for the use of language, the instrument and the support of thought. On the correct use of terms, the attainment of truth is largely

dependent. This is, above all, the case in philosophy. If, in speaking of some object that is cognizable by sense, I employ a term properly applicable to something else, I may confuse my hearers : but the thing in question is present to my imagination, and I can hardly mislead myself. But in philosophy, we deal with abstractions remote from sense. The employment of an inappropriate term, may introduce a wrong idea into the mind, without our being aware of the error ; and the result may in the long run be complete intellectual confusion. To take an example from the history of philosophy, it is impossible to overestimate the harm caused by Kant's restriction of the term 'Analytic proposition' to those judgments alone, in which the predicate is part of the intension of the subject. This terminology, involved as its consequence, that judgments in which the predicate is a property of the subject, and judgments depending on the experience of sense, were thrown into the common class of 'Synthetic propositions.'

It need scarcely be pointed out that similar ill effects will ensue, if we employ terms insufficiently defined, and the intension of which we conceive but vaguely.

The foregoing considerations may be summed up in three rules, of considerable importance for the student :—

(1) To employ the most commonly accepted terms in their most commonly accepted meanings.

(2) Where any doubt as to the meaning of a term can arise, to define carefully : and in points of great importance to make use of technical terms.

(3) Never to employ a term to which he cannot assign a precise and clearly-defined meaning.

The completeness of the Scholastic terminology, and its philosophic precision in regard to the definition of each term, were a signal merit of the mediæval schools, and lightened the labour of the student in no small degree. In these days, on the contrary, we are face to face with a score of different systems ; and the terminology of each is different. Nor is this a mere matter of words. In almost every case, a change of terminology

betokens a real difference in regard to some philosophic tenet, which is frequently of far-reaching importance. What at first sight appears a mere verbal technicality, not meriting serious attention, is found, on a closer view, to involve a fundamental cleavage of doctrine.

It will be well for the student to remember that few things contribute more to confusion of thought than a careless employment of the terminology belonging to different schools. The terms of a particular school embody its thought. By such a selection, he may commit himself to inconsistent views, and render clear thinking an impossibility to himself. He may, moreover, unconsciously mould his opinions in a fashion he little contemplates. For, just as the popular beliefs of a nation are largely determined by words and phrases that are in vogue, so too the terminology we employ, does much to fix our views. "Words," says Sir W. Hamilton, "are *the fortresses of thought*. They enable 'us to realise our dominion over what we have already 'overrun in thought." By employing the terminology of a particular school, we may find ourselves committed to the defence of the fortresses that school has erected.

* § 4. **Descartes' Rules of Method.** Descartes, in his *Discourse on Method* (Part 2), gives four general rules of method, by which, as he tells us, he believed it would be possible 'to arrive at the knowledge of whatever lay within the compass of his powers.' These were embodied in the Port Royal Logic, and since then have been cited by so many writers, that it seems advisable to make some mention of them. They run as follows :—

I. Never to accept anything for true which I do not know to be such : that is, to comprise nothing more in my judgment, than what is presented to my mind so clearly and distinctly as to exclude all grounds of doubt.

II. To divide each of the difficulties under examination into as many parts as possible, and as may be necessary for its adequate solution.

III. To conduct my thoughts in such order, that by commencing with objects the simplest and easiest to know, I may ascend by little and little, and as it were, step by step, to the knowledge of the more complex : assigning in thought a certain order even to those objects, which in their own nature do not stand in a relation of antecedence and sequence.

IV. In every case to make enumerations so complete, and reviews so general, that I may be assured that nothing is omitted.

It is remarkable that those logicians, who have cited these rules with approval, appear to assume they were intended to be simply common-sense prescriptions as to the conduct of reasoning. As a matter of fact, they embody a particular method of scientific research, which is perhaps the most characteristic feature of the Cartesian philosophy, and which none now would care to defend.

The objects, which the human intellect can know, were distinguished by Descartes into 'absolute' and 'relative.' The 'relative' can be known only by resolution into the 'absolute.' The 'absolute' are incapable of further resolution: every such object is a *simple nature*, and is of its own essence, knowable. For instance, of the three pairs of correlatives—cause and effect,—equal and unequal,—unity and multiplicity,—the first in each case is 'absolute'; the second, 'relative.' Further, axiomatic truths, such as that the whole is greater than its part, are also 'absolute' objects of knowledge. These 'absolute' truths are known by intellectual intuition. Once presented to the mind, they are immediately recognized as true; for they possess the infallible criterium of truth—*clearness and distinctness*. No induction is needed to establish these truths. Moreover, it is on them that all science must be based. Nothing may be admitted save what is derived from them, and thus shares in their clearness and distinctness (*Rule* 1).

When we undertake the examination of some phenomenon with a view to its explanation, we must analyse its circumstances, till we discover the 'simple nature' on which it depends. There our division stops. For 'simple natures' are irresolvable. Thus, if e.g. the object proposed is the refraction of light, an examination of the circumstances will convince us that the 'simple nature' concerned, is light itself. A scrutiny of our conception of light, will show us that we possess an intuitive knowledge of its nature (*Rule* 2).

The next step will be to proceed from the simple to the complex: to deduce, by a synthetic process, in which proposition follows proposition in logical sequence, the phenomena of nature from our intuitive knowledge of these primary truths. The separate sciences, however, do not stand in a relation of antecedence and sequence. Hence, it is not to be imagined that the whole of knowledge can be connected in one great concatenation. But even where truths do not admit of being arranged in connected sequence, a fixed order should be assigned them in the mind (*Rule* 3).

From time to time, our analysis or our synthesis will fail us. We shall come to a point, where there is a break in the chain

between the simple nature and the phenomenon under consideration. Then, we must have recourse to the process called by Descartes *Enumeration* or *Induction*. It is, however, more allied to Analogy than to what we term Induction. All the known phenomena of a similar kind must be enumerated, and by a consideration of them, we shall be able to determine the point at issue. Thus, for instance, when Descartes is discussing the phenomena of the rainbow, he solves the question by an appeal to what takes place when the sun's rays strike a glass ball (*Rule* 4).

This was the method by which Descartes hoped to establish the whole of science on a mathematical basis, and by which he believed he had solved the principal problems of natural philosophy. "I have essayed," he writes, "to find in general the 'principles or first causes of all that is or can be in the world, 'without taking into consideration for this end anything but 'God himself, who has created it, *and without educing them from 'any other source than from certain germs of truths naturally existing 'in our minds*. In the second place, I examined what were the 'first and most ordinary effects that could be deduced from these 'causes : and it appears to me that in this way I have found 'heavens, stars and earth, and even on the earth, water, air, fire, 'minerals and some other things of the kind." [1]

* § 5. **Leibniz's Views on Method.** It was probably due to the influence of Descartes' writings on Method, that Leibniz, at a slightly later date, turned his attention to this subject. To his mind the systematization of knowledge was of the first importance. The knowledge possessed by the human race, is, he urges, vast and various : but it is unorganized and dispersed in the works of many authors. System must be brought into the sciences. The ideal before us should be nothing less than a vast Encyclopedia of knowledge, in which every science is represented, and all the truths constituting it are deductively proved from certain first principles. To this ideal, we cannot indeed hope perfectly to attain. We shall often be compelled to rely on some principle which is not self-evident, but which we have good reason to suppose true. No harm will be done by this, if it be frankly stated that the principle is a supposition, and though apparently correct, has not been demonstrated from the primary truths on which the science is based. There are, he holds, such suppositions even in mathematics. Such is Euclid's assumption that two straight lines cannot have a common segment. Not improbably a way will afterwards be found of proving these suppositions, and then all the conclusions we have deduced from them will possess demonstrative value. Nevertheless, our aim should be to achieve a complete deductive proof from a series

[1] *Discourse on Method*, Pt. 6.

of definitions, whose truth is established by identical principles.[1] If possible, proofs resting on identical principles should be discovered even for axioms. Descartes, he considers, erred in holding that whatever he saw clearly, was true. To postulate such a criterion, is to beg the question. The ultimate criterion must be identical principles.[2]

This method must, of course, be applicable in different degrees to different sciences. In Physics it can hardly be applied ; for, since we are ignorant regarding the essential natures of material substances, we cannot hope to demonstrate their properties deductively. Yet even here, one who is versed in the matter in hand, will probably be able to judge with some accuracy as to the properties likely to belong to a substance. But although the application of this method to the physical sciences presents special difficulties, it is suited in the highest degree to moral science, the noblest object of our faculties.

Moreover, were the sciences thus systematized, it would be possible to indicate the way to develop the conclusions deducible from first principles and from established facts. This method of discovery would be as simple as are the processes of Arithmetic or Algebra. It was, he assures us, his intention to demonstrate the feasibility of this method, as soon as he should have secured sufficient examples to make its value evident. In a calculus of this kind lies the ultimate perfection of Art of Discovery (*l'art d'inventer*). This art already exists, though it be little cultivated.[3]

[1] Leibniz (*Monadology*, § 35) defines 'identical propositions' as "those 'primary principles which cannot be proved, and indeed have no need of 'proof . . . whose opposite involves an express contradiction."

[2] Cf. *De Synthesi et Analysi Universali* (ed. Gerhardt, vol. 7, p. 295), 'Itaque cujuscunque veritatis reddi potest ratio, connexio enim praedicati cum subjecto aut per se patet, ut in identicis, aut explicanda est, quod fit resolutione terminorum. Atque hoc unicum summumque est veritatis criterium, in abstractis scilicet neque ab experimento pendentibus, ut sit vel identica, vel ad identicas revocabilis.'

[3] See *Préceptes pour avancer les sciences*, ed. Erdmann, p. 165. *Nouveaux Essais*, IV., cc. 12, 21.

QUESTIONS ON LOGIC

These questions for the most part are taken from papers set at one or other of the universities of Great Britain or Ireland, or in the examinations for H. M. Indian Civil Service. The source is indicated in each case by an initial letter: O=Oxford, C=Cambridge, L=London, G=Glasgow, T.C.D.=Dublin, R.U.I.=Royal University of Ireland, I.C.S.=Indian Civil Service. For the permission to print them here, courteously accorded to me by the responsible authorities, my thanks are due.[1]

The numbering of the exercises corresponds to that of the chapters.

Exercise I.

1. What are the three parts of logical doctrine? How may they be named (*a*) with reference to language, (*b*) with reference to thought? (C. *Prev.* 1902.)

2. To what extent is Logic concerned with language? What are the respective provinces of (1) Logic, (2) Grammar, (3) Psychology?

3. Examine the following statements as to the province or function of Logic:

(*a*) The science not of Truth but of Consistency.

(*b*) The science of the operations of the understanding which are subservient to the estimation of evidence.

(*c*) A machine for combating fallacy. (L. *Inter. Arts*, 1905.)

4. Distinguish between (*a*) the conceptualist and nominalist methods of treating Logic. (*b*) Logical and psychological laws of thought. (T.C.D. *2nd year Hon.* 1904.)

5. Discuss the various attempted definitions (not descriptions) of Logic, with which you are acquainted. (O. *Mods.* 1907.)

6. Discuss the validity of the distinction between Formal and Material Logic, and comment upon other ways of expressing the same or a related distinction. (L. *Inter. Arts*, 1903.)

Exercise II.

1. Explain the nature of Abstraction. Of what importance is this operation in the formation of the Logical Concept?

[1] For leave to print questions drawn from Indian Civil Service papers I am indebted to the Controller of H.M. Stationery Office.

What do you understand by a Phantasm ? How does a Phantasm differ from a Concept ?

2. Define *Term, Concept* : and illustrate from a logical point of view the process of forming a concept.

Is every General term the name of a class ? Give your reasons. (L. *Inter. Arts*, 1906.)

3. Explain concisely the distinction (*a*) between General and Singular Terms ; (*b*) between Concrete and Abstract Terms ; illustrating your explanation by a discussion of the logical characteristics of the following terms :—The University of London, Logic, Iron, Sound, Fallacy. (L. *B.Sc.* 1906.)

4. Discuss the distinctions (*a*) between Collective and Distributive ; (*b*) between Positive and Negative Terms. Illustrate your discussion by examining the following statements :—

' All the plays of Shakespeare cannot be read in a day.'

' Any one who is not industrious must be accounted idle.'

(C. *Prev.* 1904.)

5. ' The Connotation of a name consists of the qualities on account of which the name was given.'

' The Connotation of a name consists of the qualities implied in actual discourse.'

Consider the distinction of Connotative and Non-Connotative names in the light of these propositions. (O. *Mods.* 1906.)

6. (*a*) What do you understand by a Non-Connotative term ? Is " the longest river in the world " Connotative or Non-Connotative ?

(*b*) Examine the following :—' Popocatapetl is a Non-Connotative term to us, only so long as we do not know what the Mexicans mean by the word.' (L. *Matr.* 1905.)

7. Explain and criticise the view that Extension and Intension vary inversely. (O. *Mods.* 1904.)

8. What is meant by Infinite terms ? The notion of such terms has been denied. How can it be defended ?

9. Consider carefully the distinction between Concrete and Abstract terms. Under which head (if under either) would you class Adjectives ? (O. *Mods.* 1907.)

10. Explain the division of terms into (*a*) Univocal, Equivocal, Analogous : (*b*) Terms of First Intention, Terms of Second Intention.

Exercise III.

1. ' Judgment is a synthesis of two concepts.'

' Judgment is an act of division rather than an act of synthesis. Comment on these statements.

2. Explain the significance of the logical copula.

3. " Judgment consists in comparing together two notions or ideas of objects derived from simple apprehension, so as to ascertain whether they agree or differ."

Critically examine this view, and indicate carefully the relation you take to hold between notions, judgments and inferences.

(L. *Inter. Arts*, 1905.)

4. Discuss the significance for logical purposes of the distinction between Analytic and Synthetic Judgments.

(L. *Inter. Arts*, 1903.)

5. What is the exact meaning of "some" in a logical judgment?

Express the following sentences in logical form :—

(i) When a man aims blindly, he sometimes hits the mark.
(ii) Few men get all they want.
(iii) No Scotsmen need apply.
(iv) Many a flower is born to blush unseen.
(v) A few sailors were saved.
(vi) Men alone can be members of Parliament.

(G. *M.A.* 1907.)

6. Explain the distinction between Complex and Compound Propositions, and show its importance.

(T.C.D. *2nd year Hon.* 1904.)

7. Give the sense of each of the following sentences in such a way as to show its logical quantity and quality :—

(1) No news is good news.
(2) Set a thief to catch a thief.
(3) One of the commonest of stupid misstatements is that all our public monuments are bad. (C. *Prev.* 1902.)

8. Define Exclusive and Exceptive propositions. What is the logical relation, if any, between them? Express the following propositions in logical form :—

(*a*) The only man in the house was the landlord.
(*b*) Any one but an idiot would believe it.
(*c*) No admittance except on business.
(*d*) Members alone can vote. (L. *Inter. Arts*, 1906.)

9. Reduce to logical form :—

(*a*) The older we are the more experienced we become.
(*b*) Few men really know their own mind.
(*c*) Children alone are admitted.
(*d*) Any answer is not good enough. (L. *Matr.* 1905.)

10. Give a brief account of Modal propositions as understood in the traditional Logic. Contrast this system with that introduced by Kant.

11. In what sense if any are Universal Categorical judgments hypothetical? (I.C.S. 1901.)

Exercise IV.

1. Define a Law of Thought: and explain how it is possible for a law of thought to be violated. (C. *Prev.* 1904.)

2. Discuss the logical significance of the laws of Identity, Contradiction, and Excluded Middle. (L. *Inter. Arts*, 1903.)

3. Enunciate in the form that seems to you most appropriate to the Science of Logic, the laws of Identity, Contradiction and Excluded Middle. Give reasons for preferring the form you select.

4. Discuss the claim of the Law of Identity to be a Law of Thought independent of the Principle of Contradiction.

5. Are the principles of Contradiction and Excluded Middle as stated by Aristotle, true without exception ? (I.C.S. 1901.)

* 6. Mill rejects two common theories as to the Law of Contradiction and substitutes his own. Discuss.
(T.C.D. *2nd year Hon.* 1904.)

* 7. What do you take to be the ground of validity of the laws of Contradiction and Excluded Middle ? Notice briefly any other theories regarding this subject with which you may be acquainted.

Exercise V.

1. Is the use of circles in Logic legitimate ? (O. *Mods.* 1908.)

2. What are the desiderata in a perfect system of diagrammatic representation ? How far does Euler's system fail to realize these requirements ? Is the task of finding an ideally perfect scheme capable of fulfilment ?

3. (*a*) Contradiction is the most perfect form of logical opposition. Explain this.
(*b*) Show that the principle of Contradiction does not apply to sub-contraries. (R.U.I. *2nd Arts, Hon.* 1906.)

4. State, giving examples with *S* and *P* used for terms, what can be asserted as to the truth of a proposition from :—
(*a*) the falsity of its contrary.
(*b*) the truth of its sub-contrary.
(*c*) the falsity of its subalternate. (L. *Inter. Arts*, 1905.)

5. Discuss the character of the following oppositions :—
(*a*) Men are mortal : Men are not mortal.
(*b*) All swans are white : Some swans are black.
(*c*) All swans are white : No swans are white.
(L. *Matr.* 1904.)

6. It has been asserted that the rule, ' Subcontrary propositions cannot both be false,' is erroneous. The two propositions, ' Some men (*i.e.* Europeans) are black,' ' Some men (*i.e.* Ethiopians) are not black,' are, it is urged, particular propositions of opposite quality, yet both are false. Examine the argument.

7. Assign contradictory and (where possible) contrary forms for the following propositions :—' Dead men alone tell no tales,' ' Caius said that it was all up with the army,' ' All except the idle have a right to the means of life,' ' Half the flock perished of the pest.'

Exercise VI.

1. Define Immediate Inference :—and give an example of each of the following :—Inference by Subalternation ; Conversion ; Contraposition ; Added Determinants ; Converse Relation ; giving in each case a brief justification of the process.

(L. *B.Sc.* 1903.)

2. (*a*) Express the following in strict logical form, and if possible give in each case the converse of the contrapositive :—

(1) Only ants are really industrious.

(2) Some insects only are industrious.

(3) No leaves are other than white.

(*b*) Examine the following inference :—A tortoise is an animal. Therefore an unusually swift tortoise is an unusually swift animal.

(L. *Matr.* 1904.)

3. Enumerate and illustrate the several forms of Immediate Inference, and consider to what extent they give us new truth.

(O. *Mods.* 1906.)

4. Define Conversion and Obversion. Find the obverse of the converse of (*a*) None but the brave deserve the fair ; (*b*) Some who are well educated are credulous : and the converse of the obverse of (*c*) Where there is smoke there is fire ; (*d*) Not all is gold that glitters. (C. *Prev.* 1904.)

5. Investigate the formal relations between :—

(*a*) All *S* is *P*, and All not-*S* is *P*.

(*b*) All not-*S* is *P*, and All not-*P* is *S*.

(*c*) All not-*S* is *P*, and All *S* is not-*P*.

(R.U.I. *2nd Arts*, 1896.)

6. Mention all the Immediate Inferences which may be drawn from the proposition, 'None but the brave deserve the fair.' Indicate very briefly the peculiarities of each inference you draw.

(R.U.I. *2nd Arts*, 1896.)

7. 'What is inferred in Conversion is not properly something else than the convertend, but the same thing.' Is this so ?

Explain what is meant by Immediate Inference, and state how many valid kinds might be included within the range of Formal Logic. (R.U.I. *2nd Arts, Hon.* 1901.)

8. (*a*) Define Obversion and point out the principle on which this form of inference is based.

(*b*) State the logical relation, if any, that holds between the first of the following propositions and each of the others. If any proposition cannot be inferred from the first, say whether it is consistent with it or not.

(i) Good men alone are wise.

(ii) He who is good is not unwise.

(iii) Any one who is not good is unwise.

(iv) No unwise men are good.

(v) Not all good men are wise.

(vi) It is false that all unwise men are good.

(G. *M.A.* 1906.)

9. How would you show that the denial of a denial amounts to an affirmation ? (I.C.S. 1895.)

10. Show how a statement may be expressed formally as an *A* or an *E* or an *I* or an *O* proposition. Does this change of form involve any material change of meaning, and is its application of any practical value in Logic ? (R.U.I. *2nd Arts*, 1902.)

Exercise VII.

1. What is meant by the import of Propositions ? State various views which logicians have held on this subject. Which view seems to you preferable, and why ? (L. *Inter. Arts*, 1903.)

2. "General Propositions assert a relation between classes." Discuss this statement. (O. *Mods.* 1907.)

* 3. State and discuss the doctrine of the quantification of the predicate. (G. *M.A.* 1906.)

4. Discuss the various interpretations of the logical proposition implied by the following modes of expressing the judgment 'All men are mortal' :—

(*a*) Men=men mortals.

(*b*) Every man possesses the attribute of mortality.

(*c*) The attribute of mortality constantly accompanies the attribute of humanity.

5. "Many propositions which in reference to the universe of sensible experience would be meaningless, when referred to the real universe of discourse are seen to be significant and true." Discuss this statement.

* 6. 'To say that all *A* is *B*, is in fact merely to assert that the real world contains no objects that are *A's*, but that fail to be of the class *B*.' Discuss, explaining what kind of propositions appears to you to be most truly 'existential.' (I.C.S. 1902.)

* 7. Critically examine the view that particular propositions do, and that universal propositions do not imply the existence of their subjects.

* 8. Explain what is meant by the assertion that the ultimate subject in judgment is Reality as a whole. (G. *M.A.* 1905.)

Exercise VIII.

1. What are the Predicables ? Exemplify by reference to the word 'triangle.' (G. *M.A.* 1907.)

2. Set down with explanation the heads of predicables. (C. *Prev.* 1902.)

3. How must the belief or disbelief in the existence of real kinds in nature, affect the doctrine of the Predicables ? (O. *Mods.* 1906.)

4. 'The individual substance is the ultimate subject of all predication.' What bearing has this principle on the Porphyrian doctrine of the Predicables? Is the principle defensible in the light of such a proposition as, 'The person approaching is Callias'?

5. Give an account of Property and Accident. How far are they distinguishable? (O. *Mods.* 1907.)

6. Explain and comment on the distinction of Accidents into separable and inseparable. (O. *Mods.* 1908.)

7. Can Porphyry's fivefold division of the Predicables be shewn to be exhaustive?

*8. What do you understand by Moderate Realism? Distinguish it carefully from Platonic Realism on the one hand and Conceptualism on the other.

*9. Explain the nature of the Controversy between Nominalism and Realism. Upon what questions in Logic does the controversy bear, and how? (O. *Mods.* 1908.)

*10. '*Universalia ante rem*,' '*Universalia post rem*.' Explain and discuss. (O. *Mods.* 1905.)

Exercise IX.

1. Give some account of the Categories, and of their place in Logic. (O. *Mods.* 1906.)

2. Enumerate the ten Categories, and distinguish them carefully with examples. What do you suppose to have been the origin and purpose of the list? (O. *Mods.* 1907.)

3. Distinguish carefully between the Categories as a part of logical and as a part of metaphysical doctrine.

4. What do you understand by Predicamental lines? Discuss their connexion with the logical analysis of science.

5. Explain briefly how the doctrine of the Categories throws light on the question of the Import of Propositions.

*6. Contrast the Aristotelian Categories with any other system with which you may be acquainted. Shew why as a part of logical doctrine the Aristotelian system is to be preferred.

Exercise X.

1. Discuss the question whether *things*, *concepts*, or *names* are the object of Definition.

2. Indicate and exemplify the methods by which we form definitions. State the most important rules of definition. (R.U.I. *2nd Arts*, 1906.)

3. State the meaning of Definition, and carefully explain any technical terms that you use in your statement.

Comment critically with examples on (*a*) the distinction between nominal and real definition, (*b*) the distinction between the definable and the undefinable. (L. *Inter. Arts*, 1906.)

4. "The distinction between nominal and real definitions cannot

be maintained. No definition is ever intended to explain the nature of a thing." Can this position be sustained ?

(O. *Mods.* 1904.)

5. In an examination into the nature of definition, must we distinguish between ordinary definition and scientific definition ? Are Genetic definitions properly called definitions ?

(R.U.I. *2nd Arts, Hon.* 1905.)

6. (*a*) Discuss the connexion between the theory of definition and the theory of the predicables.

(*b*) Consider carefully the following definitions :—

(1) Light is a mode of motion.

(2) A contrapositive is the converse of the obverse.

(3) A woman is a creature who cannot reason and who pokes the fire from the top. (L. *Matr.* 1905.)

7. Give examples showing the importance of defining the technical terms of a science. Why is it in defining a species, the proximate *genus* and *differentia* should be given but no proprium or accidens ? (C. *Prev.* 1904.)

8. Explain the nature of Logical Division, and indicate carefully the relations between logical part and logical whole.

9. To what chief defects is Logical Division liable ? Can they be obviated by the process of Dichotomy ? (O. *Mods.* 1905.)

10. Distinguish Logical Division from division of other kinds. Comment on the division of man into 'rational' and 'animal'; of a house into stone, timber and mortar ; of Socrates into body and soul ; of the soul into intellect and will.

11. Explain the expressions :—*fundamentum divisionis*, cross-division, division by dichotomy, division *per se*, division *per accidens*.

12. Explain carefully what the Scholastic philosophers meant when they said that all men are such in virtue of a common 'humanity.'

Exercise XI.

1. Explain the meaning of the expression 'middle-term.' Does the middle-term of a syllogism always hold that relation to its extremes which the name implies ? Is the expression understood in the same sense by all logicians ?

2. Distinguish the necessity of the consequence from the necessity of the consequent. Does Logic in any department attempt to secure both ? (R.U.I. *2nd Arts, Hon.* 1905.)

3. Give the rules of quality and of distribution, which are necessary and sufficient for securing the validity of a syllogistic argument. Give an illustration in which one and only one of the rules is violated. (L. *B.Sc.* 1903.)

4. (*a*) How is it that *EIO* is always valid, and *IEO* is never

valid, if the difference between them is one of mere order of premisses ?

(*b*) Determine the rules of the third figure, and enumerate fully the characteristics of that figure. (R.U.I. *2nd Arts*, 1901.)

5. (*a*) Why does *p* never occur in the first syllable of any named mood ? Has it the same meaning in Darapti and in Bramantip ?

(*b*) Why is it that in all figures " the conclusion must be negative if the minor premiss be negative " ; and " the conclusion is not *A* if the major term be subject ? "

(R.U.I. *2nd Arts, Hon.* 1902.)

6. What is the special use of the second figure of the syllogism ? Construct an example of a real argument (not in letters) in one of the moods of this figure. Reduce Baroco both directly and indirectly. (R.U.I. *2nd Arts*, 1902).

7. Assuming the general rules of the syllogism, deduce that *OAO* is only possible in Fig. 3. (L. *Matr.* 1905.)

8. Prove that nothing can logically be inferred from two particular premisses. (L. *Matr.* 1904.)

9. Prove that if the middle-term of a syllogism be twice distributed, the conclusion must be particular. (L. *Inter. Arts*, 1905.)

10. Define ' illicit process,' and explain with illustrations the reasons which make it illicit.

Why can an *O* proposition occur as a major premiss only in Fig. 3, and as a minor only in Fig. 2 ? (R.U.I. *2nd Arts*, 1905.)

* 11. Show that all the moods of the first figure can be directly or indirectly reduced to Barbara. (R.U.I. *2nd Arts, Hon.* 1901.)

12. Resolve some short example of a geometrical demonstration [e.g. Euclid I. 6] into its constituent syllogisms, indicating major and minor premiss in each case, and showing what axioms or previous propositions are introduced and how. (L. *B.Sc.* 1903.)

Exercise XII.

1. State the *dictum de omni et nullo*, and prove from it the ' special rules of the first figure.' (L. *Inter. Arts*, 1906.)

2. What is the object of Reduction, and how far does it alter the character of an argument ? Illustrate your answer by constructing and reducing syllogisms in Cesare and Disamis.

(O. *Mods.* 1906.)

3. Critically consider various ways in which the ultimate principle of syllogistic inference has been formulated.

" In no case can a syllogism with two singular premisses be viewed as a genuine syllogistic or deductive inference." Examine this. (L. *B.Sc. Hon.* 1905.)

4. Reduce the following, or the arguments involved in them to syllogistic form, stating figure and mood if valid : and fallacy if invalid :—

(*a*) All living things are interesting, and some interesting

things are beautiful. Therefore some living things are beautiful.

(*b*) No guest is unwelcome. Hence since some guests have not been introduced, it follows that the welcome have not in all cases been introduced. (L. *Matr.* 1904.)

5. Express the following arguments in syllogistic form, and examine their validity from a logical point of view :—

(*a*) 'Freedom means power to do or to forbear from doing any particular act, upon preference : and since the will is merely the power of preference, the question whether the will is free is an unmeaning one : therefore the only proper question is whether a man (not his will) is free.'

(*b*) 'Now for the poet, he affirmeth nothing, and therefore never lieth. For as I take it, to lie is to affirm that to be true which is false : but the poet never affirmeth : therefore though he recount things not true, yet because he telleth them not for true, he lieth not.'

(R.U.I. *2nd Arts, Hon.* 1901.)

6. 'All reasoning is from universals to particulars.'

'All reasoning is from particulars to particulars.'

What is the bearing of these two theories on the doctrine of the syllogism ? (O. *Mods.* 1906.)

7. State in general and criticise Mill's theory of the syllogism. (T.C.D. *2nd year Hon.* 1904.)

8. 'The problem of inference is something of a paradox : for we have not got inference unless the conclusion is (*a*) in the premisses, (*b*) outside the premisses.' Examine this statement. (R.U.I. *B.A. Hon.* 1901.)

9. Critically examine the opinion that there are other forms of inference besides syllogistic reasoning. Notice any views on this point defended by recent writers.

10. On what grounds and with what justice has superiority been attributed to the first figure of the syllogism ? Should a fourth figure be recognized ? (O. *Mods.* 1904.)

Exercise XIII.

1. Give the chief forms of argument involving hypothetical or disjunctive propositions : and examine whether they are governed by the same principles as the categorical syllogism. (L. *Inter. Arts, Hon.* 1903.)

2. Discuss the question whether the reasoning of the hypothetico-categorical syllogism is mediate or immediate. (T.C.D. *2nd year Hon.* 1904.)

3. 'The theory that a hypothetical proposition cannot be satisfactorily reduced to categorical form, does not apply to the hypothetical syllogism. The latter may be expressed without

material alteration, *modus ponens* in Fig. 1, *modus tollens* in Fig. 2. Discuss and illustrate these statements.

(R.U.I. *2nd Arts, Hon.* 1905.)

4. What proof would you offer of the rules of the hypothetical syllogism ? Give concrete examples in illustration of the rules.

(L. *Inter. Arts*, 1906.)

5. Explain and discuss the following assertion :—'In a disjunctive proposition the positing of one assertion does not warrant the sublating of the other, though the sublating of one posits the other.' (R.U.I. *2nd Arts*, 1902.)

6. Given that "If a student is a man, he gains his degree either through satisfying the examiners or through payment of a fee," what conclusions can be drawn from the following :—

(*a*) X has neither satisfied the examiners nor paid his fees.

(*b*) X has both satisfied the examiners and paid his fees ?

(L. *Matr.* 1904.)

7. Explain : "A dilemma is a complex hypothetical argument which is often used fallaciously, because we often overlook one of the possible alternatives." Give a concrete example of the dilemma to illustrate your remarks. (R.U.I. *2nd Arts*, 1906.)

8. Discuss the logical value (*a*) of constructing, (*b*) of rebutting a dilemma. (O. *Mods.* 1905.)

9. What are the essential characteristics of the Dilemma ? How many different kinds of Dilemma are recognized ? Give an example (in symbols) of each.

Indicate the most common sources of fallacy in dilemmatic arguments, with illustrations. (L. *Inter. Arts*, 1906.)

Exercise XIV.

1. Analyse the conception of Cause, as it is used in inductive investigation. (G. *M.A.* 1906.)

2 Define precisely what Science understands by the Cause of an event, and illustrate your definition, by a typical example of causation as determined by experiment.

What is the relation of the scientific conception of Cause to the conception of Cause as the sum-total of the conditions on the occurrence of which the event depends ?

Can you account for the divergence between the scientific and the popular views as to the Cause ? (L. *Inter. Arts*, 1905.)

3. Describe carefully the inductive process, by which from particular facts we pass to the knowledge of a universal law. Consider to what extent and for what reasons the multiplication of instances is necessary to the formation of a valid induction.

4. Explain and illustrate the nature of Induction and its relation to Deduction. Illustrate the importance of both processes in science by examples taken from any of the Natural Sciences.

(O. *Mods.* 1907.)

5. State and examine the grounds on which the following statement has been made :—' No inference is possible unless a general proposition be somewhere in the background.' (G. *M.A.* 1905.)

6. Describe the Inductive Syllogism of Aristotle. Estimate the logical value and the practical utility of the argument : and state whether in your opinion it is rightly termed an ' inductive ' argument.

7. Examine the claims of a ' Perfect Induction ' to be regarded as an inductive process. (O. *Mods.* 1905.)

Exercise XV.

1. Consider carefully the relation of the principle of the Uniformity of Nature to the inductive process. Is there a parity between the function of the *dictum de omni* in deduction, and the principle of Uniformity in Induction ?

2. State precisely what you understand by " the uniformity of nature." Is it capable of proof, and if so, of what kind ? (G. *M.A.* 1906.)

3. Comment on the following :—" The contrary of every matter of fact is still possible, because it can never imply a contradiction. That the sun will not rise to-morrow is no less intelligible a proposition and implies no more contradiction, than the affirmation that it will rise " (*Hume*). (L. *Inter. Arts*, 1906.)

4. Is a plurality of causes possible ? (G. *M.A.* 1908.)

5. " *Cessante causa, cessat effectus.*" Explain.

6. Discuss Mill's view that the principle of the uniformity of Nature is arrived at by Induction. (O. *Mods.* 1906.)

7. Define the conception of Cause as used in Science. How far is the conception consistent with the admission (*a*) of plurality of causes, (*b*) of plurality of effects ? (L. *B.Sc. Hon.* 1903.)

* 8. What is meant by the assertion of certain logicians that " the Unity of Nature " is a more correct term than " the Uniformity of Nature " ? Critically examine the doctrine presupposed by the former of these expressions.

Exercise XVI.

* 1. Explain with examples the Aristotelian Enthymeme.

2. Define :—polysyllogism, epichirema, sorites ; and give a symbolical example of each. (T.C.D. *2nd year Hon.* 1904.)

3. State the rules of the Sorites, and prove them briefly. Examine the following :—' The final good of man is to be realized only in virtuous action. But action will not be right and virtuous, unless the will be also right : and the rightness of will depends on ordered habits of the soul, and that again springs from right general principles.' (R.U.I. *2nd Arts*, 1905.)

4. Explain and illustrate the nature of analogical reasoning. What are its dangers ? (G. *M.A.* 1907.)

5. Distinguish between Induction and Analogy. Is the analogical argument syllogistic ? (R.U.I. *2nd Arts*, 1906.)

6. "In Analogy we must weigh the points of agreement, and not merely count them." How far does Mill recognize this ? How is the precept to be carried out ? (O. *Mods.* 1906.)

Exercise XVII.

1. Explain and illustrate the following fallacies :—*Petitio principii, Ignoratio Elenchi, A dicto secundum quid ad dictum simpliciter* (and the converse), Composition, Division. (L. *Inter. Arts*, 1906.)

2. Illustrate from current controversies any of the commonly recognized fallacies. (O. *Mods.* 1904.)

3. Classify the fallacies which are incident to Formal Reasoning, and give one example under each of your divisions. (R.U.I. *2nd Arts*, 1902.)

4. Examine and name three of the following inferences :—

(*a*) Slang is metaphor : metaphor is poetry : therefore slang is poetry.

(*b*) Cleon is not Socrates : Socrates is a man : therefore Cleon is not a man.

(*c*) Aristotle is a word of four syllables : Aristotle is acute : therefore acute is a word of four syllables.

(*d*) No one is born a slave, because every one is born with all his original rights.

(*e*) No one can become a slave, because no one from being a person can become a thing. (R.U.I. *2nd Arts*, 1901.)

5. Under what conditions can we infer causal connexion between two phenomena on the ground of the observed frequency of their concurrence ? Illustrate the fallacies incident to such inference. (L. *B.Sc.* 1903.)

6. Consider the following :—

Counsel : There was no written agreement for the sale of this carpet.

Plaintiff : Well, you don't have a written agreement when you buy a loaf.

Counsel : You don't cover a floor with a loaf.

Plaintiff : Neither do you eat a carpet. (O. *Mods.* 1906.)

7. Explain the distinction between logical and material fallacies, illustrating by some of the most conspicuous examples of each kind. (O. *Mods.* 1905.)

8. Define the term Fallacy as employed in Logic, and distinguish a fallacy from a false belief.

Test the following arguments, and name any fallacies they may contain :—

(*a*) Ill-managed business is unprofitable. Railways are never ill-managed. Therefore all railways are profitable.

(*b*) A vacuum is impossible : for if there is nothing between objects, they must touch.

(*c*) The governor of a country ought not to be blamed for using his influence to further his religious views, for every man has a right to inculcate his opinions.

(L. *Inter. Arts*, 1905.)

9. Write out the principal heads of one or more celebrated classifications of Fallacies, explaining the terms employed.

(R.U.I. *2nd Arts*, 1896.)

10. Logic has been described as " a machine for combating fallacy." How far does this description seem to you appropriate ?

(G. *M.A.* 1905.)

Exercise XVIII.

1. Consider critically the Aristotelian division of Philosophy into Metaphysics, Mathematics, Physics.

2. Discuss the relation between Logic and Metaphysics, giving historical references. (I.C.S. 1901.)

3. Discuss the position of Logic among the Sciences. What special view on this subject is implied by the use of the word *Organon* as a name for Aristotle's logical treatises ?

4. " Arithmetic and its fundamental principles are independent of our experiences and the order of the world." Discuss this statement. (L. *B.A. Hon.* 1905.)

5. " Logic aims at being not the Physics but the Ethics of Thought " (*Sigwart*). Discuss this statement.

6. '*Quidquid existit est singulare.*' '*Scientia est de universalibus.*' Discuss these two Scholastic principles. How far have they a claim to absolute validity ? Notice any systems which seem to you to have violated one or other of them.

7. What important features of Logic as it is at present taught, do we owe to the influence of Bacon ? How far is the novel element a legitimate development of the principles of the science ?

Exercise XIX.

1. When has Observation more advantages than Experiment ? and why is Experiment usually more advantageous than Observation ? (R.U.I. *2nd Arts, Hon.* 1902.)

2. " Observation and Experiment (supposing no aid from deduction) can ascertain sequences and coexistences, but cannot prove causation." Examine. (O. *Mods.* 1904.)

3. What do you hold to be the essentials of a truly scientific experiment ? (L. *Matr.* 1904.)

4. How far does Experiment differ from Observation ? Is the difference fundamental ? Show the dependence of Observation

on previous knowledge, and in what sense it always involves inference. (L. *Inter. Arts*, 1906.)

5. What is the logical character of the appeal to the testimony of the senses? Does this testimony really bear witness to the motion of the sun relatively to the earth? (L. *Inter. Arts*, 1905.)

Exercise XX.

1. Show exactly how Mill's Method of Agreement differs from Simple Enumeration.

On what presuppositions is the former method applicable? What are its defects and its utility in scientific discovery?

(L. *Inter. Arts*, 1906.)

2. Explain the Method of Concomitant Variations. Give examples of cases where its application is extremely profitable, and point out the limitations attending its use.

(L. *Inter. Arts*, 1906.)

3. Analyse and describe in logical terms the method by which any important discovery of recent years was made.

(O. *Mods.* 1906.)

4. Distinguish carefully between the Method of Difference and the Method of Residues. (O. *Mods.* 1905.)

5. How far does the validity of any of the Inductive Methods depend on the possibility of expressing Cause and Effect quantitively? (O. *Mods.* 1906.)

6. Examine and explain Mill's view that Plurality of Causes renders the Method of Agreement uncertain. (O. *Mods.* 1904.)

7. Explain why the Method of Agreement requires many instances, while the Method of Difference is satisfied with one precise experiment. Why is the Method of Agreement of little value, as compared with the Method of Difference?

(R.U.I. *2nd Arts*, 1905.)

8. How would Inductive Logic proceed to investigate the influence upon character of any one of the following:—(*a*) climate, (*b*) athletics, (*c*) an artistic profession? (O. *Mods.* 1905.)

9. Can the Method of Residues be fairly considered inductive in character? (O. *Mods.* 1904.)

10. Explain the Joint Method of Agreement and Difference, and give a concrete example. (R.U.I. *2nd Arts*, 1906.)

11. "The Plurality of Causes must be considered as something which actually occurs in Nature, and which as often as it occurs, ought to be capable of being discovered by the methods of induction."

Explain and discuss this statement. (O. *Mods.* 1908.)

12. What does Mill mean by Cause? How far are his methods adequate for the discovery of causes in his own sense of the word?

(O. *Mods.* 1907.)

Exercise XXI.

1. What is the object of Explanation ? Describe and illustrate its principal forms. (O. *Mods.* 1907.)

2. " The object of Science is *explanation.*"

" Science never *explains* : she only reduces complex events to simple ones of the same kind, as when she deals with certain phenomena of magnetism by supposing every ultimate unit of the substance to act as if it were a magnet."

Consider these statements. (L. *Inter. Arts*, 1905.)

3. Indicate carefully the meaning of the term ' law ' as employed in natural science. Distinguish between empirical laws, laws of nature, and ultimate laws.

' A truly universal law is not a demonstrable truth.' Discuss this. (L. *Inter. Arts*, 1906.)

4. Distinguish the different processes included in or subservient to Induction, and discuss the relations of Induction to Description, Hypothesis, and Explanation. (C. *Mor. Sc. tripos.* 1905.)

5. Bring out the full meaning of the dictum that ' Induction is the Inverse of Deduction.'

Comment on the following :—' All knowledge is founded ultimately on Induction, all reasoning on Deduction.'

(L. *B.Sc.* 1903.)

6. ' The secondary application of the Deductive Method is not to prove laws of phenomena, but to explain them.' Enumerate the different kinds of explanation recognized by Mill, and show in what way each is deductive. (O. *Mods.* 1906.)

7. Comment on the phrase *Hypotheses non fingo* as employed by Newton : and explain the expression ***vera causa.***

Exercise XXII.

1. What is an hypothesis ? How is an hypothesis verified ? Which of the empirical methods is fruitful in hypotheses ? Which might be fruitfully used for verification of an hypothesis ?

(R.U.I. *2nd Arts*, 1901.)

2. Illustrate the use of hypothesis in scientific explanation.

(C. *Prev.* 1904.)

3. Examine and illustrate the characteristics of Hypothesis as a conscious process of scientific investigation. Show that a rejected hypothesis need not necessarily have been fruitless.

(L. *Inter. Arts*, 1906.)

4. Explain and comment on the expressions :—Working hypothesis, Hypothesis of Cause, Hypothesis of Law.

5. " Men are now cured of their passion for Hypotheses " (*Hume*). " Hypothesis is induction looked at from the inside." What different views of the nature and function of hypothesis are represented by these statements ? (I.C.S. 1903.)

Exercise XXIII.

1. Explain the importance of Measurement in inductive investigation. Notice the circumstances which principally contribute to render Measurement a task of difficulty.

2. Carefully consider the statement that accurate quantitative determination is the ultimate stage in the establishment of every scientific theory.

3. "There is no such thing as Chance. To attribute an event to Chance is merely a way of saying that we are ignorant of its cause."

"There is no such thing as Chance. What we attribute to Chance, we should rightly attribute to Providence."

Discuss these statements. Does either of them contain an adequate account of Chance?

4. Determine the province of the theory of Probability in induction. If it be admitted that no result of inductive reasoning is absolutely free from doubt, does it follow that induction should be based on the theory of Probability? (L. *Inter. Arts*, 1905.)

5. Explain what is meant by the doctrine of philosophical probability. Illustrate by examples.

Exercise XXIV.

1. Discuss the value of Natural Classification as a scientific method.

2. Give the rules of Classification: and notice the chief difficulties which attend the task of establishing a valid Classification.

3. What do you hold to be the relation between Logical Division and Classification?

4. Explain and illustrate the distinction between Natural and Artificial Classifications, noticing the purpose and characteristics of each. What objections have been felt by some in admitting this distinction?

* 5. What is meant by Classification by Type, and Classification by Series?

Exercise XXV.

1. Explain the expressions:—Method of Discovery, Method of Instruction. Illustrate by examples the practical importance of this distinction.

2. In what senses have the terms Analysis and Synthesis been employed by writers on Logic? Is there anything common to these senses on one hand, and the signification which the words possess in the physical sciences on the other?

3. "The human intellect was well compared by the Schoolmen to the eye of the owl, which in the dusk can see, but is blinded by the brighter light of day. Not without reason did they

distinguish between those things which are better known to Nature, and those which are better known to us." Discuss.

4. To what extent is it possible to prescribe rules for the methodic conduct of our thought in the pursuit of truth? Examine critically any such scheme of rules with which you may be familiar.

5. How far can it be said that the attainment of truth in philosophy, depends on the employment of an accurate philosophic terminology?

*6. Give Descartes' Rules of Method. Show that they no longer can be regarded as possessing any philosophical value.

*7. Briefly explain Leibniz's theory of Method. What is to be thought regarding his views on Identical propositions?

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